The Subarea Focusing Concept for Trip Distribution in the Puget Sound Area

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This paper explores one method of reducing the computational costs associated with urban travel demand modeling. The technique investigated is a data base approach called subarea focusing. The distinction between this technique and other data simplification methods is in the detailed analysis of only a portion of a study area and the simultaneous presence of several levels of areal detail in the data base. A computerized technique for subarea focusing was developed and used for a sample trip distribution application to data from the Puget Sound region. The procedure was not unduly expensive in either manpower or computer time. When aggregate data sets were used to obtain travel demand predictions, substantial savings in computer time were realized. The error analyses indicated some sacrifice in accuracy, but not a serious sacrifice. The results appear to justify continued refinement of the aggregation procedure and investigation of the effects of subarea focusing on other demand models.

Conventional travel demand forecasting procedures have received substantial criticism from professionals and policymakers since the late 1960s. A repeated complaint [e.g., Bouchard (1)] is that conventional forecasting procedures are too time-consuming and too expensive. In response to these and other criticisms, new methods are being discussed, developed, and tested.

One method of reducing the excessive costs of computer modeling is the simplification of model inputs. Dial and others (2) have proposed three types of data base simplification: regionwide abstraction, subarea focusing, and subarea windowing. Figure 1 illustrates these three concepts. Regionwide abstraction is the uniform aggregation of network and zone information across a study region to create a district system (areal scheme b in Figure 1). Subarea focusing is the extraction of a subarea of interest (called the window) from the original data base and the abstraction of zone and network information outside the window (areal scheme c in Figure 1): All data within the window boundary are kept at a zonal level of detail. Subarea windowing also extracts a subarea of interest from the original data base but then collapses trip ends outside the window onto the window boundary much like the treatment of external stations in the original network (areal scheme d in Figure 1).

For analyses in which only a small section of an entire region is of interest, subarea focusing appears to be the best method for reducing data requirements without adverse effects on investigations inside the windowed section. It is best suited to local planning, corridor analyses, and major updates of existing regional plans when only a small number of subareas are under investigation.

Although subarea focusing has played a role in transportation planning studies for many years (3, 4), it has been formally recognized only recently with the advent of sketch planning techniques. Thus, while the concept has been applied informally for some time, there is little documentation on how this was done, and no studies have been initiated on the effects of applying it. Thus, the prime objective of this investigation was the development and automation of an aggregation methodology for subarea focusing. A second objective was the demonstration of the effects of such simplification on predicted trip flows to, from, and within a subarea of interest for a realistically sized study region.

A few recent studies have considered means of alleviating many of the time and money problems associated with conventional modeling techniques. Peat, Marwick, Mitchell and Company (5) have proposed new simulation models, network and zone aggregation techniques, and an interactive computer environment in an early but detailed look at sketch planning techniques. When operational these concepts were to have been added to the UMTA Transportation Planning System, but they have since been superseded by others (2). These techniques included an integrated multimodal demand model, a time-sharing computer environment, and the three types of data simplification described above. Mann (6), on the other hand, has developed a composite model using simplified versions of existing models with a district-level data base that achieves significant savings in computer time. The most unconventional sketch planning technique for travel demand forecasting is that of Schleifer, Zimmerman, and Gendell (7), which does not require extensive network coding since arterials are considered ubiquitous and areal data are aggregated to the community level. The model is noniterative and requires only trip end data by mode for each community. Transit estimation is presently done manually.

Two other studies were of interest to this project. One aided in the study design; the other gave some insights about the effects of zonal aggregation.

Bovy and Jansen (8) define the problem and present an experimental design for investigating the effects of spatial abstraction in transportation planning. They propose investigating the effects of abstraction on travel predictions and costs for three types of trip assignment models, three degrees of network abstraction, and three degrees of zonal abstraction. The models investigated are conventional in nature and include all-or-nothing, capacity restraint, and multipath assignment techniques. Although not clearly stated, the data simplifications seem to be of the regionwide abstraction type.

