Unsaturated Transient and Steady-State Flow of Moisture in Subgrade Soil

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A general procedure for determining the soil-water properties necessary for predicting moisture movement in subgrade soils is described. A gamma-ray transmission method was used for the nondestructive measurement of the water content and a tensiometer-pressure transducer arrangement to measure the soil-water pressure (suction); the unsaturated hydraulic conductivity, diffusivity, and soil-water characteristic functions were evaluated for AASHO A-4 subgrade soil. The soil was compacted in a column at uniform density and moisture content and tested under isothermal conditions. A water table was established at the bottom of the soil column and the transport of water through the subgrade soil studied. The mathematical procedures for determining the soil-water properties from the laboratory data are described, and the use of the experimentally determined hydraulic conductivity, soil-water diffusivity, and soil-water characteristic functions in the prediction of moisture changes in an AASHO A-4 soil is demonstrated.

In 1973 the Federal Highway Administration conducted five workshops to study the effects of moisture on pavement systems. Also in 1973, a research group sponsored by the Organization for Economic Cooperation and Development discussed the importance of predicting and controlling the effects of moisture in pavement systems and published a report (15) that recommended that the procedures shown in Figures 1 and 2 be used to develop research for predicting and controlling moisture effects in pavement systems.

The properties of soil-water systems must be known in order to predict and control moisture content and movement in pavement systems. Extensive studies of the resilient behavior of fine-grained soils, conducted by Robnett and Thompson (21), have shown the detrimental effects of moisture on the repeated-load resilient modulus. Fine-grained soils display various degrees of moisture sensitivity that depend on certain of their inherent characteristics. In general, much of their strength can be explained by changes in the soil-water pressure (suction), which is related to its water content as shown in Figure 3. The absolute magnitude of change in the resilient modulus caused by an increase in the soil-moisture content will not be constant for all soils, but will vary with the accompanying change in soil-water pressure. The understanding and quantification of soil-water properties are important not only to drainage analysis and design but also to studies of frost action.

The study of a moisture movement requires knowledge of the initial and boundary conditions that describe the specific moisture-flow situation, and of the hydraulic conductivity, diffusivity, and water-capacity functions that characterize the soil or the subgrade. With these, it is possible to predict the behavior of the flow system using an approximate analytical or numerical solution of the moisture-flow equation. A few not very successful attempts to predict the moisture conditions in a pavement profile have been made, but a satisfactory and realistic procedure for the study of moisture movement in pavements is needed to solve the engineering problems associated with the behavior of pavement systems in response to moisture changes. Therefore, the objectives of this study are

1. To investigate unsaturated transient moisture flow in subgrade soils,
2. To demonstrate a general procedure for determining the subgrade soil-moisture properties such as hydraulic conductivity, diffusivity, and the soil-moisture-suction characteristic function necessary for predicting moisture movement; and
3. To show how the soil-moisture parameters can be used to provide a comprehensive procedure for predicting moisture conditions in pavement profiles.

LITERATURE REVIEW

Darcy's Law and the Continuity Equation

In 1822, Fourier (8) published his very complete mathematical theory of heat transport in conducting materials. In 1827, Ohm (16) published his law that the rate of transport of electricity in a conductor is proportional to the electrical potential difference between its ends, i.e., that the electric current is proportional to the electrical potential gradient. In 1822, Navier (14) developed equa-

Publication of this paper sponsored by Committee on Subsurface Drainage.

tions describing the flow of viscous fluids in terms of the distribution of hydraulic potential. These equations were later derived by Stokes (23) in a more general way. By using the Stokes-Navier equations, the rate of flow can be derived in terms of the dimensions of the conductor and the hydraulic potential difference between the ends. Poiseuille's experimentally derived equation for the flow of a fluid through a tube (18) can be readily obtained from the earlier theoretical work. This equation can be written in the form

$$Q/t = (\Delta\phi/L)(\pi/8\eta)R^4 g\rho \qquad (1)$$

where

- Q = volume passing in time, t,
- L = length of the tube between the ends of which the potential difference is $\Delta\phi$ (i.e., $\Delta\phi/L$ is the hydraulic potential gradient),
- η = viscosity of the fluid,
- ρ = its density,
- g = gravitation constant, and
- R = radius of the tube.

