systems developed are those that are desired and acceptable.

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Bus Passenger Service-Time Distributions

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The characteristics of bus passenger service-time distributions are a necessary input for the transportation simulation models that are used to evaluate the operations of street transit systems. In this paper, distributions of passenger service times through bus doors (the rates at which passengers entered, passed through, and departed from the bus) are analyzed by photographic studies and simulated by an Erlang function. These mathematical expressions simulating the passenger rates of flow entering and departing from a bus are compared with the observed times; the differences are not significant at the 95 percent level. The results of this research can be used to analyze a series of bus transit-flow situations and may serve as guidelines in assisting the designer and operator in evaluating existing or proposed bus systems. Specific models could be developed to evaluate the effects of the method of fare collection on passenger queue lengths and average waiting time under different rates of passenger arrivals. The overall design of bus transit vehicles has been shown to affect passenger flows in relation to such items as fare collection and in the use of door(s) for boarding and alighting.

The characteristics of bus passenger service-time distributions are necessary for the evaluation of street transit systems by the use of simulation models. This paper analyzes photographic studies of passenger movements through bus doors and shows that an Erlang function can represent the service-time distributions in the simulation process.

The door of a street transit vehicle can be viewed as a single-server queueing model. Passengers arrive at a certain rate, pass through a service area, and depart at another rate. The rate of departure depends on how fast they pass through the service area. A simulation model that uses generalized arrival and departure rates for transit stations has been developed by Fausch (1).

Simulation models of the type developed by Fausch are tools that can be used to evaluate the operations of street transit systems. The information necessary for such a simulation includes data on the capacity of bus doors and on the arrival and service-time distributions of passengers. Under maximum capacity conditions, the alighting or boarding of passengers invariably occurs in a group. In other words, when a bus arrives at a stop, the passengers to board are already waiting and alighting passengers are waiting in the vehicle. The service-time distribution, however, is not the same for boarding and alighting passengers as it depends on factors that affect the interaction between passengers and

vehicles. Those factors include human characteristics, modal characteristics, operating policies, mobility, climate and weather, and other system elements (2).

Photographic studies were taken to aid in analyzing the service-time distributions of passengers. These involved filming individual passengers alighting and boarding from the front doors of buses in San Diego; Montreal; and New Brunswick, New Jersey, in 1974. This information is summarized below.

Item	Montreal	San Diego	New Brunswick
Service	Local	Local	Suburban
Day of week	Wednesday or Thursday	Wednesday or Friday	Monday or Tuesday
Date	July 17 or 18	December 4 or 6	April 29 and June 18
Method of fare collection	Pay-enter (cash and change)	Pay-enter (exact fare)	Pay-enter (cash and change)
Type of fare	Flat, mixed	Flat, mixed	Multiple zone, cash
Number of buses observed	30	25	23
Number of passengers observed	412	233	411
Men	125	127	326
Women	286	105	83
Children	1	1	2

All photographs were taken at a nominal speed of 18 frames/s, with an 8-mm movie camera equipped with an f/1.8 zoom lens that could vary focal lengths from 7 to 70 mm. The number of frames required for each person to pass through the portal of the bus was recorded, the passengers were classified as to men, women, and children, and the type and number of items that they carried were noted. The information was keypunched and tabulated by computer.

ANALYSIS OF DATA

There were two areas of analysis. One was the study of the time sequence of the passengers in the order that they boarded the bus, and the other was the determination of service-time distribution characteristics for boarding and alighting passengers.

Time Sequence of Passengers

The average service times of the first 18 passengers in boarding sequence in San Diego and Montreal were studied by using the analysis of variance (ANOVA) technique. As might be expected with counting data, the cell variances were not equal but were proportional to the cell means. Homogeneity of variance was obtained by using a logarithmic transformation and a two-factor ANOVA. A plot of the cell means is shown in Figure 1 and indicates that the time for the first passenger is generally less than that for succeeding passengers. This is due to the availability of storage area on the steps between the bus door and the driver.

By using the results of ANOVA and analyzing Tukey's limits for multiple comparisons at the 5 percent level, the time differences between the first and third through the nth passenger were shown to be statistically significant at the 95 percent level. The time differences between the second and all other passengers were not statistically significant at the 95 percent level.

