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Estimation of Delay at Traffic-Actuated Signals

Kenneth G. Courage, University of Florida
Paraskevi Papapanou, Rensselaer Polytechnic Institute

Field measurement of delay at traffic signals is a costly and cumbersome process, and the use of analytical models to estimate delay is, therefore, of interest to the traffic engineer. A model originally developed by Webster has gained widespread use and acceptance in the estimation of delay at pretimed signals where signal timing remains constant from cycle to cycle. The original version of this model has been modified for application to traffic-actuated signals where signal timing is determined on the basis of vehicle presence information received from detectors in the roadway. This paper describes the modifications to Webster's model, which consist primarily of the substitution of values in the second (random arrival) term based on maximum cycle length rather than on optimal or average cycle lengths. The delay calculations that result from the modified version are compared with the values for pretimed operation based on the original model. Both versions of the model are compared with a simulation model and found to produce satisfactory approximations. Delay under traffic-actuated control is lower than delay under pretimed control. The difference depends on the degree of saturation of the approach lanes. The maximum difference is observed at 75 percent saturation. No difference is observed at very low saturation levels because very little delay accrues under these conditions. The difference also approaches zero at very high saturation levels because the actuated controller becomes constrained by the maximum interval timer to operate in a pretimed mode.

Delay is well recognized by the traffic engineer as a useful measure of effectiveness in a traffic-control system. Motorists view traffic delay with great disfavor, and economists agree that delay in movement of traffic is costly. Estimation of delay is, therefore, an important topic in the analysis of transportation systems.

Delay may be estimated either by field measurement or by analytical or simulation models. Although field measurement produces the most accurate results, the procedures are somewhat costly and time consuming. Furthermore, field measurement techniques cannot be applied to hypothetical situations such as proposed signal installations. Analytical approximations are, therefore, of interest to the traffic engineer.

The best recognized analytical treatment of delay estimation has been performed by Webster (1, 2). Webster demonstrates that satisfactory delay estimates may be obtained for any signalized approach when one is given the traffic volume, capacity, and signal timing (cycle length and effective green time) for that approach. The analytical process becomes, however, substantially more complicated when the signal timing varies with demand as in the case of traffic-actuated signals. A complex stochastic queuing model evolves from this analytical process, and this complex model is not adaptable to a practical solution because of the simplifying assumptions that must be made. The purpose of this paper, therefore, is to examine an analytical model that can be used to produce a useful approximation of delay at intersections where fixed signal timing does not exist. This examination is accomplished by refining Webster's model for pretimed control rather than by developing a separate, theoretical model. This refinement technique is further investigated by simulation to determine whether the techniques can be applied in a practical sense to estimate delay at vehicle-actuated signals.

WEBSTER'S PRETIMED DELAY MODEL

Webster demonstrates (1) that delay at pretimed signals may be approximated by the sum of two separate components.

1. The component due to uniform vehicle arrivals may be derived analytically in the form

$$D_1 = [C(1 - \lambda)^2]/[2(1 - x)] \qquad (1)$$

where

D_1 = delay per vehicle, seconds,
C = cycle length, seconds,
λ = proportion of green time given to the approach, and
x = degree of saturation of the approach, volume/capacity.

This component expresses the delay that would be experienced if the traffic stream were composed of equally spaced vehicles that arrive in a uniform manner.

2. The component due to random arrivals was developed semiempirically in the form

$$D_2 = x^2/[2q(1 - x)] \qquad (2)$$

where

D_2 = delay per vehicle, seconds, and
q = flow on the approach, vehicle per second.

This component expresses the additional delay that results from the random-arrival characteristics of the traffic stream.

The total delay per vehicle may be expressed as

$$D = 0.9/(D_1 + D_2) \qquad (3)$$

where the value of 0.9 is an empirical correction factor. The D_1 and D_2 terms are commonly referred to as Webster's first and second terms respectively.

APPLICATION TO TRAFFIC-ACTUATED CONTROL

For purposes of this analysis, the control strategy is assumed to:

1. Distribute available green time in proportion to demand on critical approaches and
2. Minimize wasted time by terminating each green interval as soon as the queue of vehicles has been properly serviced.