Wilbur Smith and Associates (9) studied the effects of zonal data simplification on trip distribution and assignment models. The areal simplifications were of a regionwide abstraction type, manually created, and spanned a range of total centroids from 630 to 73. Three trip purposes in the trip distribution stage, two trip assignment models, and six different sized areal systems were analyzed for both trip assignment and trip distribution. It was concluded that major reductions in the number of centroids on a regionwide basis could be achieved before errors due to aggregation became larger than errors inherent in the trip assignment process.

PROJECT DEFINITION

Study Scope

The salient features of the subarea focusing study are presented here. A more detailed description of the experimental design, aggregation procedure, and error results is given in Miller (10).

In applying the subarea focusing concept the data simplification was limited to zonal information. Network modifications were avoided for three reasons: (a) Zonal aggregation appeared far easier to achieve, (b) although a network aggregation technique has been developed (11) it has not been successfully automated for large networks, and (c) significant regionwide reductions in areal units have been achieved without incurring unreasonable trip prediction errors (9).

In the initial demonstration of the subarea focusing procedure, one design variable, the degree of aggregation, was varied while others such as window location and size were held constant. [In an associated project (12) the sensitivity analysis is extended by examining two additional window locations for approximately the same degrees of aggregation. Both studies limit their investigations to trip distribution predictions using the gravity model.]

The study area was the three-county mainland portion of the Puget Sound region. The data base for this area consists of a base 635-zone system containing 597 traffic analysis zones and 38 external stations, a network with 9495 one-way links and 3186 nodes, projected 1990 home-based person work-trip productions and attractions, and projected impedance factors for work trips in 1990.

The Seattle CBD was the subarea of interest. There are 42 traffic analysis zones within this window. Aggregate districts were created from one or more original zones outside the window and approximately aggregated to the same level of detail regardless of distance from the window boundary. The degree of aggregation was measured by the percentage reduction in the number of centroids outside the window. Three degrees of aggregation, 55, 79, and 90 percent reductions in centroids outside the window, were analyzed. The total number of centroids present in each system was 302, 165, and 103 respectively.

A forecast trip matrix was obtained for the 635-zone system by applying the gravity model to the base data. This forecast trip matrix was considered correct, and any deviation from it was attributed to data aggregation. A computerized aggregation procedure was developed to generate aggregate district data from the base zone system. An aggregate trip matrix was obtained for each degree of aggregation by applying the gravity model to the district-level data. The fundamental comparison in the error analysis was between the forecast trip matrix (compressed into appropriate district-level dimensions) and each of the aggregate trip matrices.

Spatial Aggregation Procedure

There are several assumptions associated with the aggregation procedure developed. They are: that zonal trip and network data are available for a particular region and that some subarea of the study region is of special interest; that the network representing the transportation system need not be modified and that the zones cannot be split during aggregation; and that the conventional demand prediction methods are sufficiently accurate for subarea analysis.

Several means of achieving zonal aggregation were considered and three criteria used to select the procedure that was finally implemented. These were: simplicity, transferability, and reasonable accuracy of trip prediction with respect to the window. The three major steps of the aggregation procedure selected are given below:

Step 1. Construct aggregate district boundaries,
Step 2. Establish district centroids, and
Step 3. Assign terminal and intrazonal travel time for districts.

In step 1, outside zones were aggregated into districts based on a minimum travel time difference criterion. That is, zones that displayed the smallest differences in travel time to a common node on the window boundary were combined first. The process was then continued until the desired number of districts was reached.

In step 2, a district centroid was chosen from the set of zone centroids originally contained in the district. The chosen centroid was centrally located in the district and ranged nearest the median value with regard to travel time to the window boundary.

In step 3, a simple unweighted average approach was used to calculate terminal and intradistrict times for each district.

With the exception of minor hand adjustments, the procedure was fully automated. An example of an areal system obtained by this method is shown in Figure 2. [A more detailed discussion of the process is given by Miller (10).]