Figure 1. Research needs for prediction and control of moisture in pavement systems.

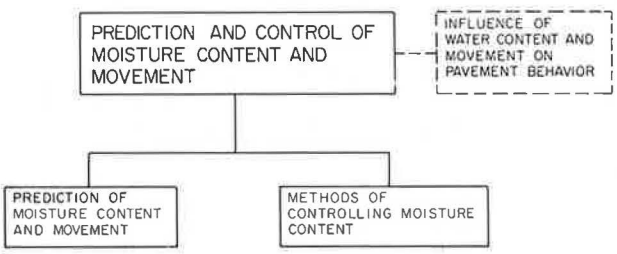


Figure 2. Approach for predicting moisture in pavement systems.

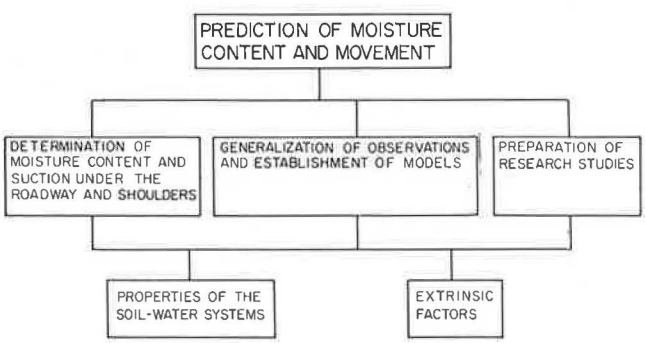
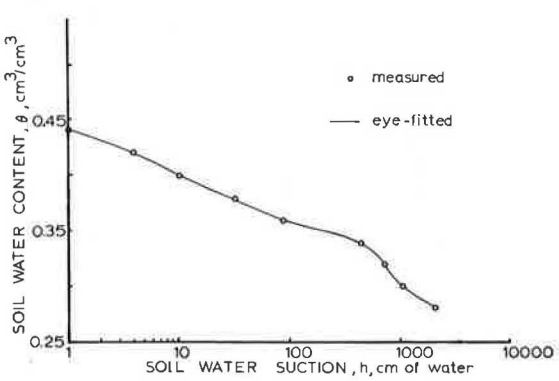


Figure 3. Soil-water characteristic function for AASHO A-4 subgrade soil.



The configuration of the pore space in a porous material such as soil is far too complicated to permit the rate of flow of a fluid to be calculated by the Stokes-Navier equations. However, Darcy (3), from experiments on the infiltration of water through filter beds of sands, formulated a law that has been widely accepted as the basis for describing the flow of a liquid in a saturated porous material. In 1950, Childs and Collis-George (2) introduced the soil-water diffusivity concept as a way to describe the flow in a water-unsaturated soil. Their theory for the unsaturated flow of water assumes that Darcy's law can be written as a diffusion-type water-flow equation in homogeneous soils, where gradients of water content rather than gradients of total potential are expressed as

$$q = D(\theta)\nabla\theta - K(\theta) \qquad (2)$$

where

- q = water flux,
- θ = soil-water content on a volume basis,
- $K(\theta)$ = soil hydraulic conductivity,
- $D(\theta)$ = $K(\theta)/C(\theta)$, the soil-water diffusivity,
- $C(\theta)$ = $\partial\theta/\partial h$, the specific soil-water storage capacity where h is the soil water pressure head having negative values, and
- $\nabla\theta$ = water-content gradient.

Both $D(\theta)$ and $K(\theta)$ are functions of the soil-water content, θ.

Equation 2 resembles Fick's law of diffusion with a concentration dependent diffusivity, except for the $K(\theta)$ term, which arises from the gravitation component of the total hydraulic head. If this equation is combined with the equation of continuity (i.e., the conservation of mass), a diffusion-type equation for flow in porous media under isothermal conditions is obtained (13). This can be written in the form

$$\partial\theta/\partial t = \partial[D(\theta)\partial\theta/\partial Z]/\partial Z + \partial K(\theta)/\partial Z \qquad (3)$$

where t is the time and Z is the vertical space coordinate. The term $\partial K(\theta)/\partial Z$ in equation 3 is normally referred to as the gravitational component. The validity of equation 3 in describing the flow of water in unsaturated soils has been demonstrated by several workers (5, 17, 22, 25), and, more recently, by Dempsey and Elzeftawy in a paper in this Record.