Service-Time Distribution

The means and variances of the service-time distributions for each successive person to pass through the vehicle door were calculated to assist in determining the mathematical function that could be used to represent the distributions in the simulation model.

Several forms of service-time distributions including the Gamma function, the Erlang function, and the uniform distribution function have been used in the development of queueing models. The most commonly used is the negative exponential function, a special case of the Erlang function (1, 3, 4).

The probability density functions for each of the distribution forms were plotted to view their general shape, and from these observations, it was hypothesized that the distributions could be described by the following Erlang function (3):

$$P(g \geq t) = \left\{ \sum_{i=0}^{K-1} [K(t-\tau)/(\bar{t}-\tau)]^i \, e/i! \right\}^{-K(t-\tau)/(\bar{t}-\tau)} \quad (1)$$

where

$P(g \geq t)$ = probability that time g is greater than or equal to time t,
K = positive integer,
t = any service time,
$\bar{t}$ = average service time, and
τ = minimum service time.

The individual means and variances were then used to calculate an Erlang function for each distribution. Integer values of K were estimated from the mean ($\bar{t}$) variance (s^2) and the minimum service time (τ) by the following:

$$K \approx (\bar{t}-\tau)^2/s^2 \quad (2)$$

These initial K-values were adjusted as necessary to improve the goodness of fit between the observed and calculated distributions. Table 1 lists the parameters of the observed passenger service-time distributions and the derived Erlang functions.

The K-value of 1 indicates that the two distributions for the one-door buses are represented by the negative exponential function, a special case of the Erlang function. Figure 2 shows the observed values and the calculated functions for alighting passengers in New Brunswick and boarding passengers in San Diego.

To check the mathematical validity of these results, the distributions of the observed and calculated functions were compared by a chi-square test. In all cases, the test results did not reject the hypothesis that the distributions were the same at the 95 percent level. Hence, it can be concluded that passenger service-time distributions can be represented by an Erlang function. It can also be inferred that K is equal to the number of doors on the vehicle and that the minimum service time (τ) is approximately half the average service time ($\bar{t}$). These results can be used to estimate any particular passenger service-time distribution if the minimum and average service times are known and K can be estimated.

ANALYSIS OF RESULTS

Two approaches were used to analyze the results of this research. One was a comparison between the data obtained by manual surveys of the time required for an entire queue to board the vehicle and the data obtained by the photographic studies of individual passenger service time. The other was a comparison between service-time distributions and stairway-service standards.

Figure 1. Average service times of passengers in the sequence in which they boarded.

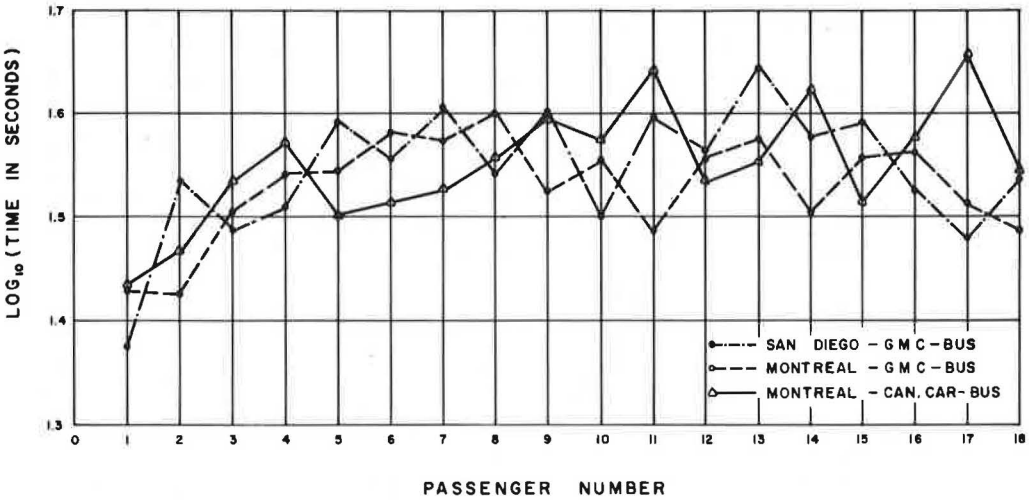


Table 1. Parameters of observed passenger service-time distributions and derived Erlang functions.

