

Basic Techniques and Examples of Computer Simulation

Richard M. Weed, New Jersey Department of Transportation

Computer simulation is one of the most useful techniques available to the statistician. However, current literature indicates that this approach has not been widely used by engineers. This paper provides the basic tools for anyone with an elementary knowledge of statistical theory and computer programming to simulate a random normal process on a digital computer. The usefulness of this technique is illustrated by two examples that involve concrete strength specifications. The first deals with the determination of the best sampling plan for a quality-assurance testing program, and the second illustrates the simulation of a statistical acceptance procedure with a reduced pay schedule. Several FORTRAN sub-routines are described for generating both uniform and normal random numbers, sorting a data array, selecting a random sample, calculating statistical parameters, and printing a histogram.

In recent years, there has been an increasing tendency to use statistical analysis to develop end-result specifications. Based on information obtained from a random sample, the product in question may be accepted, rejected, or accepted with a reduced payment. Since specifications control the ultimate quality of the product and determine the reimbursement to the contractor, it is important that they are well designed.

A useful tool for developing specifications of this type is computer simulation. Because it is basically a try-and-see approach, computer simulation enables the specification writer to see how a new specification can be expected to perform in the field. It is then possible to spot potential problems and correct them before they lead to more serious difficulties after the specification is adopted. In this manner, specifications can be written that are not only more effective but also equitable to all parties concerned.

REQUIREMENTS FOR COMPUTER SIMULATION

For the effective use of computer simulation, certain requirements must be met. An obviously basic require-

ment is a high-speed digital computer. And, if at all possible, the programming should be done from a communications terminal on a time-sharing basis that allows the user to interact directly with the computer.

Although work of this type could be done by an engineer and a programmer working together, it is more efficient if done by one individual who has an understanding of the process to be simulated and the ability to write the simulation program. A college level course in both statistics and programming should be sufficient to understand the material described in this paper.

Once the equipment and manpower requirements are met for the simulation of a real process, it is necessary to obtain some basic information that describes the variability of the real process. This information may be obtained from a variety of sources such as historical data, literature searches, preliminary field studies, or other simulations. The reliability of any simulation depends on the accuracy of the input parameters, so care must be taken to ensure that they are well defined.

The random generators used in this paper will generate either uniform or normal random numbers and, consequently, may be used only if these distributions are appropriate. Fortunately, these two distributions are suitable for most situations. The uniform distribution is used for random sampling, and the normal distribution is used for approximating the variability of most construction parameters. For those instances in which it is known that the actual distribution is not normal, the central limit theorem may have to be relied on (sample means from any population tend to be normally distributed). If necessary, another alternative is to use a transformation (such as logarithm, power, or root) to convert a skewed distribution to one that is approximately normal.

ADVANTAGES OF COMPUTER SIMULATION

Many advantages are derived from computer simulation. Of prime importance are the considerable savings in both time and expense. Programs that simulate a typical construction specification require only a day or two to write and a matter of seconds to run on the computer. In

this manner, information based on the equivalent of several months or years of field data can be obtained in less than a week.

Of possibly greater importance is the use of computer simulation to solve problems for which direct analytical solutions are not known. For example, consider the expression $Q = (XY)/Z$ in which X, Y, and Z are independent random variables, and the means and standard deviations of the variables are known. To determine the mean and standard deviation of the variable Q, some type of approximation must be used. To solve this problem by computer simulation, one would simply generate random values for X, Y, and Z and calculate Q; repeat this process many times (perhaps 1000); and store the Q values in the memory of the computer. Then, it is possible to compute the mean and standard deviation of the simulated Q values and, if desired, to print a histogram for checking the shape of the Q distribution.

Another example that is difficult to analyze by conventional means is the performance of a statistical acceptance procedure with a reduced pay schedule. In this case, computer simulation can be used to determine the distribution of pay factors resulting from any selected quality level of production. By averaging these simulated pay factors, an overall expected pay factor can be determined. The specification writer can then develop a realistic pay schedule that will provide payment commensurate with the quality received.

