Nature of Equilibrium in the Market for Taxi Services

Charles F. Manski and J. David Wright, School of Urban and Public Affairs, Carnegie-Mellon University

This paper reports an investigation of the mechanism by which demand and supply are equilibrated in the regulated taxi market. A simple model is developed in which the demand for taxi trips, as measured by the passenger arrival rate, is a function of a set of exogenous variables, including fares, and of the endogenous variable, taxi availability, is developed. The profitability of taxi operations is determined by exogenous factors and by an endogenous variable, taxi utilization. The number of taxi vehicles supplied is determined by the profit function and by the industrial structure. Taxi availability, which is measured by expected waiting time, and utilization, which is measured by the expected proportion of time the vehicle is occupied, are determined by both the passenger arrival rate and the number of taxis operating. The complexity of the taxi market, particularly its spatial and temporal aspects, makes considerable idealization necessary for its analysis. Even the simple model developed here does not admit a closed form solution for its equilibrium conditions, thereby constraining the work to a numerical example. The most striking feature of the model is its demonstration that in the taxi market supply generates demand and vice versa since an exogenously caused increase in the number of taxis operating will decrease waiting times and thus increase the passenger arrival rate, which will increase the taxi occupancy rate and thus increase profits and, hence, the taxi supply. This supply-demand interaction can be explosive but eventually must damp out.

Recognition of the value of low-capital-cost, flexible public travel modes as alternatives and complements to conventional public transport is growing among transportation planners and researchers. Inevitably, this recognition has focused increasing attention on the one low-capital, flexible mode currently in widespread use, i.e., the taxi.

Evidence of the desire to transform the taxi from somewhat of a residual mode into an integral part of the urban transportation system is found in a variety of experimental programs now ongoing or being proposed. For example, Washington, D.C., has recently instituted regulatory changes that attempt to increase taxi availability by permitting ride sharing during all hours of the day. In numerous places, there have been proposals to deregulate the taxi industry in one way or another.

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It is unfortunate that the regulatory changes implemented thus far and those now being contemplated have proceeded largely on the intuitions of their advocates and without the benefit of any real analysis of their consequences. This situation, which arises from the present absence of a coherent framework within which the operation of the taxi market may be understood, has prompted us to begin an effort to provide the necessary structure. This paper reports the first results of the work.

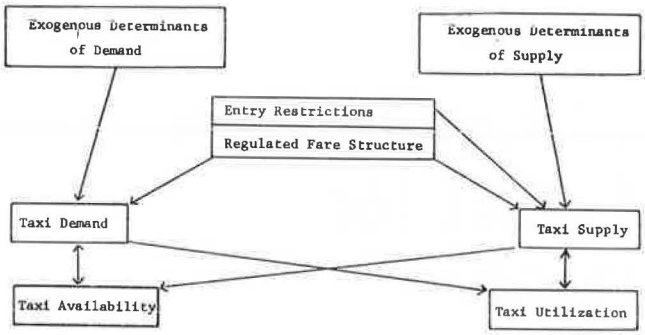
OPERATION OF THE TAXI MARKET

Many characteristics distinguish the taxi market from the idealized market of conventional economic analysis. Of these, two are of particular importance. These are the pervasiveness of regulation in this market and the inherent temporal-spatial nature of the taxi service.

Regulation of taxi fares is almost universal and regulation of entry into the industry nearly so. Ignoring welfare considerations, the presence of such regulations creates an important question about the manner in which the taxi market operates. That is, if, in this market, price, the usual short-run market-clearing variable, is exogenously fixed, how can we characterize a short-run equilibrium of the taxi market? And similarly, when entry is restricted, what is the essence of long-run equilibrium?

A second set of issues derives from the nature of a taxi service. Being one of transportation, the taxi service necessarily consumes time as well as material resources. This basic fact, coupled with an irregular pattern of customer arrivals and of travel times to destinations, introduces special concerns to both the users and suppliers of the taxi service. On the demand side, it forces potential taxi users to consider taxi availability as well as fare in making their mode-choice decisions. From the supply perspective, the taxi firm must concern itself with the rate of utilization of its vehicles as well as with trip revenues and costs. Moreover, taxi availability, through its influence on the level of taxi use, indirectly affects the vehicle utilization rate and the utilization rate, through its influence on the level of supply, in turn affects taxi availability. Hence, the

Figure 1. Relations between taxi availability and taxi utilization.



supply of and demand for taxis are intertwined in a manner that is not considered in traditional market models. The essential elements of these relationships are shown diagrammatically in Figure 1.