Location	Direction of Flow	Number of Doors on Bus	Observed Time (s)		Erlang Parameters		
			Mean	Variance	K	$\bar{t}$ (s)	τ (s)
Montreal	Boarding	2	2.097	0.727	2	2.097	0.90
Montreal	Boarding	2	2.034	0.834	2	2.034	1.25
New Brunswick	Alighting	1	1.972	1.045	1	1.972	0.95
New Brunswick	Boarding	1	3.471	3.499	1	3.471	1.75
San Diego	Alighting	2	1.472	0.403	2	1.472	0.75
San Diego	Boarding	2	2.180	0.868	2	2.180	0.75

Since the individual service times can be represented by an Erlang distribution, it should be possible to derive regression equations for the time required for varying sizes of queues to enter a bus. The validity of such derived equations could then be checked by comparing them with the equations developed from the observed data. This process was followed to develop a simulation model for use in determining the amount of time that it would take a specified number of passengers to board a bus.

Development of Simulation Model

The model to simulate the boarding of passengers at a loading area was constructed as a single sequence of blocks, as shown in Figure 3. Basically, the model generates any specified number of passengers to arrive at the bus instantaneously. The first passenger in line captures the door, leaves the line, and passes through the door on the basis of a randomly selected value from the Erlang service-time functions previously derived. He or she then frees the door and enters the bus. At that time, the next passenger captures the door and the process continues.

The second segment of the model is a timer that puts a limit on the amount of time the model can simulate. The longest limit is usually set at the amount that could be expected if all maximum service times were used.

The simulation model was run twice for each integer in the range of values of the observed regression equations; i.e., if the original observed regression equation had values between 6 and 25, then the derived regression equations were run twice for each of the values between 6 and 25. Actually, only five of the six Erlang functions were simulated because the observations of the boarding passengers in New Brunswick were too few in number and clustered in too narrow a range to develop a meaningful observed regression equation.

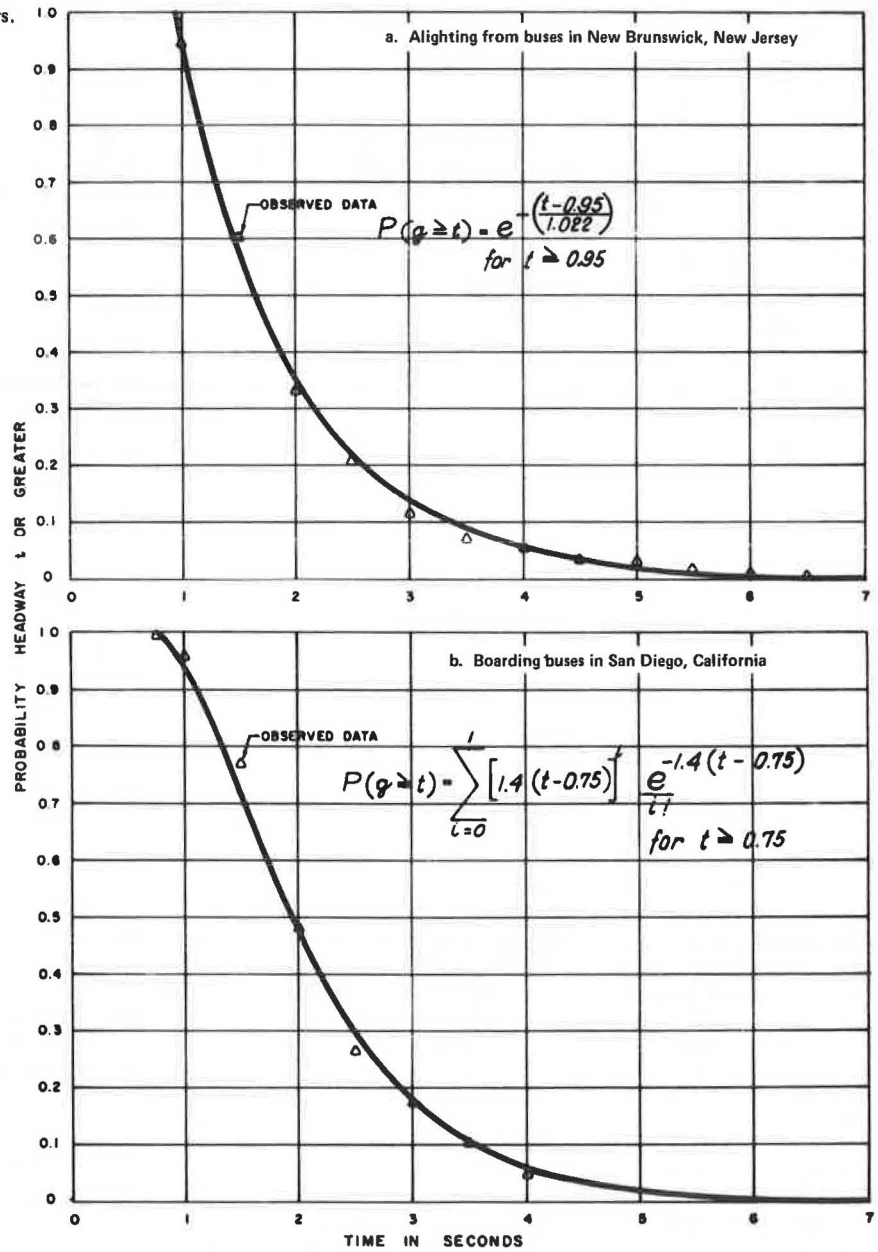
The simulated equations were compared with the observed equations by using an F-test for the variances of estimate and a t-test for the slopes. This comparison is shown in Table 2. All of the F-tests and four of the five t-tests indicated that the differences were not statistically significant at the 95 percent level. The fifth t-test indicated that the difference was not statistically significant at the 98 percent level. Thus, the simulation results were consistent with the observed data.