To assist in the study of various sampling plans, further advantages can be gained by using computer simulation. Since the simulated data are stored in the memory of the computer, it is possible to test different sampling plans on the same set of data. This technique is usually impossible with field tests because the sampling and testing process disturbs or destroys the material being tested. Also, the true mean value is never known in actual field tests and must be estimated. Because of the nature of the simulation process, the true mean is always known. Thus, it is possible to check the difference between the true mean and the sample mean in the simulation and to decide which of the several sampling plans is most accurate. Finally, if different acceptance procedures are being tested, the number of correct and incorrect decisions can be counted to determine which procedure is most often correct.

USE OF RANDOM GENERATORS

Random numbers that are generated by digital computers are not truly random because many numbers generated in this manner will eventually repeat in a sequence. However, random generators with long cycles have been developed and are satisfactory for most purposes (1).

Two subroutines, RAND and NORM, are used for simulation purposes (2). RAND generates uniform random numbers between the values of 0 and +1, and NORM generates normal random numbers with a mean of 0 and a standard deviation of 1. For both of these subroutines, a seed number must be transmitted from the main program each time the subroutine is called. In the examples presented here, the seed number is referred to as RANDSTART outside the computer program and is assigned the FORTRAN variable name NSTART within the program. Initially, NSTART should be an odd integer that does not exceed nine digits. NSTART is read into the main program once and it is automatically changed each time one of the subroutines is called. Typical coding for reading NSTART from a time-sharing computer terminal might be as follows:

```
WRITE (6,100)
100 FORMAT ('0', 'ENTER RANDSTART')
READ(5,110) NSTART
```

110 FORMAT(I9)

A free read feature is more convenient than the formatted read statement shown above.

Occasionally, the simulation will be such that it is desired to use both RAND and NORM in the main program. In this situation, a single NSTART term is sufficient for the independent operation of both random generators.

The random generators can be used in several ways. For example, it may be desired to have a simulated event occur randomly a certain percentage of the time, say, 15 percent. Since the numbers generated by RAND are uniformly distributed between 0 and +1, approximately 15 percent of them will have a value of 0.15 or less. If RUX is defined as the uniform random number and GO TO 1 is the event we want to occur 15 percent of the time, the coding for this operation might be as follows:

```
CALL RAND(NSTART, RUX)
IF(RUX.LE.0.15) GO TO 1
```

Then, at label 1 in the program, we would have additional FORTRAN statements that simulate whatever random event is desired.

The primary use of RAND concerns random sampling. If the variable SAMPLE represents a single random sample that is selected from the values stored in array X(N), the following coding might be used:

```
CALL RAND(NSTART,RUX)
IF(RUX.EQ.1.) RUX=.999999
I=IFIX(RUX*N)+1
SAMPLE=X(I)
```

In actual practice, it is more convenient to use subroutine SAMP (2) which is capable of selecting a random sample of up to 1000 from an array of up to 1 000 000.

When subroutine NORM is used, it is desired to generate values that have some specified mean and standard deviation. The term VALUE is defined as the random normal variable that has a mean and standard deviation of AVG and STDV respectively. Subroutine NORM returns two independent normal variates, RNX and RNY, each time it is called; therefore, a total of N of these values could be generated by the following coding, if N is assumed to be an even number.

```
NHALF=N/2
DO 1 I=1,NHALF
CALL NORM(NSTART,RNX,RNY)
VALUE(I)=AVG+STDV*RNX
1 VALUE(NHALF+I)=AVG+STDV*RNY
```

In those rare cases in which it is inconvenient to use more than one random number each time the subroutine is called, the second value returned by NORM may simply be ignored.

SIMULATION TO DETERMINE BEST SAMPLING PLAN

For many years, it has been the practice of the New Jersey Department of Transportation to make three compression test cylinders for every 76.5 m³ (100 yd³) of concrete delivered to the job. Although it was not specifically required, it had become common practice to take the three cylinders from the same truck. The vast majority of the concrete was truck-mix (as opposed to central-mix), and ample data were available that indicated a great deal of variability in the strength of con-

crete from truck to truck (batch to batch). Because of the large degree of batch-to-batch variability, it was theorized that the sampling plan would be more effective if the three cylinders were taken from separate trucks. It was further predicted that a more accurate measure of the lot quality of the concrete would be provided by only two cylinders taken from different trucks than by three cylinders taken from the same truck.

