THE EFFECT OF OPERATING CONDITIONS ON THE SKID PERFORMANCE OF TIRES

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This paper presents the results of a test program on the skid performance of tires. The program was conducted under Contract DOT-HS-205-2-238 from the Department of Transportation, National Highway Traffic Safety Administration. Effects of the following variables were evaluated: tire load; tire inflation pressure, road speed and surface water depth. An ASTM type control tire and a production bias belted tire were used in the tests. Both peak and locked wheel coefficients of friction were measured using two skid trailers. All tests were run on a wet surface with either the skid trailer's onboard watering system spreading 0.51 mm (0.02 in.) of water on the test track or sprinklers flooding the track surface to a depth of 3.05 mm (0.12 in). Load, speed, and inflation were varied over a range of three values. Two surfaces were used, SN 35 ± 5 concrete and SN 60 ± 5 asphalt. Of the variables tested, load and inflation had the least effect on traction over the range which they were varied. Speed and water depth interacted, causing a significant decrease in skid coefficients with an increase in both parameters.

During a simple stop on pavement a number of forces and moments are acting on a vehicle. All these forces must act through the tires to provide the proper reaction. In only a very few of the deceleration maneuvers are the tires used to the limits of their tractive ability. On wet pavement in particular these limits can be reached.

The purpose of this paper is to investigate the effect of certain operating parameters on peak and locked-wheel coefficients of friction on wet pavement. The data base used for this analysis was gathered under the Department of Transportation Contract DOT-HS-205-2-238  $(\underline{1})$ . Approximately 30,000 skids were produced at the Goodyear Proving Grounds in San Angelo, Texas, during the summer and fall of 1972.

A pair of two-wheeled towed trailers (Figure 1) was used to measure the vertical and horizontal forces acting on a tire during a brake application. These trailers evolved from earlier torque measuring trailers used primarily to measure pavement Skid Numbers. They have become valuable tire testing tools. Actual vehicle tests provide meaningful data about vehicle and tire system behavior and can

be useful in comparing tires under service conditions. Still, it is difficult to obtain absolute tire data (2) from vehicle tests. The major disadvantages of vehicle tests are (1) the difference in speed at which peak and slide occur, (2) effects of the vehicle, (3) the unavailability of reading traction coefficients directly, and (4) the large number of vehicles needed to cover the range of tire sizes.

Figure 1. Two trailers used to obtain skid test data.



The industry standard established for skid trailer testing is SAE J345a(3). DOT specified that Goodyear's data collection procedures comply with the standard.

The data gathered provide an excellent opportunity to study the parameters of load, speed, inflation pressure, and water depth on a concrete surface, SN  $35\pm5$ , and on an asphaltic concrete pavement, SN  $60\pm5$ . All testing was done on wet pavement. The original test was run using two types of bias-belted tires: therefore, our discussion will be limited to these two groups of tires only. The tires used are shown in Figure 2. The Group B test tire (G78-15 Goodyear Custom Power Cushion Polyglas) is shown on the left and the Group A control tire (G78-15 proposed ASTM) is shown on the right.

The control tire should not be confused with either the 7.50-14 E  $249(\underline{4})$  ASTM tire or the G78-15 E  $501(\underline{5})$  ASTM tire. The Group A tire used here was introduced to replace the E 249 ASTM tire and was an interim tire that aided in the develop-

Figure 2. Goodyear (Group B) and ASTM (Group A) test tires.



Figure 3. Typical  $\mu$ -slip curve.

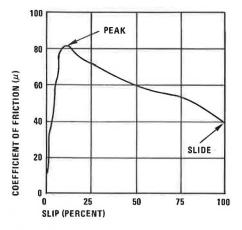


Figure 4. Test program conditions and variables.

ment of the E 501 ASTM tire. The E 501 ASTM tire was adopted as a replacement for the E 249 in 1974. The correlation between these tires has been well established (6). The Group A tire used for this program would probably exhibit wet traction properties similar to either the E 249 or the E501.

The two trailers used in this program were of similar design and function. As outlined in both the Florida Skid Trailer Correlation of 1967(7) and the Highway Research Report on calibration techniques(8), the need for standardized design, accuracy in calibration, effective watering systems, and uniform operating techniques is very important. Every effort was made to reduce any variations between units. High-speed movies were made to insure that the onboard watering systems were functioning properly. Throughout the testing, both trailers provided similar results. To insure that the trailers did not distort the data, five of the ten tires tested under each condition were run on separate trailers when possible.