PRESENTATION OF RESULTS

Time Savings and Expenditures

The reduction in computer time for trip distribution was calculated for each level of aggregation. This involved three tasks: (a) the creation of a minimum path travel time matrix that included terminal and intrazonal travel times (SKIMTREE PROGRAM), (b) the production of a line printer list of a matrix (T PRINT PROGRAM), and (c) the production of a trip matrix by the gravity model (WILSON PROGRAM). The computer execution times, the gross reductions in computer times, and the percentage reductions for the three programs are shown in Table 1.

The preparation of the data inputs for the aggregation procedure required 2 person-days. Additional costs of

10, 7.5, and 4 person-days were incurred to obtain the district-level data sets for the 55, 79, and 90 percent degrees of aggregation respectively. These manpower requirements were for the existing aggregation procedure, which should be considered a prototype. Significant savings may be realized by further automation. (The manpower requirements included learning time. Thus, it should decrease as more experience is gained.)

The computational costs of the aggregation procedure in terms of computer execution time were minimal and relatively inelastic with respect to aggregation level. The total costs for a single application of the algorithm for either 55, 79, or 90 percent degrees of aggregation were 81, 74, or 73 s respectively. When all three levels of aggregation were desired, the total CPU time was 164 s.

Errors Attributable to Degree of Aggregation

Analysis of the differences between the forecast and aggregate trip matrices was limited to the calculation of seven error measures. Three of these were regional measures. The remaining four measured matrix errors on an interchange-by-interchange basis. In calculating these four measures it was necessary to compress the forecast trip matrix to a size that comformed with each of the aggregate trip matrices. Thus, compression of the single 635-zone trip matrix created a distinct compressed trip matrix for each degree of aggregation investigated. (These will be termed compressed trip matrices.) Analysis of pairs of equally dimensioned compressed and aggregate trip matrices was performed for all matrix cells.

In addition to the comparisons of full matrix pairs, matrix quadrants were compared to isolate errors for trips to, from, and within the subarea of interest. Specifically, the three groups of interchanges of interest were: trips produced at centroids outside the window and attracted to centroids inside the window; trips produced at centroids inside the window and attracted to centroids outside the window; and trips made completely within the window. To facilitate this analysis the aggregate and compressed trip matrices were arranged so that the lowest numbered centroids were those contained within the window. The specially arranged matrices were separated into four quadrants based on the types of trip interchanges as illustrated in Figure 3.

Matrix Error Measures

Pairs of trip matrices of conformable sizes were compared on an interchange-by-interchange basis to provide a microscopic analysis of the differences between them. The error measures, absolute percent deviation and absolute deviation, are defined in equations 1 and 2 below.

$$\gamma_{ij} = |C_{ij} - A_{ij}|/C_{ij} \tag{1}$$

subject to

$C_{ij} > 0$ trips and
$C_{ij} > 3$ trips, except when $|C_{ij} - A_{ij}| > 10$ trips;

and

$$\beta_{ij} = |C_{ij} - A_{ij}| \tag{2}$$

where

C = compressed 635-zone trip matrix (dimensions $r \times r$),
A = aggregate trip matrix (dimensions $r \times r$),
C_{ij} = trips predicted from i to j for matrix C,
A_{ij} = trips predicted from i to j for matrix A,
r = number of total centroids (i.e., 302, 165, or 103),
γ_{ij} = absolute percent error for interchange i, j, and
β_{ij} = absolute deviation for interchange i, j.

The absolute percent error measure was constrained to consider only compressed trip interchanges with the number of trips greater than zero since division by zero is undefined. Compressed trip interchanges having magnitudes of less than three trips were not analyzed unless the absolute deviation was greater than ten trips because many small trips created large absolute percent errors (e.g., 2 trips ± 1 trip gives a 50 percent error). (These large percent errors were insignificant because of the small number of trips involved.)

