This paper describes a theory of access and land development containing explicit time dependence. The theory is based on concepts drawn from earlier work on the equilibrium relation between access and land development. In the earlier work, a quantity called temperature emerged, and this quantity may be calculated for each zone as a function of both its access and its development. When these temperatures are the same everywhere, equilibrium is obtained. In the dynamic extension, temperature differences between each zone and the rest of the world act to foster changes in the development of each zone at a rate related to the magnitude of the difference and in the direction of diminishing them. The behavior aspects of the dynamic theory that are visible in its formulation and in a few experimental calculations are discussed as well as primeval computational procedures and problems.

The original formulation of an access and land development (ALD) theory (1) was derived by considering equilibrium as that condition in which development everywhere is in balance with the access pressures that act on it. Nothing was said about the evolution in time of such an equilibrium or about how an old equilibrium would deform under change toward a new equilibrium. If the impulsion toward equilibrium were very strong and the response to that impulsion quick, then the time dimension in regional development would not be of great interest: A region would always be in equilibrium, and its shape would be determined by the transportation characteristics of the moment. But on a regional scale, the process of development and redevelopment is a slow one, easily overtaken by events, and most likely a region is perpetually trying to catch up with itself rather than reposing in stately equilibrium.

Although the importance of the time element in the access-development relation was perceived quite early (as were the complementary shortcomings of a pure equilibrium approach), the time element has always been set aside. During the formation of the Chicago Area Transportation Study ALD project (2, 3), Gur (4) recognized that the dynamic theory ought to be a definite work item and later contributed both impetus and groundwork to it. However, dynamic behavior was not of concern until networks, data, analysis, and computational apparatus were put into some kind of order. It was through the use of ALD in the Chicago area that the practical problems of using the equilibrium solution became more vivid.

These problems have several forms and are all intertwined with one another. A large-scale equilibrium solution can be too farsighted: It may describe some sort of ultimate state that, although believable, may be just too ultimate to care about. Also, the equilibrium solution has a poor sense of history: When applied to any system change, it immediately wants to dissolve and then rebuild the region from scratch. Basically, the equilibrium solution does not comprehend the inertia of the existing pattern: The pattern can be preserved by means of computational imperatives, but the more these imperatives do the less the model itself does. Finally, an equilibrium solution is not generally unique; one cannot always easily know which solution is the correct one or always be certain that the current computer algorithm, can find all possible equilibria, even if the algorithm is not far from the starting point (5).

These difficulties are by no means fatal—it has long been understood that there are various ways to draw useful inferences from ALD without relying exclusively on exact regionwide equilibrium solutions—and in any case the dynamic model appears to overcome them in principle because the equilibrium theory enables the model to speculate coherently about what is to be done in each zone. Both the greatest significance and the greatest practical value of ALD may well be not that it gives strict equilibrium calculations but that it states a perfectly definite equilibrium condition and thereby identifies a regional sense of direction.

**DYNAMIC FORMULATION**

The central quantity that emerged from ALD (6) is called temperature (T). The use of this in ALD is derived from its use in thermodynamics: Equilibrium occurs when the temperature is the same everywhere. T is defined as...
\[ T = \sum \Delta R_i / \sum R_i \]  

where the summation may comprise all zones in the region, only one zone, or any cluster of zones. The attractiveness of a zone \( R \) is the sum of attractiveness due to development \( R_i \) and attractiveness due to land area \( R_i \). Thus, attractiveness is defined as

\[ R = R_i + R \]  

The access integral around a zone \( I \) is defined as

\[ I = \sum G_i R_i \]  

where \( G \) is the travel or impedance function.

This equilibrium condition is not enough to completely determine a dynamic formulation; however, it does give some information about the formulation. For example, in a closed region, the rates of change of development should go to zero when temperatures are not different and tend to go together when temperatures are different. There is a strong suggestion that the rate of temperature change at any zone should be directly related to temperature differences between the zone and the rest of the world. In thermodynamics, a volume element (or zone) is sensitive only to its contiguous neighbors. This seems an unlikely model for a human context, but it is persuasive to think that each zone senses the temperature of every other zone and that the sensation becomes weaker for more distant zones.

These considerations lead directly to the idea of a kind of temperature potential acting on each zone. This temperature potential is defined as the sum of the differences between the temperature of the zone and that of every other zone, and each difference is weighted by some function of the separation between the two zones. The travel function used in the access integrals, which has already been established as having an impact on equilibrium development, is an obvious choice for this function of separation. However, it should be remembered that the choice is merely reasonable and not strictly dictated. There is one other thing that must enter into the potential, and that is the size of each zone.