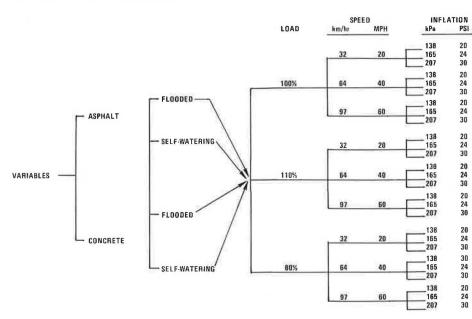
The trailers provided the information needed to analyze tire traction. Fundamental to the understanding of traction is the  $\mu\text{-slip}$  curve. Coefficient of friction ( $\mu$ ) as used in this discussion is defined as follows:

$$\mu = \frac{\text{longitudinal force}}{\text{vertical load}} \times 100. \tag{1}$$

It should be understood that the value of  $\mu$  being used here is 100 times the classic definition of coefficient of friction. This value was used to be consistent with the Skid Number notation commonly used when discussing ASTM E274 skid results. Muslip curves were developed for all test conditions. A typical curve is shown in Figure 3. Peak is the maximum coefficient obtained at low rates of slip. Slide is the coefficient of friction during lockedwheel condition.

## Test Organization

The test program outlined in the DOT contract consisted of two major parts. The first portion was designed to provide data to evaluate the repeatability of the two-wheeled towed trailer methodology. The second part provided the data analyzed in this paper. As shown in Figure 4, three levels of load,



speed, and inflation pressure as well as two levels of water depth and surface type were evaluated for each of the tire groups. The Tire and Rim Association rated load at 165 kPa (24 psi) is 626 kg (1380 lb). This value was used as the 100% load level. Ten tires of each group were tested at each of the 108 test conditions.

#### Discussion of Results

The data from each of the 27 combinations of load, speed, and inflation pressure on each surface and water depth condition were analyzed by a multiple regression program. The first step was to use a linear fit for each of the variables. The results from this phase were encouraging, but it appeared that speed may be a non-linear factor affecting the coefficient of friction. Attention was then concentrated on finding the best manner in which to express changes in skid coefficient as speed varied.

The relationship between peak and slide was also investigated to determine if peak coefficients of friction could be estimated by using the measured locked-wheel results. The relationship between the Group A and Group B tires was of interest because

Table 1. Coefficients of friction - self-watering tests on concrete.

LOAD	SPEED	INFLATION PRESSURES	GROUP A		GROUP B	
%	(km/hr)	(kPa)	PEAK	SLIDE	PEAK	SLIDE
		138	76.9	36.8	80.6	37.5
	32	165	79.6	37.4	81.1	38.0
		207	77.9	36.3	80.2	38.4
		138	77.3	34.5	80.4	35.1
100	64	165	80.8	34.1	82.8	34.7
		207	81.6	34.5	84.6	36.1
		138	73.0	30.2	80.1	31.9
	97	165	77.3	28.7	81.7	30.1
		207	77.8	28.2	80.9	30.9
		138	76.4	38.4	78.5	38.9
0.	32	165	76.2	37.9	78.4	38 R
		207	77.4	37.4	80.2	38.2
		138	76.7	34.0	80.3	35.4
110	64	165	79.6	34.5	81.7	35.6
		207	76.9	33.6	80.2	35.2
		138	75.2	31.8	77.4	32.4
Į	97	165	75.5	30.6	78.3	32.1
		207	76.3	31.4	81.1	33.6
		138	81.8	39.2	81.4	39.7
-	32	165	84.5	39.0	85.0	39.7
		207	83.8	38.1	86.9	38.6
		138	76.2	34.8	78.3	35.4
80	64	165	81.5	34.5	82.7	35.7
		207	79.7	33.6	82.6	34.5
		138	74.0	32.3	77.9	33.6
	97	165	76.7	31.2	79.6	31.9
		207	77.5	29.8	80.2	31.6

CONVERSION: 1 kPa = 0.145 psi 1 km/hr = 0.62 mph of the common practice of using a "control tire." The purpose of the control tire is to monitor uncontrollable variables, then adjust the test tire results to account for any deviations. Let us first examine the resuls from the self-watering test phase.