LABORATORY MATERIALS AND METHODS

Materials

The soil-water properties were determined for an AASHO A-4 soil, which was compacted in a plexiglass column 150 cm (60 in) high and having a cross section of 20 by 20 cm (8 in by 8 in). The composition and properties of the soil as compacted in the column are given below.

Component	Percent	Component	Percent
Sand	62.00	Plastic limit	14.70
Silt	20.00	Optimum water content, W_{opp}	11.00
Clay	18.00	W_{HYGR}	1.40
Liquid limit	22.20		

Property	Value
Maximum dry density ($\gamma_{d\,max}$)	1016 kg/m^3
Saturated hydraulic conductivity (K_s)	0.31 cm/h
Bulk density (ρ_s)	1.72 g/cm

The particle size distribution showed a predominance of sand (more than 60 percent); however, the clay content (18 percent) and the compacted nature explain the low value of the saturated hydraulic conductivity.

Equipment

The gamma-ray attenuation method (11) can be used experimentally to investigate most moisture-movement problems in soils and subgrade materials. As shown in Figure 4, this method uses a radioactive source on one side of the soil column and a scintillation counter on the other for the nondestructive measurement of the water content. The assembly of source and detector must be positioned at the elevation at which the measurement of the water-content change is to be made. The general design requirements of the traversing mechanism are as follows:

1. The mechanism must be strong enough to move the source and detector, together with the heavy shielding involved, up and down the column with a minimum of vibration and with accurate horizontal positioning of the collimation slit relative to the column center line.
2. The source and detector assembly must be capable of being stopped precisely at any graduation mark on the column.
3. The movement from one elevation to another must be as rapid as possible (keeping in mind the accuracy of positioning required by 2 above).

Water Content Measurements

The full and desirable specifications for water content measurements for the unsaturated transient-flow type of problem are

1. Measurement over a small thickness of the sample to approximate as closely as possible a planar measurement,
2. Determination by nondestructive means,
3. Measurement over a very short time interval, and
4. Rapid means of making measurements at different parts of the soil column.

Ferguson (7), Rawlings (19), and Gurr (11) have shown that the principle of gamma-ray absorption can be used to infer the moisture content of the soil from changes in its density. The attenuation equation for a moist soil, and for a collimated, monoenergetic beam is

$$I_m/I_o = \exp - [(\mu_\theta \rho_\theta \theta) + (\mu_s \rho_s)x] \qquad (4)$$

where

I_m/I_o = ratio of incident to transmitted flux for moist soil,
μ_θ = mass attenuation coefficient for water,
μ_s = mass attenuation coefficient for soil,
x = thickness of soil,
θ = volume of water/unit volume of soil,
ρ_s = bulk density of the soil, and
ρ_θ = density of water.

Study of equation 4 shows that its use in determining θ requires (a) values of the mass attenuation coefficients, (b) a uniform and known bulk density of the soil in the column (this can be achieved by special packing techniques), and (c) a constant thickness dimension along the column length.

Figure 4 also shows the source-detector assembly. A cesium-137 source, a scintillation probe, and a scaler unit are used to determine changes in the moisture content of the soil along the length of the column. The ^{137}Cs source and scintillation probe-preamplifier unit, with the required lead shielding, is moved along the length of the soil column by the use of three threaded screws that are part of the supporting frame. The scintillation probe, which absorbs the gamma radiation after it has passed through the soil column, is connected to a decade scaler and a pulse-height analyzer.

Soil-Water Pressure Measurement

A tensiometer system for the measurement of rapidly changing soil suction (negative pressure) should have the following features: (a) a high gauge sensitivity, (b) a response time of the order of a few seconds or less, (c) rapid means of reading, and (d) convenience of recording. The achievement of a very rapid response in the tensiometer system to pressure changes in the soil inherently implies a negligibly small exchange of water between the soil and the tensiometer. Klute and Peters (13) and Watson (24) have reported the satisfactory use of a pressure transducer for soil-water measurements requiring rapid response with minimum transfer of water between the soil water and the measuring system. Their approach has been further developed in the present tensiometer-transducer system, which can be used for a suction head range up to 900 cm of water (350 in).