There have been a number of past attempts, particularly those of De Vany (1), Douglas (2), and Orr (6), to study the difficult analytical issues posed by the taxi market and to examine the market under alternative regulatory conditions. In addition, Douglas and Miller (3) and Mohring (5) have considered similar problems in their analyses of the airline market and bus routings respectively. Each of these authors has recognized in one way or another that the demand for and supply of taxi services are connected through the intervening variables of availability and utilization. However, for a simple yet subtle reason, none of these studies succeeds in obtaining a satisfying model of the demand-availability-utilization-supply relation. That reason is their common attempt to analyze a temporal problem within the inherently static framework of conventional supply-demand analysis.

In contrast, we have developed an explicitly temporal market model through the use of elementary queuing theory. In this model, the demand for taxi trips is measured by the passenger arrival rate (λ), which is a function of exogenous variables including fare and of the endogenous variable taxi availability. The profitability of taxi operations is determined by exogenous factors and by the endogenous variable taxi utilization. The number of taxi vehicles supplied (S) is then determined by the profit function and by the industrial structure. In the model, taxi availability is measured by expected waiting time [E(W)] and taxi utilization by the expected fraction of time a taxi is occupied [E(U)]. The two variables E(W) and E(U) play the market-clearing role normally assigned to price.

The complexity of the taxi market makes considerable idealization necessary for its analysis. Even the simple model developed here does not admit a closed-form solution for its equilibrium conditions. As a result, much of the discussion of the properties of the model is through a numerical example.

This work, nevertheless, offers interesting insights into the operation of the taxi market. Conceptually, the queuing framework is a far more satisfying perspective for viewing the market than that given by static demand-supply analysis. Moreover, we can, even at this early stage, offer some tentative policy-relevant findings. In particular, the analysis suggests that, at least within some range, an increase in the number of taxi vehicles operating will be beneficial to both consumers and firms.

In what follows, the model is presented, the taxi market equilibrium in a semirealistic example is determined, and the findings are discussed.

MODEL OF A TAXI MARKET

Assumptions

1. Trip origination—All taxi trips are initiated at a single origin at which a cabstand is located. At this stand, taxis queue for passengers and passengers for taxis.
2. Demand for trips—Passenger arrivals are Poisson distributed with a mean arrival rate of λ. Where $\lambda > 0$, λ is a linear function of a vector of exogenous variables including fares and of the endogenous expected waiting time. Otherwise, $\lambda = 0$.
3. Trip times—Travel times to passengers' destinations are exponentially distributed with a mean of $1/\mu$, where μ is exogenously determined. The return to the cab stand of a vacant taxi takes the same time as the outbound trip. Loading and unloading times are insignificant.
4. Taxi cost function—Taxi operations have constant returns to scale, both in fleet size and in the length of time a given taxi operates daily. Costs for vacant and occupied vehicles are identical. All taxi vehicles are identical with respect to costs. An infinitely elastic supply of taxi drivers exists.
5. Fare structure—The regulated fare structure is such that trip prices are proportional to travel times.
6. Industrial structure—Market equilibrium under three alternative regulated industrial structures will be examined. These are (a) monopoly franchise, (b) competitive firms restricted in number by medallion limitations, and (c) competitive firms with free entry. Firms are always assumed to be expected-profit maximizers.

Queuing Process

The above system is a queuing process with servers in parallel, a single queue with Poisson arrivals, exponentially distributed service times, first in-first out service order, and unlimited queuing capacity. Such processes have been extensively studied and the results are well-known.

Let n be the number of passengers in the system, both those waiting for a taxi and those being served. (For purposes of the queuing model, a passenger will be considered to be being served until his taxi returns to the cabstand. This convention simplifies the presentation.) Let S be the number of taxis operating. Let P be the probability that all taxis are in service, that is, $P \equiv \text{Prob}(n \geq S)$. Let $\rho \equiv \lambda/\mu$.