To test these conclusions, a computer program was written to simulate the three sampling plans for many lots of concrete of varying average strengths. The best sampling plan will be the one that produces a sample mean that (a) deviates the least from the true lot mean and (b) overestimates as often as it underestimates the true lot mean. In statistical terms, this is a minimum-variance, unbiased estimator.

In the simulation of a measurement process, it is usually necessary to separate the total variance of the field data into the component associated with the measurement process itself and the component represented by the true variability of the material being measured. The variability of the measurement process is determined from repeated measurements of material from the same batch. The variability of the material is determined by subtracting the variance of the measurement process from the variance of the measurements made on many different batches of material. The measurements of many sets of cylinders taken from single trucks provided a pooled estimate of variance for the measurement process. The measurements of single cylinders selected randomly from different trucks were used to calculate an overall variance from which the measurement process variance could be subtracted to obtain the true batch-to-batch variability of the concrete.

The general techniques previously described are the basic building blocks of the simulation program. After the initial parameters are entered, it is necessary to have the computer create a large population of strengths for batches of concrete similar to the strengths occurring during construction. Subroutine NORM randomly generates the strengths by using the standard deviation that represents only the variability of the material. The batches are then divided into lots, and subroutine SAMP samples the lots as prescribed for the sampling plans being tested. An additional random component of variability that represents the sampling-and-testing error (i.e., variability of the measurement process) is added by using subroutine NORM to each simulated cylinder strength. Since the true strength of each simulated batch is known, the true mean of each lot can be calculated and compared to the mean estimated from the sample. For each lot and each sampling plan, the algebraic difference between the sample mean and the true mean is calculated and stored. After the desired number of lots have been sampled, the algebraic differences are analyzed. For a good sampling plan, the distribution of the algebraic differences will have a mean close to zero and a low standard deviation.

The steps in this program are summarized as follows:

1. Enter into the computer desired number of lots, batches (trucks) per lot, mean strength of concrete, batch-to-batch standard deviation, sampling-and-testing standard deviation, and number to start random generator;
2. Calculate the total number of simulated batches required and generate this number of "true" concrete batch strengths (use mean strength and batch-to-batch standard deviation);
3. Divide the batch strengths into lots, calculate the true lot strength for each, print a histogram of all batch strengths, and count and print percentages of batch

strengths and lot strengths that are below design strength;

4. In accordance with the provisions for each plan (three cylinders from the same truck, three cylinders from separate trucks, and two cylinders from separate trucks), randomly sample each lot (adding on the variability due to sampling-and-testing error), calculate the estimated strength for each lot, and store the estimated lot strengths in separate arrays for each plan; and

5. Compare the estimated lot strength for each plan with the true lot strengths to see which plan is best and calculate and print all statistical parameters of interest.

A typical printout of the simulation for determining the best sampling plan is shown in Figure 1. The entry variables STDV(BB) and STDV(ST) refer to the batch-to-batch and sampling-and-testing standard deviations respectively. The rest of the printout is self-explanatory and provides the information necessary to determine which sampling plan is performing best.

To thoroughly test the three sampling plans over a wide range of average strengths, 17 runs of 100 lots each were made for average strengths ranging from about 20.7 to 31.0 MPa (3000 to 4500 lbf/in²). The lot size and truck size were assumed to be 76.5 m³ (100 yd³) and 7.65 m³ (10 yd³) respectively. These measurements gave a value of 10 trucks per lot to be entered into the program. The 17 runs represented the equivalent of sampling 130 000 m³ (170 000 yd³) of concrete for each of the three plans.

It was known that a high value for batch-to-batch variability (σ_{BB}) and a low value for sampling-and-testing variability (σ_{ST}) would accentuate the differences in the results obtained from the different sampling plans and would bias the simulation in favor of the predicted results. Therefore, a slightly low value of $\sigma_{BB} = 4.14$ MPa (600 lbf/in²) and a slightly high value of $\sigma_{ST} = 2.07$ MPa (300 lbf/in²) were used in the program. As an additional check, a few runs were made with different values, and the results were essentially the same.