The mean and standard deviations for the two error measures for each level of aggregation are given in Table 2. The computer program that calculates these measures also stratifies the error by row and column for the full matrix and for particular matrix quadrants. These results are not included here but are available for future, more detailed analyses and refinements. Such analyses might isolate centroids with unusually high error magnitudes and study their properties for further refinement of the aggregation procedure.

The maximum error for a single trip interchange in the full matrix and in each matrix quadrant was determined for the three degrees of aggregation. These errors are shown in Table 3. In all instances the maximum error found in quadrant IV is also the maximum error found in the entire matrix. Since the aggregation criteria were purposely biased against quadrant IV trips, this is an expected result. (Only a subset of quadrant IV trips, namely through trips, are of interest to the subarea planner. These are discussed later.) The number of trips listed for each error measure is the number of trips for the associated compressed trip interchange. For example, the full 302-by-302 matrix had a maximum percent interchange deviation of 304 based on six trips. [These percentages were calculated before trips were rounded off. In terms of whole trips this was (24 - 6)/6 = 300.]

Error Length Distributions

The error length distributions indicate the bias of deviations for trips of different lengths. Two error measures, the mean absolute percent deviation per time interval and the mean absolute deviation per time interval, were used to calculate these distributions. They are given by

$$\lambda_M = \frac{\sum_{i,j \ni t_{ij} \in M} [(|C_{ij} - A_{ij}|)/C_{ij}]}{N} \tag{3}$$

and

$$\rho_M = \frac{\sum_{i,j \ni t_{ij} \in M} |C_{ij} - A_{ij}|}{N^2} \tag{4}$$

where

C_{ij} = trips predicted from i to j for matrix C,
A_{ij} = trips predicted from i to j for matrix A,
M = time interval,
N = number of trip interchanges in time interval M,
t_{ij} = travel time from i to j in the aggregate system,
λ_M = mean absolute percent error for time interval M, and
ρ_M = mean absolute deviation for time interval M.

Figure 1. Types of data base simplifications.

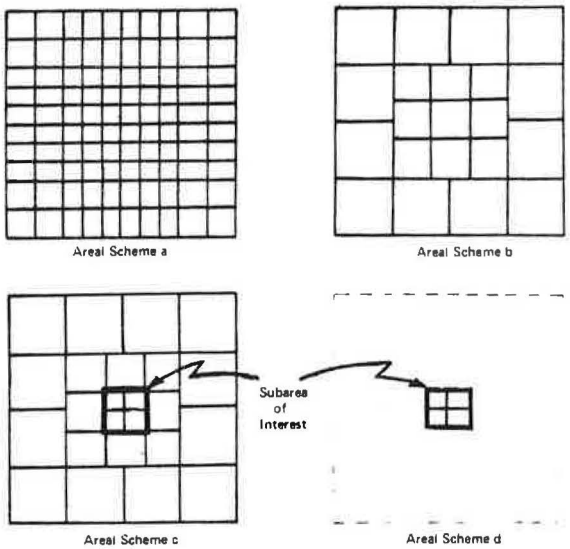


Figure 2. Aggregate traffic analysis districts.