Then, a steady-state queue distribution will exist so long as $\rho/S < 1$ and E(W) will be as follows:

$$E(W) = P/(\mu S - \lambda) \tag{1}$$

where $P = \left[1 + (S!/\rho^S)(1 - \rho/S)\sum_{j=0}^{S-1} \rho^j/j!\right]^{-1}$. E(U) will be as follows:

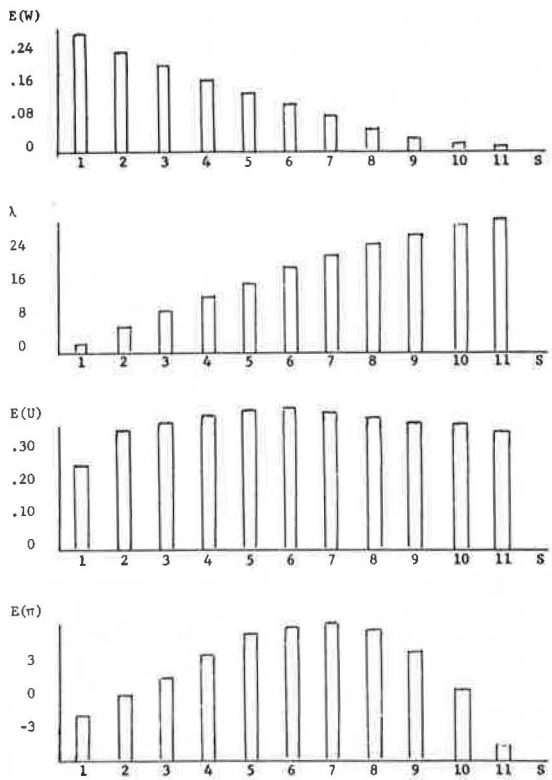
$$E(U) = \rho/2S \tag{2}$$

The factor two in the denominator of this equation arises because the taxi, in this model, is occupied only on the outbound journey but is in service for the entire round trip.

Behavior of Consumers and Firms

By the second assumption, consumer behavior is fully summarized by the following statement: For any given values for the regulated fare and other exogenous vari-

Figure 2. Example of a solution of the system for S = 1, . . ., 11.



ables, there exist α_1, $\alpha_2 > 0$ such that

$$\lambda = \alpha_1 - \alpha_2 E(W) \quad [\text{if } \alpha_1 - \alpha_2 E(W) > 0]$$
$$= 0 \quad (\text{otherwise}) \tag{3}$$

This form for the utility guarantees that, for a given S, a steady-state queue distribution exists. To see this, observe that $\rho/S \geq 1 \rightarrow E(W) = \infty \rightarrow \lambda = 0 \rightarrow \rho/S = 0$. Hence, there is a contradiction and ρ/S must be less than one. Thus, the condition $\rho/S < 1$ is necessary and sufficient for a steady state to exist. This argument only ensures a steady state conditioned on S. Proof that a steady state continues to exist when S is allowed to vary in accordance with taxi firms' behaviors is not available. In the numerical example, a steady state unconditional on S is obtained.

Firm behavior, which determines S, depends on the industrial structure assumed. Let F be the fare for a trip of unit time length and let C be the operating and prorated capital cost per taxi per unit time. Then the expected industry profits per unit time [$E(\pi)$] are

$$E(\pi) = [E(U)F - C]S \tag{4}$$

Under a monopoly system, the single expected-profit maximizing firm will select a fleet size (Sm) as follows:

$$Sm = \{\text{any } S\text{: } [E(U)F - C]S \geq [E(U)F - C]S', \text{ all } S' \geq 0\} \tag{5a}$$

A set of Sm values may satisfy these conditions and E(U) is, by Equations 1, 2, and 3, implicitly a function of S. In a competitive free-entry market, the resulting supply (Sc) will satisfy the following condition:

$$Sc = \{\text{any } S\text{: } [E(U)F - C]S \geq 0 \text{ and } [E(U)F - C](S + 1) \leq 0\} \tag{5b}$$