### Self-Watering Tests

The self-watering phase was run using the onboard watering system on each trailer to create the desired water depth of 0.51 mm (0.02 in.). High-speed films were taken of the trailer traveling over a glass plate embedded in the road. Measurements of the wetted width of water were taken with a film analyzer at each speed. Since the width was not a linear function, the flow control valve could be adjusted to provide the proper flow rate at each speed. This adjustment seemed to contribute favorably to providing the proper water depth for this phase of testing.

The results of the self-watering testing on asphalt and concrete are given in Tables 1 and 2. As given the coefficients are higher on asphalt than concrete. The concrete had a Skid Number of SN 35  $\pm$  5 as compared with the asphalt, which was SN 60

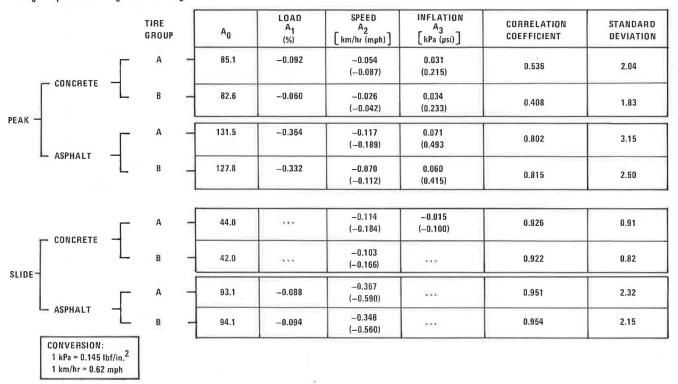
Table 2. Coefficients of friction - self-watering tests on asphalt

LOAD	SPEED	INFLATION PRESSURES	GROUP A		GROUP B	
%	(km/hr)	(kPa)	PEAK	SLIDE	PEAK	SLIDE
	Ī	138	103.6	75.6	103.3	76.2
	32	165	102.5	74.8	104.0	76.2
	L	207	107.7	74.5	106.6	76.4
		138	101.9	60.9	102.6	62.1
100	64	165	104.9	60.8	104.1	63.0
		207	107.3	61.3	105.9	63.5
		138	95.6	49.4	98.2	51.5
	97	165	97.9	50.0	99.3	51.4
		207	103.8	53.8	104.3	55.0
	ſ	139	97.5	69.8	96.1	69.9
	32	165	96.8	70.0	96.1	69.7
		207	99.1	67.8	99.7	70.5
		138	90.6	57.4	93.0	58.3
110	64	165	94.7	56.3	94.6	58.8
	L	207	97.0	57.1	97.4	60.8
	[	138	84.4	49.7	89.2	51.2
	97	165	89.3	49.7	93.7	52.7
	L	207	95.6	49.7	96.4	51.9
	ſ	138	109.0	75.2	107.1	75.4
	32	165	109.8	75.7	108.0	76.3
		207	111.8	75.3	109.3	76.7
	[	138	105.9	63.0	105.6	62.7
80	64	165	106.9	60.8	107.3	64.6
	L	207	106.3	62.5	108.2	65.9
		138	98.0	47.8	99.6	50.4
	97	165	102.7	49.0	104.8	51.2
		207	102.4	47.2	104.5	50.5

CONVERSION: 1 kPa = 0.145 psi 1 km/hr = 0.62 mph

Table 3. Regression constants for self-watering tests.

 $\mu$  =  $A_0$  +  $A_1$  x LOAD +  $A_2$  x SPEED +  $A_3$  x INFLATION



 $\pm$  5. The assumption should not be made, however, that the coefficients on asphalt are always higher than concrete, since asphalt can also have a low Skid Number.

Results of the regression analysis are shown in Table 3. Only parameters that proved to be sound using the "F-test" (90% confidence level) are represented. The results of the curve fit for peak are less favorable than those for slide. The correlation coefficients are lower and the standard deviations are higher for the peak equations than those for the slide equations. The calculated equations generally fit the test data satisfactorily.

The peak results on concrete were less conclusive than those on asphalt. To determine the usefulness of the regression line for concrete, the variation of the 27 peak data points about the mean was calculated for each tire. The standard deviation was 2.81 for Group A and 2.23 for Group B. As shown in Table 3, there is a reduction in the spread of the data when it is fit to the regression line; but it is not as much as would be desired. Correspondingly, the standard deviation in peak data measured on the asphalt surface was reduced by more than one-half when it was fit to the regression equation.