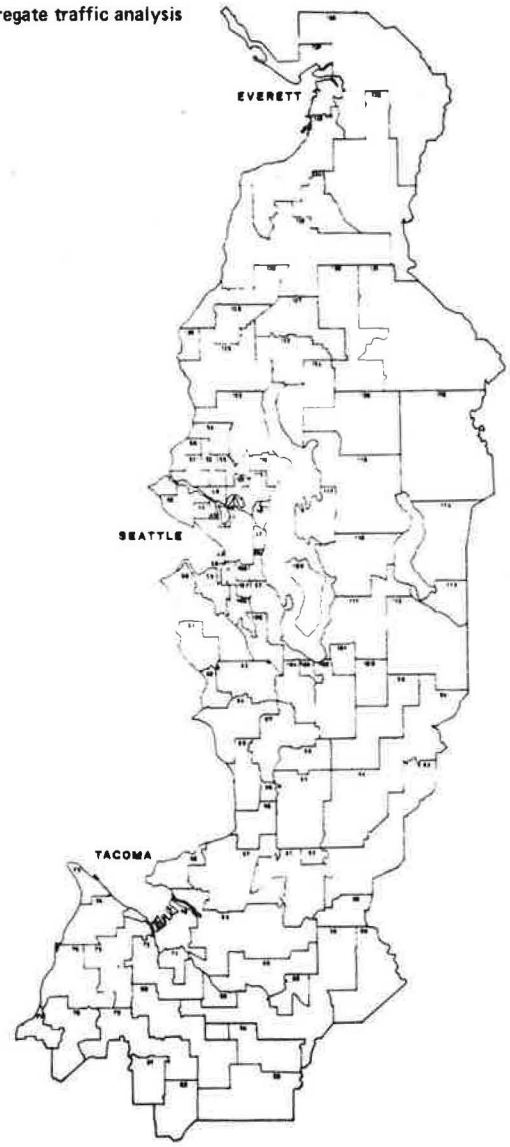


Table 1. Comparison of computer execution times for trip distribution programs.

Degree of Aggregation (%)	No. of Centroids	Skimtree[a] Program	Wilson[b] Program	Skimtree and Wilson Programs	Tprint[c] Program
Computer Execution Times					
0	635	822	360	1182	218
55	302	370	55	425	49
79	165	213	19	232	15
90	103	137	9	146	6
Gross Reduction in Computer Time[d]					
55	302	452	305	757	169
79	165	609	341	950	203
90	103	685	351	1036	212
Percentage Reduction in Computer Time[d]					
55	302	55	85	64	77
79	165	74	95	80	93
90	103	83	98	88	97

Note: All time is measured in CPU sec for CDC 6400. These may be converted to dollar terms by the following conversion unit: 0.0583 (dollars/CPU sec).

[a] See (13).
[b] Doubly constrained gravity model carried through four interactions of balancing procedure.
[c] Print out program that prints 50 × 28 entries per page.
[d] Based on 635 zone system execution time.

Figure 3. Quadrant definition.

Trips	To Centroids Inside Window	To Centroids Outside Window
From Centroids Inside Window	I Totally Within	II From Inside to Outside
From Centroids Outside Window	III From Outside to Inside	IV Completely Outside

Table 2. Trip matrix error summary.

No. of Centroids	Mode of Analysis	Absolute Percent Error: Mean	Absolute Percent Error: Standard Deviation	Absolute Deviation: Mean	Absolute Deviation: Standard Deviation
302	Full matrix	9.93	11.57	1.94	20.61
	Quadrant I	1.02	0.70	0.80	0.38
	Quadrant II	6.26	6.47	0.15	1.12
	Quadrant III	5.82	6.24	1.00	3.52
	Quadrant IV	11.77	12.66	2.43	23.87
165	Full matrix	12.45	18.17	7.68	60.88
	Quadrant I	1.63	1.01	0.28	0.63
	Quadrant II	9.00	7.84	0.51	2.70
	Quadrant III	8.73	15.16	2.35	5.29
	Quadrant IV	15.77	20.28	12.81	81.22
103	Full matrix	11.84	15.63	21.75	266.09
	Quadrant I	1.90	1.09	0.31	0.60
	Quadrant II	11.91	11.20	1.46	6.77
	Quadrant III	8.95	11.17	5.27	10.44
	Quadrant IV	18.02	19.71	57.23	447.01

Table 3. Maximum trip matrix error magnitudes.