Table 1 gives the average differences from the true mean lot strengths for each plan and each run, the standard deviations for these differences, and the t-values for the average differences. In addition, the overall average and pooled standard deviation are given for each sampling plan.

Plan 1 and plan 2 are considered unbiased because none of the average differences is significantly different from zero. Plan 3 may be slightly biased because 3 of the 17 differences are significant at the 95 percent confidence level and have the same algebraic sign. Although there is no reason to expect that this plan is biased, since the likelihood that as many as three significant t-values with the same sign would be obtained is fairly slim (about 0.016). However, from a practical standpoint, the degree of bias is small and can probably be ignored.

The pooled standard deviations associated with the three plans are quite different because the distributions of the individual standard deviations are separate from one another. Plan 2 (three cylinders from separate trucks) is the best, plan 1 (three cylinders from the same truck) is the worst, and plan 3 (two cylinders from separate trucks) is nearly better than plan 1.

The data given in Tables 2 and 3 show the incorrect decisions that are tabulated and averaged for each plan. Although all three plans seem to have about the same percentage of incorrect decisions for individual cylinders, the lot strength results serve to confirm the conclusions obtained from the data given in Table 1. That is, plan 2 is the best because it has the fewest incorrect decisions; plan 1 is the worst because it exhibits the greatest number of incorrect decisions; and plan 3 falls between plans 1 and 2.

The computer time for this simulation was approxi-

mately 11 s for each 100 lot run. The writing of and adjustments to this program required about 5 person-days and 400 s of running time on the computer.

SIMULATION OF A STATISTICAL ACCEPTANCE PROCEDURE

To obtain a greater degree of compliance with the concrete specification, a statistical acceptance procedure with a reduced pay schedule was considered. The procedure is based on the range method of Military Standard 144 (3). Lot sizes and sample sizes are specified for each class of concrete and a quality index (Q_L) is calculated for each lot as follows:

$$Q_L = (\bar{X} - \text{design strength})/R \quad (1)$$

where

- \bar{X} = sample average = $(X_1 + X_2 + \dots + X_N)/N$,
- N = sample size,
- X = compressive strength, and
- R = sample range = $X_{\text{maximum}} - X_{\text{minimum}}$.

Based on historical data for structural concrete, the percentages for acceptable quality level (AQL) and rejectable quality level (RQL) were determined to be 10 and 43 percent below design strength respectively. In other words, if no more than 10 percent of a lot of con-

crete is below the design strength, the concrete is at least equal to the quality obtained in the past and is, therefore, considered acceptable. On the other hand, if 43 percent or more of the lot is defective, the concrete is considered seriously deficient and would warrant no payment or would possibly have to be removed and replaced.

The Military Standard 144(3) provides a procedure for determining critical Q_L limits that indicate when either the AQL or the RQL has been obtained. These levels are used to set Q_L ranges that correspond to a graduated series of pay factors. A set of ranges that might be used for sample sizes of 7 and 4 is as follows:

Q _L Range		Pay Factor (%)
N = 7	N = 4	
> 0.28	≥ 0.38	100
0.22 to 0.27	0.30 to 0.37	95
0.17 to 0.21	0.23 to 0.29	90
0.12 to 0.16	0.16 to 0.22	80
0.07 to 0.11	0.09 to 0.15	50
≤ 0.06	≤ 0.08	0

For any sample size, the largest Q_L value corresponds to the AQL and a pay factor of 100 percent, and the lowest Q_L value corresponds to the RQL and a pay factor of zero. Between these two extremes, there are four intermediate pay levels. These pay levels have been weighted toward the high (100 percent) end of the pay