This control strategy closely approximates the operation of the traditional traffic-actuated controller that has been properly timed. The delay estimates will, therefore, reflect the best operation that can be expected from traffic-actuated control. Inappropriate setting of operating parameters (initial interval, extension interval, and so forth) will degrade performance of the controller.

Delay will be lower under traffic-actuated control than under pretimed control throughout most of the volume/capacity (v/c) range for two reasons.

1. Cycle length will tend to be shorter under traffic-actuated control since individual phases will be terminated as soon as queues are serviced.
2. Cycle failures will be fewer in which termination of green signal before a queue is completely serviced causes extra delay to waiting vehicles.

Both of these factors must be taken into consideration in the development of a model for estimating delay at actuated signals. The question of cycle length is addressed by Webster, who derives the optimal cycle for pretimed operation as

$$C_0 = (1.5L + 5)/(1.0 - Y) \qquad (4)$$

where

C_0 = optimal pretimed cycle for minimum delay,
L = sum of all lost times due to starting and stopping critical movements on each cycle, and
Y = overall degree of saturation of critical movements (i.e., the proportion of green time required for the movement of traffic).

For traffic-actuated operation, the appropriate cycle length is the average cycle length that will ensure that all of the excess time (beyond that which is needed for the movement of traffic) is dissipated in the starting and stopping process. The proportion of excess time available may be determined as 1.0 - Y, where Y is the proportion of time required. Therefore, the average cycle length may be expressed as a single ratio of the starting and stopping time to the proportion of time available for starting and stopping or

$$C_a = L/1.0 - Y \qquad (5)$$

where C_a is the average cycle length. The optimal cycle length for pretimed operation will, therefore, always be higher than the average cycle length under actuated operation. The extra time allocated to C_0 will appear as slack time, necessary to provide for stochastic variation in the number of vehicles that must be serviced on each cycle. This slack time will reduce the efficiency of the operation and result in an increased delay.

The question of cycle failures is addressed in Webster's second term, which takes into account the probability of a given phase being terminated before the queue is serviced. This probability is much lower under traffic-actuated control because the termination of the phase is initiated by the satisfaction of the queue. In fact, premature termination should only occur when the preset maximum green time is reached.

A reasonable approximation of delay under traffic-actuated operation should, therefore, be achieved by assigning a maximum cycle length to the operation and by basing the values used in Webster's second term on the maximum cycle length rather than on the optimal or average cycle lengths. This procedure will lower the estimated delay by increasing the effective green time used in the second term.

Based on this analysis we expect that under low to moderate volumes the delay caused by a vehicle-actuated signal will be lower than the delay caused by a pretimed signal. This lower volume can be explained by the fact that, when volumes are low to moderate, the signal responds to demand and does not allow slack time between phases or queues at the end of green. When volumes increase, however, we expect that the actuated signal will often operate under its maximum time settings and, when the volumes reach the saturation level, the operation of a vehicle-actuated signal will not differ from a pretimed signal because the signal will be continuously operating under maximum settings.

The following table illustrates the use of different cycle lengths for the two types of signal control.

Type of Signal	Cycle Used in First Term	Cycle Used in Second Term
Pretimed	Optimum	Optimum
Actuated	Average	Maximum

The first term (delay due to uniform arrivals) gives approximately the same delay for both types of control when the cycle length of the pretimed signal is equal to the average cycle length of the actuated signal. In such cases, the delay between pretimed and actuated signals is caused by the randomness of arrivals (delay expressed by second term). In actuated signals, small demand fluctuations do not cause as much random delay as in pretimed operation because the green times can be extended until demand is satisfied. If, however, these fluctuations cause the green to be extended to its maximum without satisfying the demand, then the benefits of the actuated operation no longer exist.

The solution to the problem of minimizing delay is, therefore, long cycles to accommodate random fluctuations (minimize random delay) and short cycles to accommodate regular demand (minimize uniform delay). This solution can only be applied to the vehicle-actuated signals, but in pretimed signals a compromise between average and maximum cycle can be made. The results are as expected: As long as there is a difference between average and maximum cycle length, actuated signals will result in less delay; but, when cycle lengths become equal, the resulting delays are the same for both types of control. Figure 1 demonstrates the variation of vehicle-actuated delay for maximum cycle lengths in the range of 90 to 150 s. Total intersection volumes from 800 to 1600 were considered, and corresponding intersection delays were calculated by using the model described in the table. Figure 1 shows that delay at a vehicle-actuated signal is less dependent on maximum cycle length when volumes are low to moderate (v/c ratio from 0.44 to 0.72). The maximum cycle length, however, becomes increasingly significant at higher volumes (v/c ratio higher than 0.75). At low volumes the maximum cycle length is rarely reached and, therefore, the random delay is very small. When the volumes increase, however, the maximum cycle length is reached more often, and the random delay increases significantly. Under these conditions the maximum cycle length becomes the actual operating cycle instead of simply a limiting condition.