A porous cup with as large a pore size as possible, yet with an air entry value greater than the maximum pressure (or suction) to be encountered, should be used to increase the response of the measuring system. The ceramic porous cups (fine porosity) used here as tensiometers were 1.0 cm (0.39 in) in diameter and 3.0 cm (1.2 in) in length with an air entry value (bubbling pressure) of approximately 800 cm (315 in) of water. The tensiometers were installed at eight positions and were saturated with boiled, distilled water. All of the tensiometers were connected by 1.6-mm ($^1/_{16}$-in) OD nylon tubing to a single pressure transducer by way of a common rotary switching valve. The output signal of the pressure transducer was measured by a demodulator and continuously recorded on a strip chart recorder.

A constant-head water column was used as a standard to calibrate the transducer. The ratio of the signal voltage to the differential pressure was 1 mV/cm of water pressure head.

The response-time constant of the tensiometer-transducer system when the flow system was water-saturated was 0.5 s, but became quite large (≥20 s) when the flow system was unsaturated. Richards (20) has defined the tensiometer conductance as the volume of water passing through the tensiometer cup per unit of time per unit of pressure difference: For a given time and pressure difference, the cup conductance depends primarily on the area of contact with the soil and the pore size of its porous material. He has also defined the gauge sensitivity s as the pressure change per unit volume of displacement. Watson (24) has shown that the gauge sensitivity, for the equipment used here, could more precisely be described as the transducer sensitivity. The response-time constant is thus related to the tensiometer cup conductance and the gauge sensitivity by the equation

$$\tau = 1/ks \quad (5)$$

where τ is the response-time constant, k is the tensiometer conductance, and s is the gauge sensitivity. [The pressure transducer used throughout this study has a volumetric displacement of 4.92×10^{-3} cm^3 (3.0×10^{-4} in^3); for a maximum pressure of 850 cm (335 in) of water and a gauge sensitivity of 1.7×10^5 cm^{-3} (1.1×10^6 in^{-3}), the tensiometer cup conductance can be calculated from equation 5 for any specified response-time constant.]

RESULTS AND DISCUSSION

The measured gamma-ray mass adsorption coefficient for water, μ_0, of 0.0832 ± 0.0006 cm^2/g agreed with the theoretical value of 0.0857 cm^2/g calculated by Grodstein (10). In water content calculations an average bulk density ρ_s value of 1.72 g/cm^3 (107.1 pcf), and an optimum soil-water content W_{opp} of 11.0 percent, as reported by Gurr (11), were used.

Figure 3 shows the soil-water content on a volume basis as a function of the soil-water suction expressed as cm of water head for AASHO A-4 soil. The solid line was eye-fitted to connect all of the measured values of the $h(\theta)$ relation. (The particular data shown are for the case of transient water flow during the wetting of the soil column.) A value of 1.0 cm of water pressure head (suction) was considered to be the value at which the soil was saturated. The water content of this soil at saturation, θ_s, was 0.44 cm^3/cm^3 (in comparison to 0.28 cm^3/cm^3 at -2066 cm (-813.4 in) of soil-water pressure head, h. Thus, the soil has lost approximately 36 percent of its water content in response to a pressure head of -2066 cm of water.

Campbell (1) has shown that if the water characteristic function $h(\theta)$ can be expressed by the equation

$$h = h_e(\theta/\theta_s)^{-b} \quad (6)$$

where h_e is the air entry water potential and b is an empirically determined constant, then the hydraulic conductivity is given by

$$K = K_s(\theta/\theta_s)^{(2b+3)} \quad (7)$$

where K_s is the saturated hydraulic conductivity of the soil. Since equation 6 is assumed to describe the water characteristic curve for the AASHO A-4 soil, equation 7 can be expected to give valid estimates of $K(\theta)$. The measured and calculated unsaturated hydraulic conductivity of the AASHO A-4 soil is shown in Figure 5. $K(\theta)$ was calculated by using equation 7 of Campbell (1) and by the Elzeftawy and Mansell method (4), which is a modification of the Green and Corey method (9) that includes a spline function technique (6) to provide a smooth continuous soil-water characteristic function. The measured value of the hydraulic conductivity corresponding to water saturation (θ_s = 0.44 cm^3/cm^3) was used as a matching factor to determine the calculated curve for the $K(\theta)$ function. Figure 5 shows that the revised method (4) calculating K versus θ gives better agreement with the measured values of the unsaturated hydraulic conductivity than does that of Campbell (1). The divergence of the Campbell method from the measured conductivity may be related to the assumption that the pore size distribution function is the same throughout the porous body.