Figure 1. Printout of simulation to determine best sampling plan.

```

ENTER LOTS, TRUCKS/LOT, MEAN, STDV(BB), STDV(ST), RANDSTAPT
      100      10      3500      600      300      9999977

DISTRIBUTION OF SIMULATED CONCRETE BATCH STRENGTHS

UCL      FREQUENCY
1869      4 X
2091      11 XXXX
2314      15 XXXXX
2537      43 XXXXXXXXXXXXX
2759      47 XXXXXXXXXXXXXXX
2982      75 XXXXXXXXXXXXXXXXXXX
3205      137 XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
3427      145 XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
3650      150 XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
3873      118 XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
4095      112 XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
4318      71 XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
4541      34 XXXXXXXXXXXXX
4765      21 XXXXXXX
4986      12 XXXX
5209      2 X
5431      3 X

MEAN = 3466      STD. DEV. = 604      STD. ERR. = 19
TOTAL OF 1000 BATCHES, 20.3 PERCENT BELOW THE DESIGN STRENGTH OF 3000
TOTAL OF 100 LOT AVERAGES, 0.0 PERCENT BELOW DESIGN STRENGTH

PERFORMANCE OF PLAN 1 (THREE CYLINDERS PER LOT FROM SAME TRUCK)

MEAN DEVIATION OF TEST FROM TRUE LOT AVERAGE = 5
STANDARD DEVIATION = 627      T-VALUE FOR PLAN MEAN = 0.08
INDIVIDUAL CYLINDERS BELOW DESIGN STRENGTH = 24.3 PERCENT
TESTS (LOT AVERAGES) BELOW DESIGN STRENGTH = 23.0 PERCENT
CYLINDERS FALSELY REJECTED = 8.3 PERCENT
CYLINDERS FALSELY ACCEPTED = 6.0 PERCENT
LOTS FALSELY REJECTED = 23.0 PERCENT
LOTS FALSELY ACCEPTED = 0.0 PERCENT

PERFORMANCE OF PLAN 2 (THREE CYLINDERS PER LOT FROM SEPARATE TRUCKS)

MEAN DEVIATION OF TEST FROM TRUE LOT AVERAGE = 13
STANDARD DEVIATION = 348      T-VALUE FOR PLAN MEAN = 0.37
INDIVIDUAL CYLINDERS BELOW DESIGN STRENGTH = 21.7 PERCENT
TESTS (LOT AVERAGES) BELOW DESIGN STRENGTH = 11.0 PERCENT
CYLINDERS FALSELY REJECTED = 5.7 PERCENT
CYLINDERS FALSELY ACCEPTED = 4.7 PERCENT
LOTS FALSELY REJECTED = 11.0 PERCENT
LOTS FALSELY ACCEPTED = 0.0 PERCENT

PERFORMANCE OF PLAN 3 (TWO CYLINDERS PER LOT FROM SEPARATE TRUCKS)

MEAN DEVIATION OF TEST FROM TRUE LOT AVERAGE = 8
STANDARD DEVIATION = 477      T-VALUE FOR PLAN MEAN = 0.17
INDIVIDUAL CYLINDERS BELOW DESIGN STRENGTH = 25.0 PERCENT
TESTS (LOT AVERAGES) BELOW DESIGN STRENGTH = 19.0 PERCENT
CYLINDERS FALSELY REJECTED = 7.0 PERCENT
CYLINDERS FALSELY ACCEPTED = 3.0 PERCENT
LOTS FALSELY REJECTED = 19.0 PERCENT
LOTS FALSELY ACCEPTED = 0.0 PERCENT
    
```


Table 1. Results of simulation to determine best sampling plan.