Equation 5b essentially expresses the zero profit criterion for competitive equilibrium but recognizes that, since S is integer valued, equilibrium profits may be positive. Again, Sc may be set valued. The equilibrium conditions expressed by Equation 5b may be said to imply a certain myopia on the part of firms. In particular, the conditions state that new entry will cease whenever the addition of a single vehicle would be unprofitable. It is possible, and indeed occurs in the numerical example, that the addition of one taxi is unprofitable but the addition of two taxis is profitable. Under a regulatory structure allowing only owner-drivers and no fleets, the conditions of Equation 5b are fully realistic. In a competitive market constrained to a maximum number of licenses (L), the supply (S_1) will be

$$S_1 = [\text{any } S \leq L\text{: } S \in Sc] \cup L \text{ if } [E(U)F - C]S \geq 0 \quad (S = L)$$
$$= [\text{any } S \leq L\text{: } S \in Sc] \quad (\text{otherwise}) \tag{5c}$$

Market Equilibrium

An equilibrium in the market for taxi services is a set of values for E(W), E(U), λ, and S that simultaneously satisfies Equations 1, 2, 3, and, depending on the industrial structure, 5a, 5b, or 5c. The complexity of this system, particularly Equation 1 and the set Equations 5a, 5b, and 5c, makes analytical efforts to discover its properties difficult. Therefore, this paper is largely limited to an example in which the system is solved for given values of the parameters μ, F, C, α_1, and α_2.

Solution of the System—An Example

Assume the following values for the parameters of the model system: $1/\mu = 0.25$ h/passenger, F = \$20/passenger-h/taxi, C = \$7/h/taxi, $\alpha_1 = 30$ passengers/h, and $\alpha_2 = 100$ passengers/h^2. For any given value of S, the subsystem of Equations 1, 2, 3, and 4 may be solved for E(W), λ, E(U), and $E(\pi)$ respectively. Under the assumed values for the system parameters, each such solution exists and is unique. The solutions for S = 1, . . ., 11 are shown numerically below and graphically in Figure 2.

S (taxis)	E(W) (h)	λ (passengers/h)	E(U) (passenger-h/h)	$E(\pi)$ (\$/h)
1	0.28	2.1	0.26	-1.8
2	0.24	5.6	0.35	0
3	0.21	9.1	0.38	1.8
4	0.17	12.6	0.39	3.5
5	0.14	16.0	0.40	5.0
6	0.11	19.2	0.40	6.0
7	0.08	22.1	0.39	6.3
8	0.05	24.6	0.38	5.5
9	0.03	26.6	0.37	3.5
10	0.02	28.1	0.35	0.3
11	0.01	29.0	0.33	-4.5

The equilibrium fleet size (S), and hence the equilibrium E(W), λ, and E(U), are determined by the industrial structure through Equations 5a, 5b, and 5c. The various possible equilibria are

1. Monopoly franchise: Sm = 7;
2. Competitive free entry: Sc = 0 or 10; and
3. Competitive licensing: $S_1 = 0$ if $L \leq 1$; = 0 or L if $1 < L \leq 10$; = 0 or 10 if $L > 10$.

It is of some interest to observe how the value of a medallion changes with L. Let T be the number of operating hours in a year and r be the discount rate. If L is the equilibrium number of taxis under competitive licensing, then the expected discounted profit stream to a taxi owner, i.e., the value of a medallion, is

$$V = E(\pi)T/Lr = [E(U)F - C]T/r \tag{6}$$

and V is maximized at the point where the expected profits per taxi (and the utilization rate) are maximized. In the example, if it is assumed that T = 4000 h and r = 0.10, then the medallion values for L = 2, . . . , 11 are as shown below.

L	V($)	L	V($)	L	V($)
2	0	6	40 000	9	16 000
3	24 000	7	36 000	10	1 200
4	35 000	8	28 000	11	0
5	40 000				

Discussion of Results

The most striking feature of the above example is the concomitant increases in taxi availability, utilization, and profits as the number of taxis increases from zero to six. This seemingly curious result is a manifestation of an important property of many queuing processes. That is, simultaneous increases in the arrival rate and the number of servers will, if not too disparate, both decrease the customer waiting time and increase the server utilization rate.