From the equations it appears that peak is a function of load, speed, and inflation for these two groups of tires and surfaces. Figure 5 and 6 show the plotted equations for  $\mu$  versus speed at a constant load of 100% and an inflation pressure of 165 kPa (24 psi). Speed has less influence on peak than it does on slide. When load increases from 80% to 110%, the peak coefficient on concrete is reduced by 1.8 for the Group B tire and by 2.8 for the Group A tire. A similar reduction on asphalt would be as much as 10.0 for Group B and 10.9 for Group A. The effect of load on peak is

shown in Figure 7. Increasing the inflation pressure from 138 kPa (20 psi) to 207 kPa (30 psi) would cause an increase in peak  $\mu$  of approximately 2.25 on concrete and 4.5 on asphalt. The effect of inflation on peak is shown in Figure 8.

Speed seems to be the main parameter affecting the traction developed during the locked-wheel test phase. Only one tire surface combination was sensitive to changes of inflation, but this effect can be considered minor. The effect of load variations on slide was evident on the concrete surface only, and likewise appeared to be immaterial. A 30% increase in load would cause only about a 3% reduction in slide.

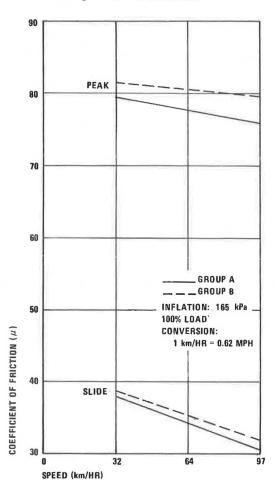
One of the interesting aspects brought out by this analysis is that asphalt in general was more sensitive than concrete to changes in load, speed, and inflation. The variation about the regression lines for both peak and slide was larger on asphalt than concrete.

Peak is a transient measurement; slide is a steady state measurement. Slide is the coefficient averaged over a one second period of time. Because of the trailer's suspension, the vertical load during a test varies substantially. Once brake force is applied to the tire, the load decreases. The strong load dependence shown here could be a contributing factor to the large variations that appear in peak measurements.

### Flooded Tests

The flooded portion of the testing was done on the same surfaces used for the self-watering phase. The water was applied to the surface by means of a pipe running along the edges of the test track. With this method, the depth of water varied, largely

Figure 5. Coefficient of friction versus speed - self-watering tests on concrete.



because of surface irregularities. The average water film thickness, measured with an electronic depth gauge, was 3.05 mm (0.12 in.). Seldom is a water film of this depth encountered in normal driving.

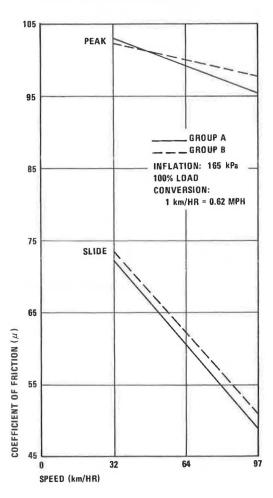
The results of the flooded test phase are given in Tables 4 and 5. As with the self-watering phase, the results on the asphalt surface were higher than the results on concrete.

Table 6 summarizes the multiple regression analysis. The equations describe the peak and slide data accurately. All the correlation coefficients approach unity. Both the first- and second-order representations of speed were used. With two exceptions the second-order equation for speed gave the best fit. For these two cases the linear speed portion of the equation was not statistically significant. In those two cases, the equation utilizing speed squared provided the best results. Speed that was varied from 32 km per hour (20 mph) to 97 km per hour (60 mph) had the largest effect on  $\mu$ . Figures 9 and 10 illustrate this relationship.

On concrete the effect of load on peak is greater for the flooded surface than it was for the self-watering surface. The effect of load could not be determined for the flooded asphalt surface. Changes in load had only a minor effect on slide.

In comparison to the effects of load and speed, changes in inflation cause only a minor effect on either the peak or slide coefficient of friction.

Figure 6. Coefficient of friction versus speed - self-watering tests on asphalt.



The differences between surface sensitivity are not as evident for flooded conditions as they are for self-watered.