Mean Strength of 100 Lots (MPa)	Plan 1			Plan 2			Plan 3		
	Avg Difference	Standard Deviation	t-Values	Avg Difference	Standard Deviation	t-Values	Avg Difference	Standard Deviation	t-Values
20.65	-0.26	3.85	-0.69	0.29	2.31	1.24	0.37	2.70	-1.38
21.33	-0.66	4.38	-1.51	-0.12	2.40	-0.49	0.50	3.20	1.59
22.26	0.44	3.85	1.14	0.03	2.32	0.12	0.03	2.91	0.10
22.56	-0.65	4.27	-1.52	0.04	2.48	0.17	0.52	3.18	1.65
23.37	0.55	4.47	1.23	0	2.56	0.0	0.49	3.03	1.65
23.90	0.03	4.32	0.08	0.09	2.40	0.37	0.06	3.29	0.17
24.15	-0.15	4.36	-0.35	-0.17	2.53	-0.65	-0.35	3.06	-1.16
24.77	-0.34	4.03	-0.84	0.20	2.52	0.78	-0.24	3.14	-0.76
25.59	0.27	3.99	0.67	0.16	2.50	0.64	0.81	2.90	2.79*
26.17	0.61	4.16	1.47	-0.16	2.18	-0.72	0.37	3.07	1.23
26.94	0.23	4.36	0.52	-0.10	2.38	-0.40	0.12	3.10	0.38
27.51	0.28	4.44	0.63	0.23	2.27	1.00	-0.18	2.83	-0.63
28.39	0.19	3.78	0.49	-0.16	2.17	-0.72	0.64	2.65	2.45*
28.96	0.43	4.21	1.02	0.13	2.50	0.53	0.46	2.96	1.56
29.52	0.21	4.26	0.48	0.19	2.32	0.82	-0.12	2.76	-0.42
30.64	-0.03	4.12	-0.08	0.17	2.30	0.73	0.60	2.63	2.29*
31.14	-0.21	4.12	-0.52	0.12	2.45	0.50	0.08	3.16	0.24
Avg	0.055			0.055			0.245		
Pooled		4.18			2.39			2.98	

Note: 1 Pa = 0.000 145 lbf/in².

*Significantly different from zero at 95 percent confidence level.

Table 2. Incorrect decisions for individual cylinders.

Mean Strength of 100 Lots (MPa)	Plan 1 (¢)		Plan 2 (¢)		Plan 3 (¢)	
	Rejected	Accepted	Rejected	Accepted	Rejected	Accepted
20.65	7.0	8.7	9.3	8.7	8.0	10.5
21.33	8.7	5.0	8.0	5.0	7.0	8.0
22.26	6.3	5.3	7.0	6.7	10.5	7.0
22.56	6.3	7.3	8.0	4.0	6.5	5.0
23.37	6.3	3.7	5.0	4.3	6.0	3.0
23.90	8.3	6.0	5.7	4.7	7.0	3.0
24.15	6.0	4.3	5.7	3.7	6.0	3.0
24.77	9.7	2.3	5.7	4.0	9.0	1.5
25.59	5.0	0.3	4.0	1.3	5.5	2.5
26.17	2.3	1.3	3.7	2.3	4.5	2.5
26.94	2.3	0.7	4.7	1.0	2.0	0.5
27.51	3.0	1.7	1.7	1.7	2.5	1.5
28.39	3.7	0.3	2.3	0.7	2.0	1.0
28.96	2.3	0.3	2.7	0.3	2.5	1.0
29.52	0.3	0	0	0	0.5	1.0
30.64	1.7	0.7	0.3	0.3	1.0	0.5
31.14	2.7	0	0.7	0	1.5	0
Avg	4.8	2.8	4.4	2.9	4.8	3.0

Note: 1 Pa = 0.000 145 lbf/in².

Table 3. Incorrect decisions for lot strengths.

Mean Strength of 100 Lots (MPa)	Plan 1 (¢)		Plan 2 (¢)		Plan 3 (¢)	
	Rejected	Accepted	Rejected	Accepted	Rejected	Accepted
20.65	24.0	17.0	16.0	19.0	18.0	11.0
21.33	35.0	12.0	24.0	14.0	23.0	9.0
22.26	28.0	6.0	23.0	3.0	21.0	3.0
22.56	37.0	3.0	19.0	1.0	19.0	5.0
23.37	24.0	0	14.0	0	12.0	0
23.90	23.0	0	11.0	0	19.0	0
24.15	24.0	0	13.0	0	18.0	0
24.77	20.0	0	7.0	0	14.0	0
25.59	9.0	0	2.0	0	4.0	0
26.17	8.0	0	1.0	0	6.0	0
26.94	6.0	0	2.0	0	3.0	0
27.51	6.0	0	1.0	0	1.0	0
28.39	2.0	0	0	0	0	0
28.96	2.0	0	0	0	0	0
29.52	3.0	0	0	0	0	0
30.64	0	0	0	0	0	0
31.14	1.0	0	0	0	1.0	0
Avg	14.8	2.2	7.8	2.2	9.4	1.6

Note: 1 Pa = 0.000 145 lbf/in².

scale to ensure that the pay for satisfactory concrete would never be severely penalized.