For the whole scheme to behave sensibly, a larger zone should have more impact on a smaller zone and in such a way that the effect is independent of partitioning it or the arbitrary way in which zones happen to be drawn. That is, if a number of small zones are aggregated into one large zone, the contribution to any potential of the aggregate should be the same—except for precision in the travel function—as the aggregate contribution of its constituents. This problem can be solved with both certainty and ease whether one starts with one big zone or many little ones, for temperature weighted by \( R \) is from definitions an aggregable quantity. Thus, a well-behaved temperature potential may be defined as

\[ P_i = \sum (T_j - T_i) G_{ij} \]  

The symbol \( r \) replaces \( R \), to avoid clumsy subscripting. The simplest and most straightforward way of using this potential is to assert the dynamic equation as follows:

\[ T_i = k P_i \]  

The dot, of course, signifies differentiation with respect to time, and \( k \) is some constant.

Equation 5 expresses a condition in which zone temperature changes are in the direction of environmental temperature and go to zero in equilibrium. However, this equation is not complete because it ignores the regional effects not directly related to temperature differences. These regional effects act to manage the collective ambitions of all the zones and keep them consistent with global constraints on total change. A useful hypothetical case in point is that of a region currently in perfect equilibrium but expected to grow a certain amount in the next few years. If the development in this region is to change, then some of the temperatures within it must also change. However, according to equation 5, the temperature cannot change because the rates of change are all zero.

Clearly, there should be an additive term in the potential that expresses the motion correlated with the state of change in the whole regional mass. Since there is no reason why the region should not remain in equilibrium as it changes, which implies the uniformity of \( T \), the additive term may be construed as a system constant. Thus, the dynamic equation becomes

\[ T_i = k (P_i + C) \]  

From the definition of temperature, it is easy to see that

\[ T_i = (\Delta R_i/\Delta R) - (\Delta T_i/\Delta R) \]  

where \( \rho \) is density \( R_i/\Delta R \), and, from the definition of \( I \),

\[ I = \sum G_{ij} \]  

if the travel function does not change continually over time. Combining equations 6 and 7 and applying some light algebra yield the dynamic equation in terms of rates of change of development rather than temperature:

\[ i_i = -k r_i P_i - k n C + P_i R_i/\Delta R \]  

where \( I \) is \( I^* \) with the intrazonal contribution extracted, and

\[ D_i = i_i - G_i R_i \]  

The constant \( k \), which measures sensitivity to temperature potential, is an empirical parameter, but \( C \) is exactly determined through the required auxiliary condition on change in total regional development. That is,

\[ \sum i_i = R_\Delta \]  

The value of \( C \) could also be determined from an auxiliary condition on change in regional temperature, if that were deemed an appropriate exogenous quantity.

Equation 9 represents a set of simultaneous differential equations: one equation for each zone in the system and one more for the auxiliary condition. However, this set cannot be solved by numerical methods. Even though each \( r \) cannot be written as an explicit function of time, each \( r \) can be calculated numerically over a series of time increments. At the starting point (time zero), each \( r \) and \( T \) and so forth is known, and the equations form an unexceptional simultaneous set that is linear in the unknowns: the derivatives \( r \), and the parameter \( C \). The value of each \( r \) can be obtained by ordinary means, and, since the value will not change much over a short enough time interval, it can be used to increment each corresponding \( r \):
The new time-1 \( r \) leads to a new \( T \) and, thus, to a new time-1 system of simultaneous equations that can be used to compute the time-1 \( r \). This process continues through time 2, time 3, and time \( n \).

Equation 9 is based on the essential correctness of ALD as any number of other dynamic equations might be that range from variations on the same theme to wholly different formulations. One variation worth bearing in mind is allowing the sensitivity measure (\( k \)) to be a zonal parameter rather than a constant and thus causing it perhaps to be inversely proportional to density in order to reflect a greater difficulty in inducing change in development that is already dense. However, equation 9 is probably the simplest formulation consistent with current empirical knowledge. Both working experience and sharper reasoning may impel refinement or wholesale revision, but equation 9 is a starting place.

**DYNAMIC BEHAVIOR**

The behavior of a system governed by equation 9 may be seen to some extent by inspection. First, development changes are all functions of the initial configuration. New development is a function of old development, and the existing pattern is embodied in the evolving pattern. Furthermore, because the mathematics are linear in \( r \), the rates of change are uniquely specified, and the system single-mindedly heads off in some definite direction. There is no ambiguity about which ultimate equilibrium the destiny of the region lies in. In general, that ultimate equilibrium is of less interest because intermediate points in time are now accessible, and the equilibrium is merely the asymptotic state of a closed system that probably will not stay closed.