No. of Centroids	Mode of Analysis	Absolute Percent Error: Maximum Error	Absolute Percent Error: Number of Trips	Absolute Deviation: Maximum Error	Absolute Deviation: Number of Trips
302	Full matrix	304	6	2 836	3 573
	Quadrant I	4	11	4	216
	Quadrant II	37	25	57	188
	Quadrant III	57	6	96	553
	Quadrant IV	304	6	2 836	3 573
165	Full matrix	704	3	2 976	1 965
	Quadrant I	5	7	10	288
	Quadrant II	41	5	106	326
	Quadrant III	191	10	76	43
	Quadrant IV	704	3	2 976	1 965
103	Full matrix	214	317	15 195	29 485
	Quadrant I	5	4	7	275
	Quadrant II	62	3	124	607
	Quadrant III	101	11	146	821
	Quadrant IV	214	317	15 195	20 485

Figure 4. Error length distribution results for 302-zone system: Seattle CBD window.

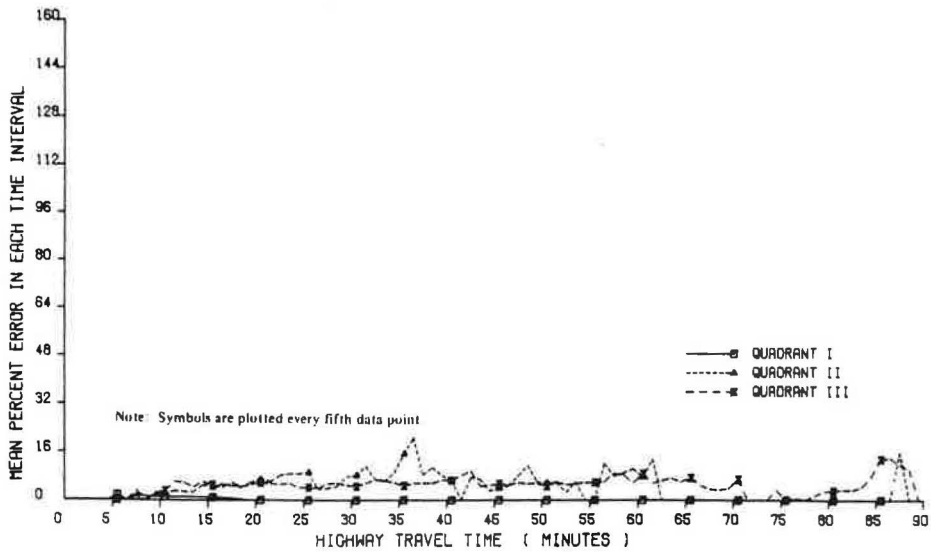


Figure 5. 1990 home-based person work-trip distribution gravity model output: Seattle CBD window.

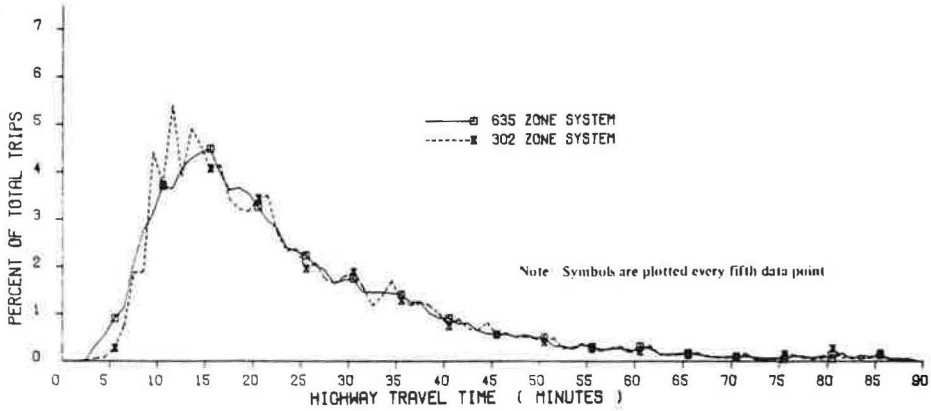
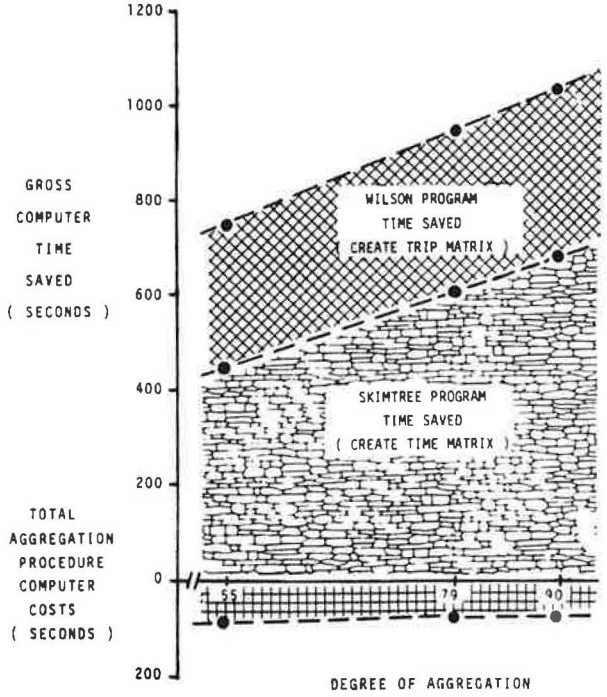