To produce a consistent supply of AQL concrete or better, the average strength must be considerably higher than the design strength. For example, if the design strength is 25.5 MPa (3700 lbf/in²) and the producer possesses a 15 percent coefficient of variation, the concrete must have an average strength of about 31.6 MPa (4580 lbf/in²) to be rated AQL. A producer must attempt to achieve this overdesign value for all batches of concrete to assure that no more than 10 percent of the batches will fall below the design value of 25.5 MPa (3700 lbf/in²).

The computer simulation for this example served three purposes. Primarily, it was a check to see if the specification was properly accepting lots of AQL and

better and rejecting lots of RQL and poorer. It also emphasized the importance of using an adequate over-design value when work is performed under this specification. And finally, it demonstrated that a contractor who overdesigned properly would, on the average, achieve a pay factor close to 100 percent.

This simulation program is similar to that of the first example except that two separate components of variance were not required. In this second example, the computer must generate a distribution of strength tests that have a specific mean and coefficient of variation. These parameters are used to determine the standard deviation and then subroutine NORM is used. A Q₁ value is calculated for each lot. These values are then compared to the ranges previously given, which have been programmed into the memory of the computer, to determine the pay factor. The simulation also contains a minimum strength requirement for studying the effects of this provision.

The steps in this program are as follows:

1. Enter into the computer desired number of lots, mean strength of concrete, coefficient of variation, design strength, minimum allowable strength, number of tests per lot, and number to start random generator;
2. Calculate the total number of simulated strength tests required, generate this number of tests by using the mean strength and coefficient of variation, and print a histogram of these values;
3. Calculate and print the mean, standard deviation, and coefficient of variation of these tests;
4. Calculate and print the number of tests below the minimum level, the number of lots having one or more tests below the minimum level, and the overall percentage of material below design strength;
5. Calculate the Q₁ value and pay factor for each lot, print a table showing the distribution of lots by Q₁ and pay factor, and calculate and print the overall expected pay factor for all lots.

A typical printout of a simulation of a statistical acceptance procedure is shown in Figure 2. The overall expected pay factor in the printout is simply the average pay factor for all lots. Table 4 gives the results of 20 runs made

Figure 2. Printout of simulation of statistical acceptance procedure.

```

ENTER LOTS, MEAN, CV, DESIGN, MINIMUM, TESTS, RANDSTART
      100  4580  15  3700  2900    7    9999993

DISTRIBUTION OF SIMULATED CONCRETE STRENGTH TESTS

UCL   FREQUENCY
2600  2 X
2879  1 X
3158  9 XXXX
3437  21 XXXXXXXXX
3716  31 XXXXXXXXXXXXX
3995  78 XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
4274  79 XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
4553  104 XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
4832  108 XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
5111  110 XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
5390  74 XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
5669  43 XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
5948  31 XXXXXXXXXXXXXXXXXXXXXXXX
6227  7 XXX
6505  1 X
6784  1 X

MEAN = 4595   STD. DEV. = 675   COEFF. OF VAR. = 14.7
TOTAL OF 700 TESTS WITH 4 BELOW MINIMUM LEVEL OF 2900
TOTAL OF 100 LOTS WITH 4 HAVING ONE OR MORE TESTS BELOW 2900
AMOUNT OF MATERIAL BELOW DESIGN STRENGTH = 9.1 PERCENT

DISTRIBUTION BY PAY FACTOR OF SIMULATED QL VALUES

QL RANGE      PAY FACTOR      FREQUENCY
>.27          100              91
.22-.27       95               2
.17-.21       90               4
.12-.16       80               1
.07-.11       50               1
<.07          0                1

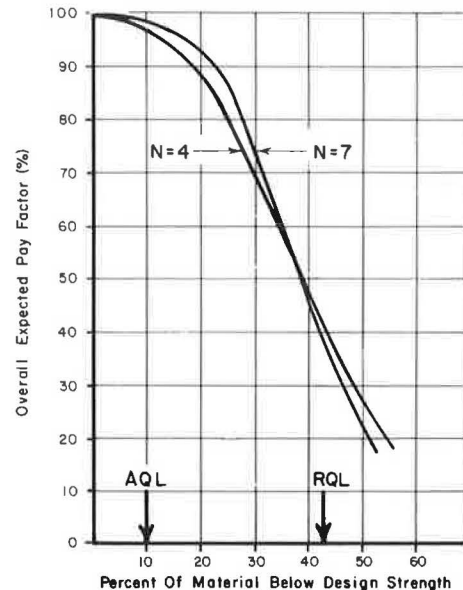
OVERALL EXPECTED PAY FACTOR FOR ALL LOTS = 97.8 PERCENT
    
```