Comparison of Service-Time Distributions With Stairway-Service Standards

Fruin has shown that the maximum flow volumes for persons ascending and descending stairs are 62 to 66 persons/min/m (18.9 and 20.0 persons/min/ft) of stair width respectively (5). These results are similar to the values of 62 and 69 recommended by Hankin and Wright as design criteria for the London Subways (6). The average service times previously discussed were transformed into similar flow rates, as shown in Table 3.

For both directions of flow, the maximum observed values are less than those observed by Fruin and recommended by Hankin. These results are logical for a number of reasons. The riser height on bus stairs is normally higher than that on building stairs [23 to 25 cm versus 15 to 20 cm (9 to 10 in versus 6 to 8 in)] and would be expected to result in slower climbing speeds. Since the fare or method of fare collection should have no effect on alighting, the different flows shown in Table 3 for alighting are probably due to the effects of baggage. The boarding flows are different because they are affected by baggage, fare, and method of fare collection.

Figure 2. Service-time distribution of passengers.



APPLICATIONS

The results of this research can be used by the transit operator, the terminal designer, and the transit-vehicle manufacturer to evaluate certain aspects of existing or proposed systems. Specific models can be developed to evaluate the effects of different methods (such as pay-enter versus pay-leave) of fare collection. Other models can be developed to evaluate the effects of using single-flow doors versus double-flow doors and of using various combinations of front and rear doors for boarding and alighting. As an example, the following model was developed to evaluate the effects of method of fare collection on queue length and average waiting times under varying rates of passenger arrivals.

Example Simulation of Terminal Loading Platform

At a typical terminal loading platform, empty buses arrive at the platform every 5 min. Each bus begins to receive passengers as soon as it has stopped and its front door has been opened. Passengers continue to board for 4.5 min. Then the last passenger is permitted to finish boarding, the door closes, and the bus departs. The next bus arrives 5 min after the first had arrived.

Passengers arrive at the platform in a Poisson stream at a rate of 300 to 1100 passengers/h. They board the empty bus until all 50 seats are filled and then continue to board at a slower rate due to the effect of the standees (it is assumed that boarding times are 20 percent greater when standees are present). Both boarding service-time distributions are represented by the following negative exponential function:

$$P(g \geq t) = \exp[-(t - \tau)/(\bar{t} - \tau)] \quad (3)$$

The parameters used for each function in the simulation model are shown below.

Method of Fare Collection	Presence of Standees	Parameters $\bar{t}$	τ
Pay-enter	No	7.06	3.53
	Yes	8.50	4.25
Pay-leave	No	3.70	1.85
	Yes	4.44	2.22

Figure 3. Block diagram of simulation model for validation tests.