To see how this may occur, recall the equations for expected waiting time and taxi utilization (Equations 1 and 2). Let S and λ increase proportionately so that the traffic intensity ($\rho/S = \lambda/\mu S$) remains constant. It is apparent from Equation 2 that E(U) is unchanged as S and λ increase in this manner and can be seen from Equation 1 that E(W) must eventually decrease. This follows because the term P in the numerator of Equation 1 is bounded from above by ρ/S while the term ($\mu S - \lambda$) in the denominator becomes larger as S and λ proportionately increase. {To demonstrate that P is bounded from above by ρ/S, observe that

$$\sum_{j=0}^{S-1} \rho^j/j! \geq \rho^{S-1}/(S-1)! \tag{7}$$

which implies that

$$P \leq [1 + (S!/\rho^S)(1-\rho/S)\rho^{S-1}/(S-1)!]^{-1} = [1 + (S/\rho) - 1]^{-1} = \rho/S \tag{8}$$

This, however, states only that E(W) will eventually decrease. It does not determine how P changes as S and λ increase proportionately. This ambiguity prevents a stronger statement about the behavior of E(W).}

The above result and the continuity of Equations 1 and 2 in λ and S imply that, within some regions, we can simultaneously achieve a decrease in E(W) and an increase in E(U) by increasing λ and S appropriately. In particular, if λ increases by a slightly larger factor than S, so that the traffic intensity increases, then E(U) will necessarily increase and, as long as λ does not increase too quickly relative to S, E(W) will still decrease. It is a phenomenon of this kind that the example manifests. A similar phenomenon was noted by Dreze (4) in an abstract discussion of markets operating under stochastic demand conditions.

Another interesting aspect of the example is that the competitive, free-entry industrial structure admits two equilibria, 0 and 10 taxis. Clearly, equilibrium at 0 is inferior from a welfare perspective, but it can occur if firms are myopic or are prevented from owning fleets. Where there exists a single firm, as in the monopoly franchise system, the interdependencies that make the utilization rate of a given taxi dependent on the number of taxis operating are internalized so that 0 is no longer a plausible equilibrium.

In constructing the above example, an attempt was made to assume realistic values for the input parameters μ, F, C, α_1, and α_2. It is encouraging that the resultant values for the endogenous variables E(W), λ, E(U), S, and V also appear to be realistic. It would, nevertheless, be inappropriate to view the example as more than an illustration.

FUTURE RESEARCH

The nature of equilibrium in the market for taxi services clearly differs from that of traditional economic markets. This notwithstanding, the operation of the taxi market is amenable to analysis. The taxi market model developed here is obviously only a first step. Its lack of realism reflects the early stage of the work rather than any inherent limitations in the approach. However, further consideration of some of the assumptions will be necessary to place the model in perspective and to demonstrate its ultimate policy relevance.

1. The present characterization of taxi demand, through the equation $\lambda = \alpha_1 - \alpha_2 E(W)$, is restrictive in a number of important ways. The least significant is its linearity in expected waiting time [E(W)]. The assumption that demand depends only on the expected waiting time and not also on higher moments of the waiting time distribution is more important, and the introduction of variance of waiting time as a determinant of demand would make the specification more realistic. The implicit assumption that all individual consumers possess identical Poisson arrival rate distributions is another strong restriction, but if the individual arrival rates are each Poisson but not identical, then the aggregate arrival rate distribution will not, in general, have the Poisson form. The assumption that the distribution of trip times, as captured by the average trip time ($1/\mu$), is independent of E(W) may not be valid Since waiting time represents a smaller fraction of total travel time for long trips than for short ones $1/\mu$ may increase with E(W).

2. The regulatory power to fix both the form and level of taxi fares strongly shapes the characteristics of present taxi markets, but the work to date has not explicitly considered the consequences of alternative fare levels and forms. Among the numerous such issues the most important are the questions of assessing the consequences of setting a uniform regulated fare at alternative levels so that individuals having a high value of time could presumably choose to pay more and wait less than customers having a low value of time; of examination of the manner in which the fare structure, together with the composition of demand, determines the waiting time distribution at each fare level; and of analysis of the operation of taxi markets in the absence of fare regulation.

3. The work thus far has implicitly assumed that regulatory policies affect the operation of the taxi market but not vice versa. For short-run analysis, this is a reasonable assumption, but for long-run analysis, regulatory behavior should be considered to be part of the taxi system rather than external to it.

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