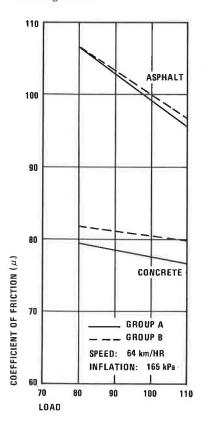
Group B as a Function of Group A

Many times when conducting tire tests, one type of tire will be used to monitor the effects of non-controlled variables. Such variables may be temperature, surface wear, or equipment and procedural differences between testers. Generally, the control tire will have about the same tractive ability and will be run under the same conditions as the test tires. The primary objective of this analysis was to investigate the relationship between Group A and Group B tires when the parameters of load, speed, and inflation were varied. A linear equation of the following form was used:

Coefficient B = 
$$A_0 + A_1 \times Coefficient A$$
. (2)

The results in Table 7 are somewhat confusing. The peak results on the concrete surface have a very low correlation coefficient in comparison with the other conditions. The standard deviations are all very low except for the flooded peak results, which are about four times as large as the rest. Graphs

Figure 7. Peak coefficient versus load - self-watering tests.



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Figure 8. Peak coefficient versus inflation - self-watering tests.

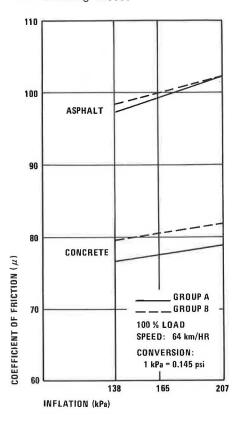


Table 4. Coefficients of friction - flooded surface tests on concrete.

LOAD	SPEED	INFLATION PRESSURES	GRO	DUP A	GRO	OUP B
%	(km/hr)	(kPa)	PEAK	SLIDE	PEAK	SLIDE
		138	79.4	33.4	73.0	32.0
	32	165	79.3	34.2	74.6	33.0
ſ		207	81.6	34.3	77.9	33.4
- 1		138	66.2	23.3	56.7	22.0
100	64	165	67.3	25.0	59.6	24.2
		207	67.5	25.6	61.9	25.3
		138	24.4	8.1	24.8	9.0
	97	165	26.2	9.6	22.8	10.2
•		207	29.9	11.3	25.4	11.9
		138	88.5	34.2	81.9	32.9
	32	165	88.2	34.3	82.0	33.7
		207	96.2	34.3	82.2	34.4
		138	61.7	20.9	45.2	19.4
110	64	165	68.6	23.2	53.4	22.3
		207	73.5	24.3	60.1	23.9
		138	27.4	7.6	26.7	8.4
	97	165	26.5	7.4	25.4	8.4
-		207	30.2	11.0	25.5	11.2
		138	87.0	35.9	80.7	34.3
100	32	165	87.3	36.9	81.6	35.8
		207	83.8	35.8	79.9	35.4
80		138	80.5	27.2	66.6	24.1
	64	165	79.6	28.1	66.3	25.2
		207	82,1	27.8	76.6	25.4
		138	27.0	7.9	28.7	8.3
	97	165	26.1	9.0	26.8	9.9
•		207	36.2	11.4	29.6	12.0

CONVERSION: 1 kPa = 0.145 psi 1 km/hr = 0.62 mph

of the equations are displayed in Figures 11 and 12. Note that the slopes of the slide curves for the self-watering and both the peak and slide curves for the flooded results are of the same magnitude. The peak curves for the self-watering results are different from the others, because the intercepts do not approach zero and the slope of the curve is comparatively lower.

The common practice is to use a one-to-one relationship for adjusting data. With the exception of peak self-watering, this practice seems reasonable for this pair of tires. However, that adjusting should be done only over small ranges. The accuracy of the peak adjustments on flooded surfaces is questionable.

# Peak as a Function of Slide

It would be useful to be able to accurately predict peak traction coefficients by using the slide results. Possible application might be (1) correcting peak tire data for changes in the surface's skid number, (2) obtaining peak on torque trailers (cannot currently be done due to inertial

Table 5. Coefficients of friction - flooded surface on asphalt.