Equation 9 also shows that more remote and less developed parts of the region tend to change more slowly, irrespective of what their final equilibrium development might be. This slow change is something of a relief, since the equilibrium solution has a pernicious habit of flooding every nook and cranny of the region with development, which can amount to a massive total even if the typical densities are quite modest. It would appear that, in the dynamic model, the equilibrium effects propagate more or less along fronts moving outward from poles of existing development rather than steep uniformly everywhere. However, it also appears that a remote or lightly developed area can be excited into rapid growth by being nucleated with a flat development of size—perhaps a major shopping center, subdivision, airport, or new town. The possibility exists, however, that an injudiciously placed nucleus could turn out to have a negative time derivative and just dwindle away.

Figure 1 shows the dynamic behavior on a very simple level. The graph shows for one zone in a two-zone system the derivative development time as a function of its own development. Each zone has an \( R \) of 1; the total \( R \) is 10 and does not change with time. The intrazonal travel function has a value of 2, and the interzonal has a value of 1. The three equilibrium solutions are 1.54, 5.00, and 8.46 for \( r_1 \), and 8.46, 5.00, and 1.54 for \( r_2 \). The derivative, which is plotted in arbitrary units on the Y-axis, passes through zero at each of the solution points through a way that the zone always moves toward an equilibrium. Once the zone is in equilibrium, it will stay in that state because a deviation in any direction results in a restoring motion. This is just as much the case for the 5, 5 equilibrium solution as for the others, which is noteworthy because this solution would never be discovered by the equilibrium algorithm unless the algorithm happened to be the initial condition. There is some reason to believe that in complicated systems there may be many equilibria that the algorithm stubbornly refuses to seek; the recognition of this possibility has been slow in coming.

Figure 1 shows discontinuities at about 2.7 and complementarity at about 7.3. In both cases, the derivative goes to \( -\infty \). These discontinuities are to be expected because, in a system that admits multiple solutions, there ought to be watershed points on one side of which a zone goes to one equilibrium and on the other side to another equilibrium. (The infinities are not an essential feature and do not generally occur in many-zone systems.) These discontinuities are well-behaved in the sense that a zone cannot approach them on its own volition and always flees from them. However, there is a clear possibility of pushing a zone, by means of flat development, from one side of the watershed to the other and switching the mode of the zone from one of decline toward a low equilibrium to one of growth toward a higher equilibrium.

Figure 2 shows the growth over time of a zone in this system. The start of this growth is shown from three different initial amounts of development. The Y-axis is development in the zone, and the X-axis is time in arbitrary units. In the first case, the zone starts out virtually undeveloped (development of 0.01), enters a phase of moderate growth, and ends quite a while later approaching its equilibrium. Both of the other cases start in the vicinity of a discontinuity where the derivatives are extremely high. But they quickly drop to more normal levels, and the effect is a sudden spurt of growth followed by a gradual approach to equilibrium. The zone starts farther from its equilibrium in the second case than in the third case and takes longer to reach equilibrium, but both the second and third cases settle down to equilibrium in less time than it takes the first case to really get moving.

Some experimental calculations of many-zone systems have been made (7), but the behavioral complexities of these systems are not easy to analyze or even to describe since experience with them is limited. The behavior of these systems generally follows along lines that can be anticipated from the more visible qualities of equation 9 and the two-zone case. However, the derivative curve for any individual zone has more system modulation in it and much less the appearance of a purely mathematical construction in the many-zone system than in the two-zone system. However, there is one phenomenon that was discovered in the course of these calculations: It is possible for a zone to enter a kind of bound state in which the zone will oscillate around a point rather than quietly come to rest on the point. This condition, meta-equilibrium, is apparently stable, and the zone cannot escape unless the rest of the system changes enough to set it free.

Meta-equilibrium occurs when a watershed discontinuity becomes reflected and a segment of a time-derivative curve of a zone looks like that shown in Figure 3. Unlike the zone in Figure 1, the zone in this region of its curve is constantly driven toward discontinuity until it finally jumps across to the other side where it is driven back toward the discontinuity again. Although straightforward numeric integration leads to a finite oscillation, in strict mathematics the amplitude should be next to zero, thereby giving the effect of a peculiar kind of equilibrium into which the zone leaps rather than gently slides. However, there is one very crucial difference: The \( r \) of a zone in the vicinity of meta-equilibrium does not go to zero, and this means, referring to equation 9, that in general every \( r \) of the other zones cannot go to zero either. Thus, the system as
a whole cannot subside into complete equilibrium so long as there is a meta-equilibrium anywhere within it. The zone must move in a way that will eventually purge or iron out the meta-equilibrium, or it must live with a systemwide meta-equilibrium of its own. The latter might take the form of a pulsating state in which the pulsations probably radiate from the meta-equilibrium site and diminish with distance from it.