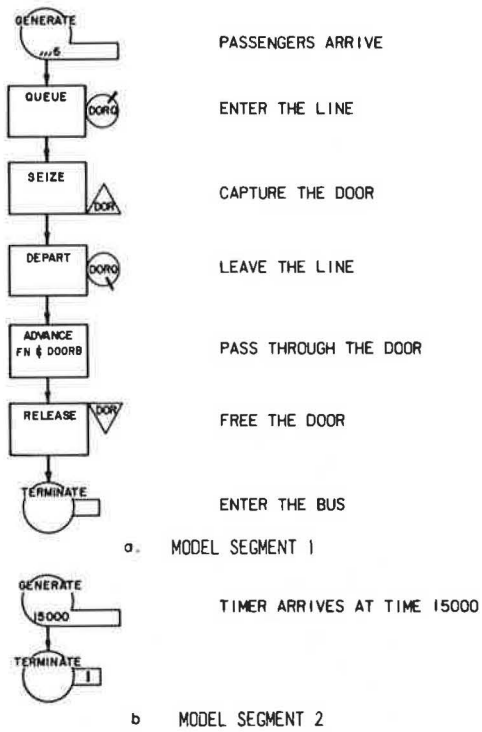


Table 2. Comparison of test statistics with tabular values for validation tests.

Location	Direction of Flow	F-test Value	F-test Statistic at 95% Level	t-test Value	t-test Statistic at 95% Level
Montreal	Boarding	1.00	2.91	1.72	2.02
Montreal	Boarding	1.46	2.15	2.33	2.39[a]
New Brunswick	Alighting	1.56	1.75	1.26	2.00
San Diego	Alighting	1.19	2.25	0.60	2.03
San Diego	Boarding	1.71	1.73	0.10	1.99

[a]At 98% level.

Table 3. Maximum observed flow rates through front door of bus.

Direction of Flow	Fare	Passengers Carrying One or More Items (%)	Maximum Observed Flow (persons/min/m)
Alighting (down)	Flat, exact fare[a]	55	53.5
	Multiple zone, cash and change[a]	82	39.6
Boarding (up)	Flat, exact fare	52	36.1
	Flat, cash and change	81	38.7
	Multiple zone, cash and change	78	22.6

Note: 1m = 3.3 ft.

[a]No fare was collected from alighting passengers.

The minimum service times were assumed to be one-half the average service time, which is consistent with the above information. The sequence for boarding is on a first-come, first-served basis. Persons who arrive at the stop while a bus is loading will be able to board during the 4.5-min period that the bus accepts

Figure 4. Example—block diagram for passenger model.

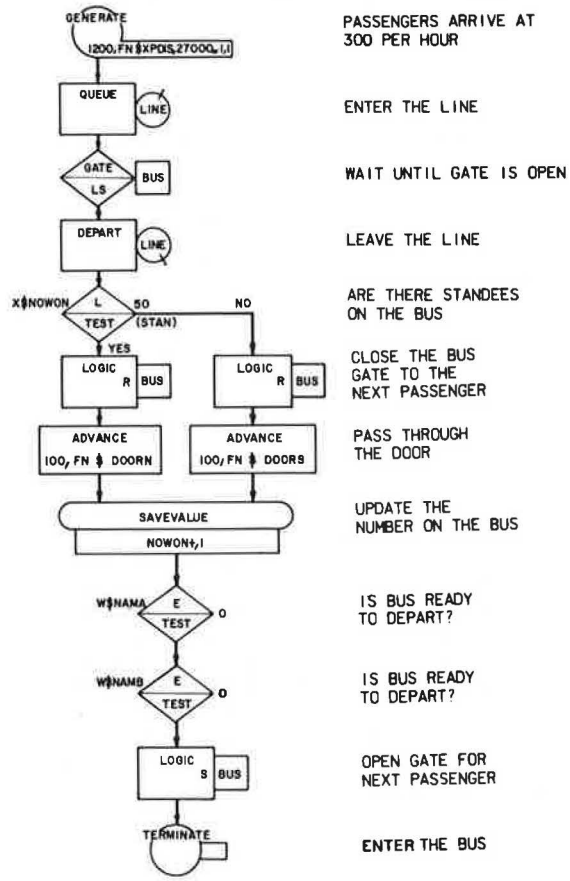


Figure 5. Example—block diagram for bus model.