LOAD	SPEED	INFLATION PRESSURES	GRO	DUP A	GRO	UP B
%	(km/hr)	(kPa)	PEAK	SLIDE	PEAK	SLIDE
		138	98.2	51.3	90.6	49.2
	32	163	102.1	50.3	93.8	49.2
		207	106.4	52.1	98.9	51.4
		138	83.5	42.1	68.7	40.5
100	64	165	84,9	41.0	72.9	40.1
		207	86.3	41.0	72.6	40.0
		138	24.9	16.4	23.6	16.2
	97	165	21.4	16.7	21.7	17.3
		207	24.3	20,2	22.6	20.5
		138	95.7	48.2	92.6	48.0
	32	165	96.8	47.9	92.6	48.6
		207	98.4	47.6	94.2	48.7
		138	78.0	39.2	66.8	37.8
110	64	165	80.1	38.0	75.7	39.0
		207	77.9	39.3	67.6	38.8
		138	23.1	17.6	23.2	17.6
	97	165	19.0	15.3	19.9	17.2
		207	21.9	17.5	20.9	18.5
		138	96.2	49.4	92.5	50.6
	32	165	91.3	48.9	90.8	49.9
		207	96.4	47.1	91.7	48.3
		138	83.4	40.1	75.8	39.4
80	64	165	78.6	40.0	66.4	40.5
		207	78.6	40.6	66.6	41.4
		138	27.3	18.6	28.2	20.2
	97	165	29.9	20.1	29.2	21.0
		207	33.0	20.4	26.8	21.9

CONVERSION
1 kPa = 0.145 psi
1 km/hr = 0.62 mph

forces), (3) analyzing vehicle skid results, and (4) computer modeling of vehicle stopping. Results of a linear regression of the data base are shown in Table 8. The equation of the form

$$Peak = A_0 + A_1 \times slide$$
 (3)

was used for the linear regression analysis.

The correlation between peak and slide for the self-watering tests is very poor (shaded portion of Table 8). This should prevent the use of a set peak-slide relationship at low water depth conditions. Correlation for the flooded conditions appears very good. This is probably due to the large decrease in both peak and slide traction as speed increases. The similarities in the results between the Group A and Group B tires as shown in

Figure 9. Coefficient of friction versus speed - flooded concrete tests.

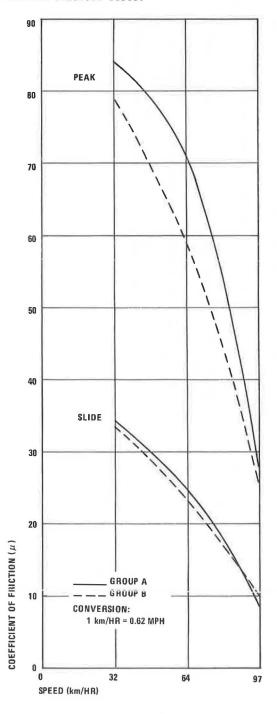
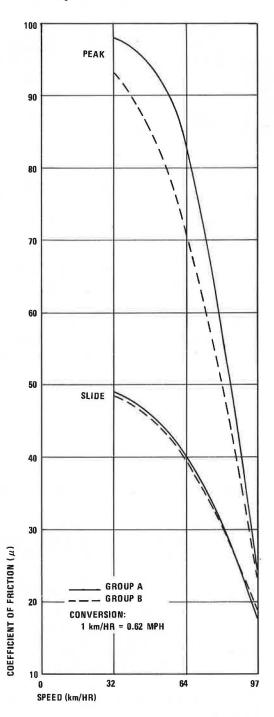


Figure 13 are very encouraging. The only major difference is in the Y intercept, A<sub>0</sub>. From these results, it can be concluded that for a surface with high water depths [approximately 3.05 mm (0.12 inch)], the slope of the regression line for peak as a function of slide is approximately 2.28. Therefore, the only apparent effect of varying the surface or the tires used would result in a change in the Y intercept, since the slopes of the four curves are similar.

Figure 10. Coefficient of friction versus speed  ${\sf -}$  flooded asphalt tests.



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Figure 11. Group B versus Group A results - self-watering surface.

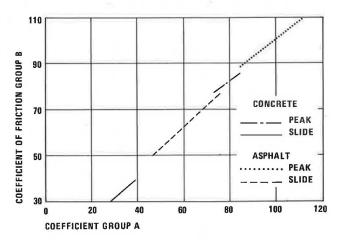


Figure 12. Group B versus Group A results - flooded surface.