Figure 2 illustrates the variation of delay for pretimed and vehicle-actuated signals for a range of total intersection volumes from 800 to 1600 vehicles/h. The delay for pretimed signals was calculated by using Webster's delay model, but the delay for actuated signals was calculated by using the modified version. Figure 2

Figure 1. Effect of maximum cycle length on intersection delay under different volume conditions.

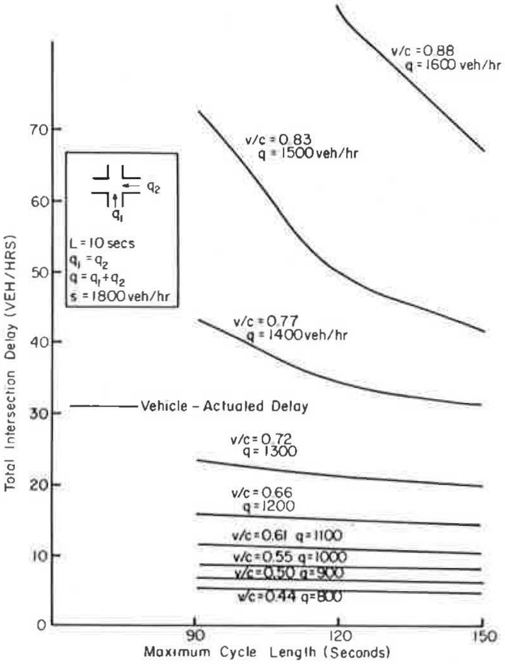


Figure 2. Relationship between pretimed delay and vehicle-actuated delay at an intersection with equal volumes at each approach under different volume conditions.

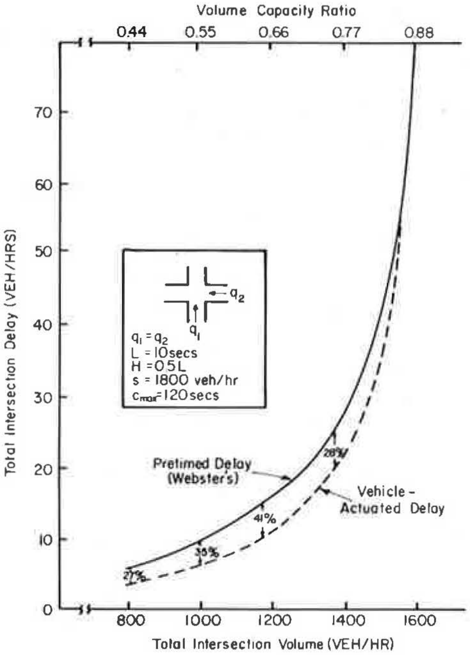


Figure 3. Relative and absolute benefits of vehicle-actuated signal control over pretimed signal control to v/c ratio.

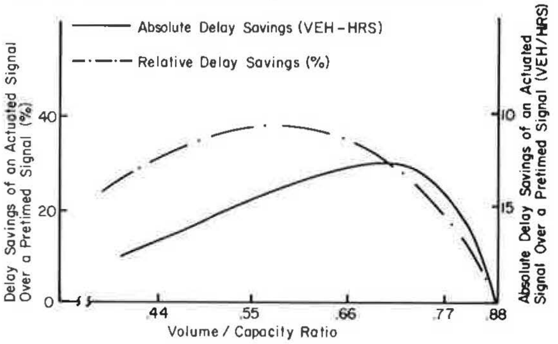


Figure 4. Comparison of simulation model and Webster model using equal approach volumes.