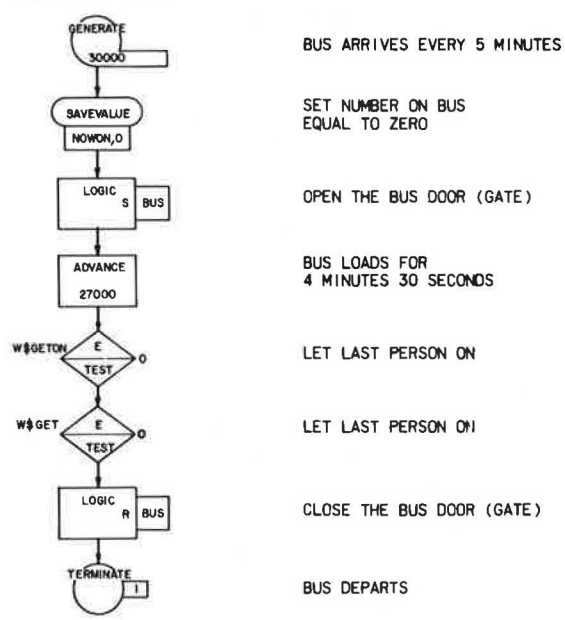


Table 4. Example—program output for pay-enter method of fare collection.

Output	Passengers per Hour						
	300	400	500	600	700	800	900
Total passengers arriving	318	407	495	582	682	778	884
Total passengers on bus	318	391	478	563	574	574	574
Number of standees	0	2	2	10	13	13	13
Number of zero entries	61	7	1	0	0	0	0
Percentage of zero entries	19.1	1.7	0.2	0	0	0	0
Maximum queue	30	43	33	55	117	204	310
Average queue	5.08	15.67	15.10	26.15	61.08	104.90	149.51
Average time per passenger in queue, s	62.76	149.04	118.10	173.88	346.57	521.81	654.54

Table 5. Example—program output for pay-leave method of fare collection.

Output	Passengers per Hour								
	300	400	500	600	700	800	900	1000	1100
Total passengers arriving	313	407	498	582	683	778	884	987	1095
Total passengers on bus	313	407	498	582	682	778	842	845	845
Number of standees	0	0	2	32	89	178	242	245	245
Number of zero entries	170	200	214	189	142	63	2	0	0
Percentage of zero entries	54.3	49.1	42.9	32.4	20.7	8.0	0.2	0	0
Maximum queue	6	8	7	10	13	30	49	143	255
Average queue	0.29	0.52	0.51	1.15	2.41	6.50	25.83	68.15	114.95
Average time per passenger in queue, s	3.62	4.93	4.00	7.62	13.69	32.36	113.20	267.30	406.41

passengers. Those not able to board that bus must wait in the queue until the next bus arrives and opens its door. (The model does not limit the number of passengers that can board due to the capacity of the bus. If bus capacity is a critical factor, then appropriate changes to the model should be made. Bus capacity was not a critical factor in this example because of the passenger flows used.) The simulation model is built in two segments. Model segment 1 simulates the passengers who arrive at the bus stop, wait for the bus, and board. Model segment 2 simulates the bus. The generalized programming service standards (GPSS) entities are defined below.

GPSS Entity	Interpretation
Transactions	
Model segment 1	Passenger
Model segment 2	Bus
Functions	
XPDIS	Exponential distribution function
DOORN	Distribution of boarding service times—no standees
DOORS	Distribution of boarding service times—standees
Logic switches	
BUS	When set indicates that the bus is at the stop and ready to load passengers
LINE	Queue in which people wait until the bus comes and they can board
Savevalues	
NOWON	Counter to keep track of the number of people on the bus
Tables	
INQUE	Table used to estimate the in-queue residence time distribution

Model Segment 1

Passengers are generated at a rate of 300 to 1100/h by 100-passenger increments. The first passenger arrives 4.5 min from the start of the simulation. This permits queueing for 0.5 min until the bus arrives; thereby a steady state condition is reached almost immediately. Passengers are given a priority level of 1 so that, if a passenger arrives at the time that the bus is ready to close its door, the passenger has priority and is permitted to board. The number of parameters for each transaction (boarding passenger) has been reduced to one to minimize the core storage that is used.