Just as the flow of heat can be expressed in the form of a diffusion equation with a diffusivity expressed in terms of the thermal conductivity, density, and specific heat of the material, so Darcy's law may be put into a diffusionlike form with the water diffusivity $D(\theta)$ given by

$$D(\theta) = K(\theta)/C(\theta) \quad (8)$$

where $C(\theta) = \partial\theta/\partial h$, the specific water capacity of the soil. Figure 6 shows the AASHO A-4 soil-water diffusivity as a function of the soil-water content for a wetting case. As the soil-water content increases from 0.28 to 0.42 cm^3/cm^3, the soil-water diffusivity increases from 3.2×10^{-1} to 5.4×10^1 cm^2/h. For the same range of water content, the soil hydraulic conductivity increased from 3.1×10^{-7} to 1.9×10^{-1} cm/h (Figure 5). Recent studies (21) with a large number of soils from the Midwest, Oklahoma, Georgia, and the Carolinas have shown that a water-content change of 1 or 2 percent by weight can have considerable influence on the strength of the AASHO Road Test subgrade soil.

Dempsey and Elzeftawy, in a paper in this Record, have used numerical solutions of equation 3 to develop a moisture model to predict moisture movement in subgrade soils. The model can be used for one-dimensional or two-dimensional moisture flow through homogeneous or multilayered subgrade soil and pavement systems under iso or nonisothermal conditions. The experimentally determined relations of h versus θ (Figure 3), K versus θ (Figure 5), and D versus θ (Figure 6) were used in this model to predict the upward moisture movement into compacted subgrade soil columns having uniform initial water contents and a water table at the bottom of the soil column. Figure 7 shows the calculated and measured soil-water distributions for AASHO A-4 subgrade soil as a function of soil height after 6, 13, 36, and 60 d from establishing a water table at the bottom of the soil column. The soil-water content at the water table position increased from the initial value of 0.21 cm^3/cm^3 to 0.44 cm^3/cm^3, which is equal to the saturated soil-water content. After 13 days, the water front reached 40 cm (15.75 in) above the water table. The decreasing soil-water content with height at a specific time ($t > 0$) is due to the increase in the negative soil-water pressure (suction). At equilibrium, the soil-water content distribution profile should be similar to that of the soil-water characteristic function, $h(\theta)$. The agreement between the measured and calculated soil-water content distributions in AASHO A-4 subgrade soil is good, and thus the Dempsey-Elzeftawy moisture model can be used to predict the water movement in a subgrade soil.

SUMMARY AND CONCLUSIONS

A water table was established at the bottom of a compacted subgrade soil column, and the movement of water through the soil was studied under isothermal conditions. A gamma-ray method was used for the nondestructive measurement of the water content, and a tensiometer-pressure transducer arrangement to measure the soil-water pressure (suction); the unsaturated hydraulic conductivity, diffusivity, and soil-water characteristic parameters were evaluated from these data and used as input data for the Dempsey and Elzeftawy moisture model to predict the movement of moisture through subgrade soil. The following conclusions were made:

1. Darcy's law and the continuity equation can be used to describe and explain soil-moisture flow through compacted subgrade soil in both saturated and unsaturated flow.
2. Soil moisture moves through subgrade soil under unsaturated transient-flow condition.

Figure 4. Soil-moisture column and source-detector assembly.