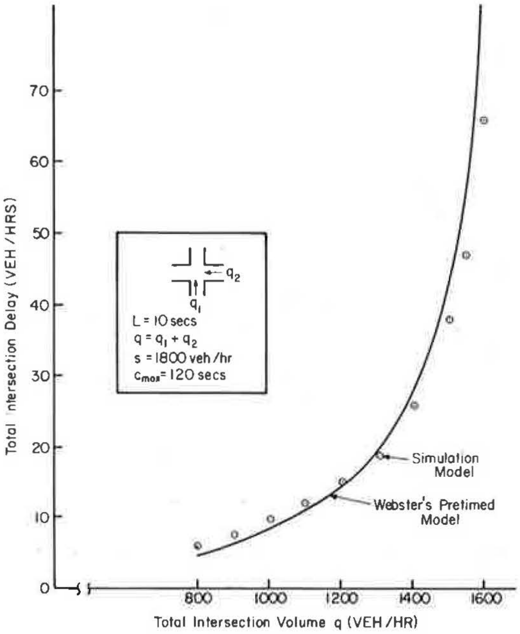


Figure 5. Comparison of simulation model and Webster model using unequal approach volumes.

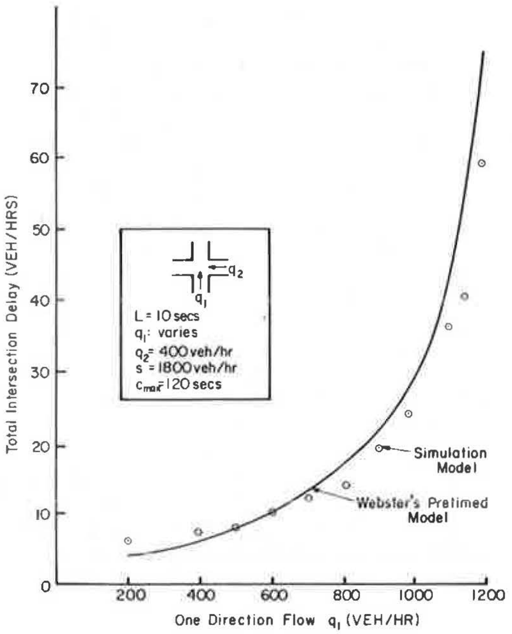
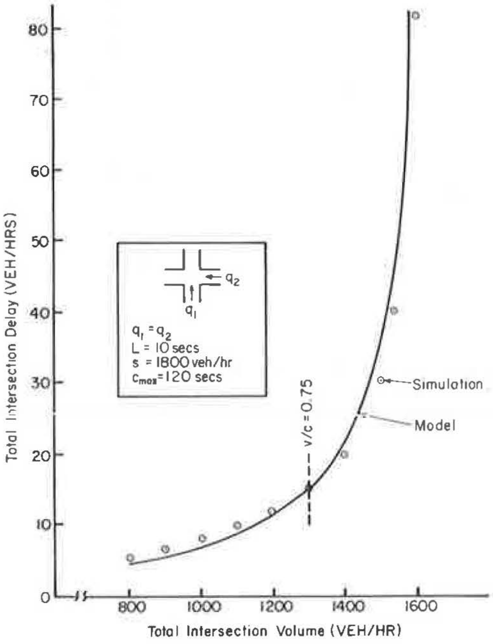


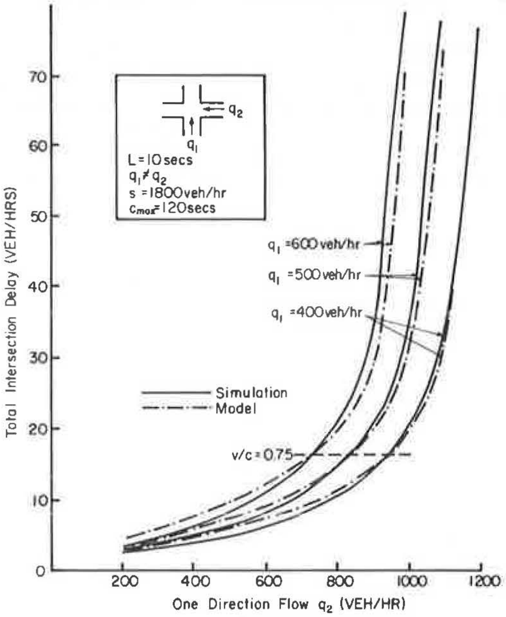
Figure 6. Comparison of simulation model with vehicle-actuated model using equal approach volumes.



shows that the delay savings due to actuated signal control are small in low volumes and keep increasing up to a maximum savings of 41 percent at 1200 vehicles/h and v/c ratio of 0.66. After this point the savings start decreasing until they become zero at 1600 vehicles/h and v/c ratio of 0.88. In this particular example, for a v/c ratio of 0.44 and of 0.88, the delay savings under vehicle-actuated control lie within 27 to 41 percent and have an average equal to 26 percent.