What a system containing a meta-equilibrium would do and under what conditions it would do it are not known. There has been little exploration of this phenomenon to date and just as little analysis of its mathematical, practical, and physical significance. Perhaps meta-equilibrium is related in some simple way to normal equilibrium; perhaps it is not. Meta-equilibrium may be merely something that happens in the kind of artificial calculations in which it was discovered, but that should not be counted on to happen. Certainly, meta-equilibrium

Figure 1. Rate of change of development as a function of development.

Figure 2. Development as a function of time for three starting points.
depends on the presence of rather dense development.

It is assumed that meta-equilibrium occurs in systems that are intended to represent reality, then the question arises as to whether meta-equilibrium occurs in the reality that the systems are intended to represent.

There are not likely to be any data that bear on the question with clarity and yet, if meta-equilibrium is in fact characterized by pulsation, there might be some nebulous traces. People have remarked, speaking of one of the older cities, that some particular district seems to be perpetually caught up in a cycle of growth and decline.

Figure 3. Segment of a time-derivative curve for a zone.

The current computational methods used in solving dynamic systems have been ordinary and hardly deserve mention. An experimental computer program (7) has been put together that follows the straight-line course of solving the simultaneous set implied by equation 9 for each r. This program increments each r, updates all tables that depend on each r, and then solves the new simultaneous set. The incremental time interval may be any predetermined amount (the typical range has been 6 months to 2 years), or the computer may be allowed to choose an interval such that no zone undergoes more than some preset change. The simultaneous equations are solved with a library routine based on successive elimination of unknowns.

This program has performed satisfactorily but has not been used for systems of more than 75 or 80 zones. Large systems may introduce problems of core management and speed. System behavior, meta-equilibrium in particular, may require greater subtlety in the numerical methods. It may prove desirable to impose constraints on each r to reflect the fact that there can be a physical limit to how fast development can grow or, more especially, decline in any one zone. This seems impossible to do with standard simultaneous equations.

Some thought has been given to the solution of the simultaneous equations. Almost certainly, the method that would be computationally convenient and would also handle constraints with ease is a simple iteration. Starting with the arbitrary first trial values of the r's, r1 is computed from equation 2 and its first trial value is replaced with the second trial value. The same procedure is followed with r2 and so on until there is no longer a change. Unfortunately, this procedure can be guaranteed to converge only under certain conditions that are not fulfilled by this set of equations. However, a number of ways for solving a specific computational problem can usually be found. One example that might be useful is discussed below.

A fictitious variable (S) may be invented such that equation 9 may be rewritten as

$$r_i = (-k_c r_i + k_c C_i + S_i r_i)/D_i$$

This makes r (and also C and C) a function of S. However, the only values of interest are when S = 1. The r's can now be expanded in a Taylor's series around S = 0 with the understanding that the series is to be evaluated at S = 1; thus,

$$r_i(1) = r_i(0) + [r_i'(0)] + [r_i''(0)] + [r_i''(1)] + \ldots$$

In equation 14, the primes signify differentiation with respect to S. Differentiating equation 13 gives

$$r_i = (-k_c r_i + k_c C_i + S_i r_i)/D_i$$

The terms in equation 14 can immediately be written as

$$r_i(0) = -[k_c r_i(0) + k_c C(0)]/D_i$$

Since the system constraint on total development change (R) is an external constant that is independent of S, there is an auxiliary condition for each equation in equation 16:

$$\sum r_i(0) = R$$

Every term in equation 14 can now be computed directly and easily. Each r(0) is a function of only fixed system quantities and not of any other r. Once each r(0) has been computed, each r(0) follows. Also, each r(0) becomes a function only of entirely known quantities. Higher derivatives are similarly computed.

Thus, the simultaneous system has been converted to a recursive one. One virtue of a recursive system is that the whole volume of system information does not have to be maintained in core at the same time. The recursive system might also turn out to be faster than the standard simultaneous methods; however, this is difficult to judge. Because the series form gives partial information about each r in advance of a complete solution and may permit other approximate inferences as well, the series form could prove convenient for dealing with constraints and maybe even help out with other numerical problems.

Other series expansions of equation 9 and probably entirely different solution procedures can undoubtedly be developed. This series has had only a casual tryout in a few two-zone cases in which it worked, but no attention has been given to its properties of large system convergence.

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