Figure 6. Comparison of gross savings in computer time with total computer time required by the aggregation procedure.



The error length distributions were calculated for the full matrix and all four quadrants for each degree of aggregation. An example of the mean percent error distribution is illustrated in Figure 4. In all instances, large percent errors were associated with small numbers of trips when compared with the absolute deviation distributions.

Trip Length Distributions

Trip length distributions were calculated for the forecast matrix and for each aggregate matrix. Figure 5 illustrates a sample comparison of trip length distributions for the 635 and 302-centroid systems. Since quadrant IV errors are of less relative importance than errors in the other three quadrants the trip length distribution for each quadrant was also calculated. This permitted the isolation of the types of trip interchanges that caused the largest deviations in the trip length distribution curve.

Regional Trip Length Measures

The final measures calculated were the mean and standard deviations of the trip length and person-kilometers traveled in the study region. The mean trip length for the 635-zone base system was 23.98 min with a standard deviation of 14.66 min. The mean trip lengths for the 55, 79, and 90 percent aggregation levels were 24.09, 23.71, and 25.01 min respectively, with corresponding standard deviations of 14.61, 13.84, and 13.59. The

total person-hours of travel was 507 790 for the base system and 510 202, 502 193, and 529 598 for the respective aggregation levels.

INTERPRETATION OF RESULTS

The significant computer time savings achieved by subarea focusing were shown in Table 1. The data indicate that it is possible to perform six 103-centroid, four 165-centroid, or two 302-centroid skimtree runs in approximately the same computer time as required for one 635-centroid run. Similarly, either forty 103-centroid, nineteen 165-centroid, or six 302-centroid gravity model runs can be made for approximately the same cost as one 635-centroid run. Therefore, more alternatives can be examined by subarea focusing than by conventional methods for the same costs, at least with regard to computer time.

The costs of preparing the three degrees of aggregate district data appear to be modest in terms of both manpower and computer time. The total manpower expended in the creation of the aggregate district data sets ranged from approximately 6 to 12 person-days. A substantial portion of the manual tasks could be automated in further refinements to the aggregation procedure. The total computer time consumed in preparing the aggregate district data sets was small relative to the computer time saved during trip prediction. This is shown graphically in Figure 6. For example, it required 81 s to run the aggregation programs for the 302-centroid system, but the computer time saved in skimtree and gravity model runs for the same system was 757 s. (TPRINT was not included in this calculation since it was unlikely that a full trip matrix would be printed for each run.)