After the passenger is generated, he or she enters the queue and waits until the bus gate is opened. After the gate is opened, the passenger leaves the line and a test is made to determine whether more than 50 passengers are on the bus. If there are fewer than 50, the passenger boards according to the DOORN distribution. If there are more than 50 (indicating standees), the passenger boards according to the DOORS distribution. After the passenger has boarded, the number of persons on the bus is updated by one and the passenger gate is opened for the next passenger if the bus is not ready to depart. A block diagram of model segment 1 is shown in Figure 4.

Model Segment 2

A bus arrives every 5 min, opens its door, and loads for a period of 4.5 min. A test is then made to see whether the last person is still boarding. The bus waits until the last person has boarded and then the bus gate is closed. The bus then leaves the model. A block diagram for the bus model is shown in Figure 5.

Program Output

The simulation model was run for both the pay-enter and pay-leave methods of fare collection. In each case, 12 buses were loaded with passenger arrivals ranging from 300 to 1100/h at 100-passenger increments.

The output of each of the model runs includes the total number of passengers in the queue, the total number of passengers on the bus, the number of standees, the number of zero entries (i.e., the number of passengers that could board without waiting in the queue), the percentage of zero entries, the maximum queue, the average queue, the average in-queue residence time in seconds, and a frequency distribution of in-queue residence time. This information, except for the distribution of in-queue residence time, is given in Tables

4 and 5 for each of the model runs. As would be expected, the total number of passengers on the bus, the number of standees, the maximum queue, the average queue, and the average in-queue residence time increased with increased passenger demand. Conversely, the number of zero entries (i.e., the number of passengers that could board without waiting in the queue) decreased with increased passenger demand.

This information is useful for planning and evaluating the operations of a boarding platform. Table 4 indicates that, for passenger flows above 300/h, with a pay-enter method of fare collection, not all passengers will be able to board a bus and that the maximum number of persons that can board the 12 buses is 574 passengers. Similar data from Table 5 for the pay-leave method of fare collection are 800 and 845 passengers/h respectively.

The information about average and maximum queues can be used to design adequate loading platforms or to change operating procedures to avoid overcrowding on an existing platform. The values of average time per passenger in the queue can be compared with desired service standards and appropriate operational changes made if necessary.

With the use of GPSS, models can be developed to simulate the operation of other bus stops. The model developed here is an example, not a model for all cases. However, it can be adapted to other cases by changing the distributions of passenger arrival and service times as well as the time allocated for each bus to load passengers.

CONCLUSIONS

From the analysis of photographic studies of bus passengers described here, it can be concluded that

1. There is no difference in the average service time for each successive passenger to board, except that the first passenger may require less time due to the ready storage area on the steps between the bus door and the driver; and
2. The distribution of service times for individual passengers to pass through the vehicle door can be represented by an Erlang function in which the value of K seems to be equal to the number of doors on the vehicle and the minimum service time is approximately half the average service time.

These results can be used as inputs with simulation models to analyze a series of bus flow situations for the development of guidelines to assist the terminal designer and street transit operator in evaluating their existing or proposed system. Specific models can be developed to evaluate the effects of the method of fare collection (for example, on queue length), the average waiting time under varying rates of passenger arrivals, the use of both front and rear doors for boarding, or the use of the front door for boarding and the rear door for alighting.

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Differential Time-of-Day Transit-Fare Policies: Revenue, Ridership, and Equity

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This paper examines the financial, ridership, and equity implications of premium rush-hour fares of seven transit systems in New York State. Using 1973 data and demand equations that establish a relation between fare and ridership, calculations are made to estimate changes in ridership and revenue in each of the cities for various peak and off-peak fare combinations. Graphs are plotted for each of the cities to determine the fare combinations that maximize ridership without decreasing revenue more than 5 percent and still improve equity. The results showed that, in all of the cities studied, no differential fare combination increases both revenue and ridership simultaneously. Certain combinations improve equity while increasing either ridership or revenue with a less than 5 percent loss in the other. In Albany-Schenectady-Troy, Rochester, Syracuse, and Binghamton, combinations that increase passengers at the expense of a less than 5 percent decrease in revenue are attractive because of their flexibility. In New York City and Buffalo, combinations that increase revenue rather than passengers are attractive because no fare combination would increase passengers more than 5 percent without a loss of 15 percent or more in revenue.