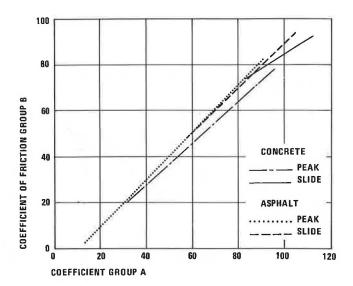
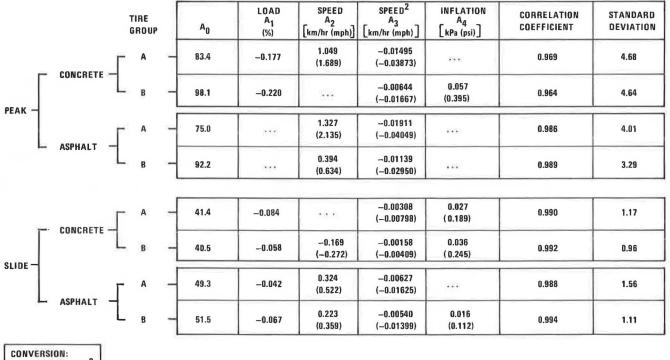


Table 6. Regression constants for flooded surface tests.

$$\mu$$
 = A<sub>0</sub> + A<sub>1</sub> × LOAD + A<sub>2</sub> × SPEED + A<sub>3</sub> × SPEED<sup>2</sup> + A<sub>4</sub> × INFLATION



CONVERSION: 1 kPa = 0.145 lbf/in.<sup>2</sup> 1 km/hr = 0.62 mph

Table 7. Regression constants for coefficient B versus coefficient A.

# COEFFICIENT B = A<sub>0</sub> + A<sub>1</sub> x COEFFICIENT A

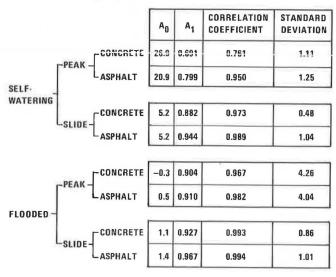
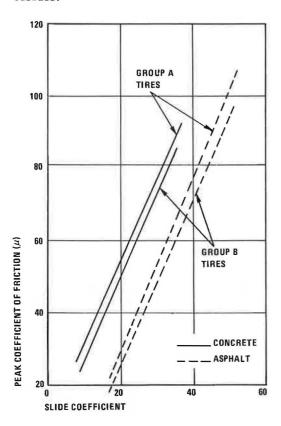


Table 8. Regression constants for peak versus slide.

PEAK = A0 + A1 x SLIDE

		TIRE Group	A <sub>0</sub>	A <sub>1</sub>	CORRELATION COEFFICIENT	STANDARD DEVIATION
	001100000	GROUP A	62.8	0.447	0.261	2.47
SELF	CONCRETE	GROUP B	72.8	0.226	0.085	2.17
WATERING	LASPHALT	GROUP A		0.383 0.282		5.52 4.83
		GROUP A	9.5	2.259	0.957	5.31
	CONCRETE	GROUPB	5.2	2.237	0.962	4.60
LOODED -		GROUP A	17.0	2.375	0.987	3.68
- 1	- ASPHALT	GROUP B	18.5	2.249	0.985	3.64

Figure 13. Peak friction versus slide on flooded surface.



## Summary

Some interesting results are provided by the multiple regression computer analysis of the DOT program data. The water film thickness of 0.51 mm (0.02 in.) used in the self-watering phase of testing is typical of both tire testing and field service conditions. The flooded water depth of 3.05 mm (0.12 in.) is greater than that normally experienced on public roads (9). The asphalt and concrete surfaces used in the tests are typical of highway construction.

For the self-watered surfaces, the regression equations represented the slide data better than the peak. Load and inflation were the dominant factor for peak, while speed the largest effect on slide. Asphalt was more sensitive than concrete to changes in all parameters.

Speed was the dominant variable for both peak and slide when the surfaces were flooded. In comparison, the effects of load and inflation variations were minor. Both Group A and Group B tires reacted similarly to changes in the parameters. Thus, the correlation between the two tires was good.

There appeared to be no direct relationship between peak and slide on the self-watered surface. This was probably due to the fact that load and inflation are the primary factors affecting peak, and speed is the primary factor affecting slide. On the flooded surface, speed is the primary factor affecting both peak and slide; so the linear correlation between the two is good.

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