The loss of accuracy between each of the three aggregate trip matrices and the 635-zone trip matrix appears acceptable. The average percent errors for the quadrants of special interest (e.g., I, II, and III) for the 302, 165, and 103-centroid systems were 6, 9, and 12 respectively. These error magnitudes are within the acceptable range (9) and agree with other results (7). Similarly, the largest average absolute deviations among the key quadrants are one, two, and five trips respectively. The results for trip interchanges with the worst error magnitudes indicate that large percent errors exist but are associated with small numbers of trips. The largest percent error found among the three primary quadrants for all three degrees of aggregation is 190 based on 10 trips in the compressed trip matrix [i.e., $(10 - 29)/10 = 190$]. The largest absolute deviation found for these quadrants is 146 trips based on 675 trips (i.e., $|675 - 821| = 146$ trips).

The results of the error length distribution analyses do not indicate any systematic bias with regard to travel time, and the magnitude of errors in these results is not serious. [Serious error is defined here as any error over approximately 15 to 20 percent (7, 9).] Only a single 1-min time interval (70 to 71 min, quadrant II, 103-centroid system) showed a large percent error, but, since this time interval contained only five trips, the total error was insignificant. It appears, therefore, that a balance has been achieved for quadrants I, II, and III and that the simple choice of unweighted averages for calculating district-level terminal and intrazonal times is not overly damaging. Of course, more complicated weighted average computations could lead to incremental improvements.

The results indicate that the trip length curve becomes less dispersed about the mean as the degree of aggregation increases. This is to be expected since fewer trips of very short or very long duration will occur as the number of centroids is reduced. This shift of short and long trips occurs at the 103-centroid system. The large peaks in intermediate length time intervals appear to offset this shift so that average trip length varies only slightly.

The mean trip length and the person-hours of travel for the 302 and 165-centroid systems closely correspond to those measured for the 635-zone system. Even at the highest degree of aggregation (103 centroids) both the mean trip length and person-hours of travel are only 4 percent in error. Since trip distribution models are usually calibrated within a 5 percent error of the origin-destination matrix, these errors are quite small.

CONCLUSIONS

The subarea focusing concept as applied to trip distribution appears to be both economical and reasonably accurate:

1. The computer time cost of the aggregation procedure is very low in comparison to the computer time savings realized for trip prediction.
2. The procedure is not unduly expensive in terms of the manpower required to develop an aggregate data set.
3. Substantially more alternatives may be examined by subarea focusing than by conventional methods for the same cost.
4. At very high degrees of aggregation subarea focusing may be suitable for preliminary screening of subarea alternatives (i.e., sketch planning) even with conventional models.
5. Data base simplification can be limited to zonal information and still achieve significant savings in computer time.
6. Assuming subarea focusing is applied, the additional savings in computer time that might be realized by network aggregation appear to be small.

The last conclusion, the potential of network aggregation given subarea focusing, requires more substantiation. Both the gravity model and the printout process will not be affected by network aggregation. Thus, the only means of varying computer time in the trip distribution phase involves the minimum travel time path program. Consider the example of the highest degree of zonal aggregation. If it is assumed that the network aggregation is 80 percent successful in reducing computer time (the percent reduction realized for zonal aggregation), there will be a saving of approximately 134 CPU s. This saving will, of course, be diminished by the costs of applying the network aggregation technique and the additional uncertainty brought into the demand predictions. But even ignoring these costs, the results still compare poorly with the savings attributable to zonal aggregation alone. If the costs of creating the aggregate district data are included, the computer time saving for one run is 612 CPU s. Since subarea focusing by definition requires zonal aggregation, additional savings due to network aggregation do not appear to merit the probable added costs and uncertainties in the trip predictions.

The error investigation was initially limited to one urban region (the Puget Sound region), one particular window type and location (the centrally located Seattle CBD), one window size (42 zones), three degrees of aggregation (55, 79, and 90 percent reduction in the number of centroids outside the window), and one travel demand model (gravity model). Thus, only preliminary conclusions about accuracy can be made. The initial results indicate that subarea focusing is sufficiently accurate for conventional planning at the first two aggregation levels and for sketch planning at the highest aggrega-

tion level. Subsequent analyses of additional windows and through-trips (12) support these preliminary conclusions.

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