Table 4. Results of simulation of statistical acceptance procedure.

Mean Strength (MPa)	Sample Size	Coefficient of Variation (%)	Material Below Design Strength (%)	Overall Expected Pay Factor (%)
34.23	7	21.1	10.9	98.3
31.68	7	14.7	9.1	97.8
29.65	7	16.1	20.3	93.3
27.77	7	15.0	30.6	71.7
26.92	7	20.2	40.1	44.8
26.48	7	19.6	42.0	44.3
26.20	7	14.3	42.3	38.0
26.07	7	15.2	44.0	36.8
25.29	7	15.1	51.6	19.8
34.55	4	20.4	9.5	96.5
31.85	4	14.6	8.8	97.6
29.54	4	16.0	20.5	85.3
27.83	4	15.3	30.5	68.6
27.01	4	15.2	37.0	54.0
26.82	4	19.3	40.8	48.0
26.68	4	20.2	40.8	47.6
26.60	4	15.9	39.5	46.8
26.42	4	15.0	42.0	41.3
26.17	4	15.5	43.8	44.3
25.24	4	15.6	52.5	21.0

Notes: 1 Pa = 0.000 145 lbf/in².
 Design strength = 25.50 MPa; overdesign strength = 31.60 MPa at CV = 15 and 34.34 MPa at CV = 20.

Figure 3. Operating characteristic curves for statistical acceptance procedure.



for a single class of concrete that has a design strength of 25.5 MPa (3700 lbf/in²).

The operating characteristic curves shown in Figure 3 were determined from the data given in Table 4. These curves show the relation between the percentage of material below design strength and the overall expected pay factor. For AQL concrete (10 percent of material below design strength), the expected pay factor should theoretically be 100 percent. In reality, it is approximately 98 and 97 percent for sample sizes of 7 and 4 respectively. At the other extreme, RQL concrete (43 percent of material below design strength) was intended to have an expected pay factor close to zero. However, because many rejectable lots actually receive partial payment, the overall expected pay factors at the RQL are approximately 36 and 41 percent for the two sample sizes. If the sample sizes remain unchanged, any attempt to lower the expected pay factors at the RQL by modifying the pay levels previously given will have the adverse effect of also lowering the expected pay factors at the AQL. It remains a matter of judgment to decide whether certain refinements should be made. The computer simulation has served its purpose by providing the information necessary to make these decisions.

The computer time for this simulation was approximately 5 s for each 100-lot run. About 3 person-days and 200 s of running time were required to develop and adjust this program.

REFERENCES

1. R. E. Shannon. Systems Simulation, the Art and Science. Prentice-Hall, 1975.
2. R. M. Weed. An Introduction to Computer Simulation. New Jersey Department of Transportation, 1976; Federal Highway Administration, U.S. Department of Transportation.
3. Sampling Procedures and Tables for Inspection by Variables for Percent Defective. U.S. Department of Defense, Military Standard 414, 1952.