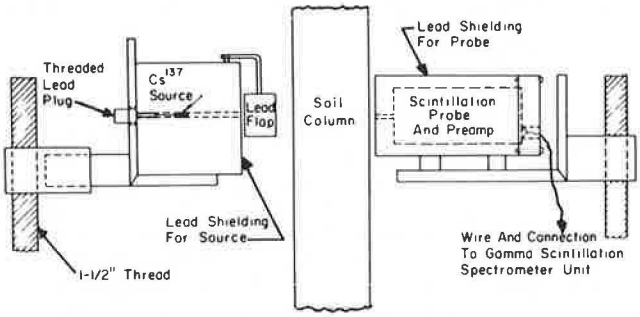


Figure 5. Measured and calculated unsaturated hydraulic conductivity of AASHO A-4 soil.

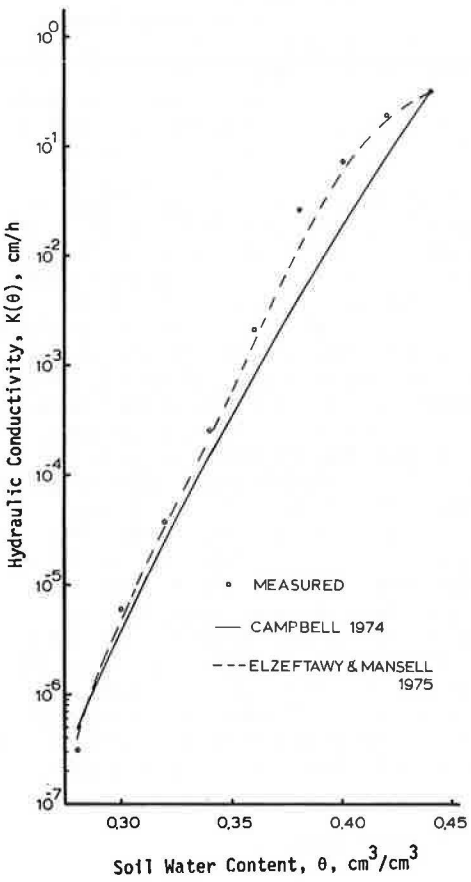


Figure 6. Soil-water diffusivity for AASHO A-4 subgrade soil.

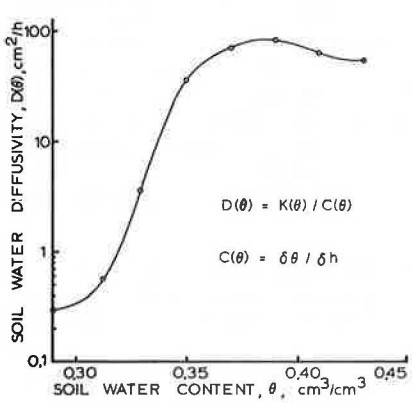
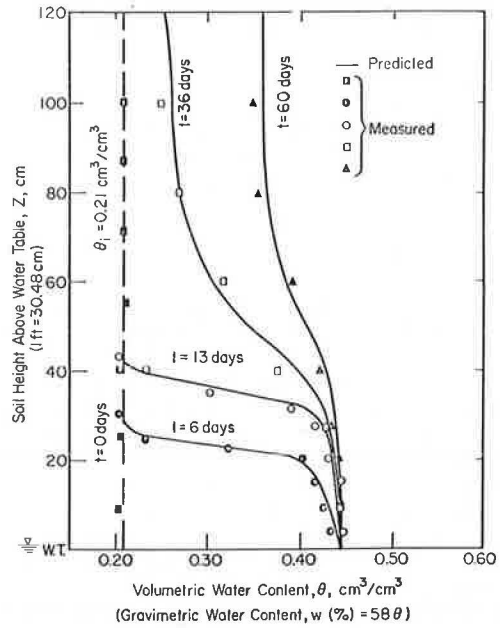


Figure 7. Soil-water distribution for AASHO A-4 subgrade soil as a function of soil height.



3. The experimentally evaluated soil-moisture parameters are necessary for the prediction of moisture content and its movement in subgrade soil.

ACKNOWLEDGMENTS

This report was prepared as a part of the Illinois Cooperative Highway Research Program, Moisture Movement and Moisture Equilibria in Pavement Systems, by the Department of Civil Engineering, Engineering Experiment Station, University of Illinois at Urbana-Champaign, in cooperation with the Illinois Department of Transportation and the Federal Highway Administration, U.S. Department of Transportation.

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