Figure 3 shows the delay savings for this example plotted as a function of the v/c ratio. From Figure 3 one can estimate that under low to moderate volumes there is an average savings of 34 percent when compared

Figure 7. Comparison of delay model results with simulation results using unequal approach volumes.



with pretimed delay. This percentage, however, drops sharply after the v/c ratio of 0.66 and becomes zero at a ratio equal to 0.88. In addition, in Figure 3 the absolute delay savings are also plotted as a function of the v/c ratio. Here the maximum absolute delay benefits occur at a v/c ratio of 0.77 although the maximum relative delay savings occur at a v/c ratio of 0.66. This change in savings happens because the delays are higher at a v/c ratio of 0.77 and, therefore, the absolute benefits are higher also. After this point, the absolute benefits drop sharply and become zero at a v/c ratio of 0.88.

MODEL VALIDATION

In the development of the delay model for vehicle-actuated signals, several assumptions and approximations were made. We felt, therefore, that the model should be tested under various conditions to investigate the model's validity and applicability to realistic situations. This testing was accomplished by exercising the model under various volume levels and comparing these results with the result produced by a simulation model for the same volume levels.

The simulation model used here consists of two submodels: the intersection simulator and the traffic signal emulator. The intersection simulator generates the vehicles in the system and records system variables such as length of queue and time in queue. The emulator superimposes either the pretimed or the vehicle-actuated traffic-signal operation.

The intersection simulator generates arrivals according to a Poisson distribution and, depending on the status of the signal given by the emulator, allows arrivals to stop or depart.

The simulator scans the system every second, records the new arrivals and departures, calculates the number of vehicles in the queue in each approach over the entire simulation period, and provides the total intersection delay for the given period.

Two kinds of traffic signal emulators were used: pretimed and vehicle-actuated. The pretimed signal emulator simulates a pretimed signal that displays green, amber, or red at fixed intervals; however, the vehicle-actuated signal emulator allocates right-of-way in the same manner as a traditional, actuated controller.

The simulation model is based on a four-legged intersection of two one-way streets. The green times, cycle lengths, and other inputs to the pretimed emulator were calculated by using Webster's method.

The validation proceeded according to the following strategy. First, the simulation model was tested against Webster's pretimed delay model by using a pretimed signal emulator. Because Webster's delay model has gained widespread use and acceptance, comparison of the simulation with Webster's model should provide sufficient evidence of the validity of the simulation model. The actuated signal delay model was then tested against the validated simulation model under various conditions.

Delays at a pretimed signal were calculated for a range of total intersection volumes from 800 to 1600 vehicles/h (the intersection becomes oversaturated after this point) and equal volumes in both directions. Simulation was performed for the same volume ranges; the results are plotted in Figure 4. Figure 4 shows that under the entire range of volumes the delays obtained by simulation are very close to the delays obtained by the delay model. The simulation model was also tested against Webster's model for unequal volumes in two directions, and the results are shown in Figure 5. Again, the results demonstrate trends that are similar to the case of equal volumes in the two directions. Based on these two comparisons, we concluded that the simulation model is successful in reproducing the delay estimates provided by Webster's pretimed model and is, therefore, a useful tool for validating the modified version for traffic-actuated operation.

In validating the modified version, the model was first tested with equal volumes in both directions. The results are plotted in Figure 6. Figure 6 shows that the model results are very close to the simulation results. The model tends to underestimate the delay slightly under low volumes and to overestimate slightly under heavy volumes. The average difference, however, lies within a 10 to 15 percent range and is reduced to zero when the total intersection volume is approximately 1350 vehicles/h and the v/c ratio is 0.75.

Another series of simulation runs was performed to test the model for different volumes in two directions. Figure 7 illustrates the results of these runs for various volume levels. For each curve, vehicles per hour in one direction is shown. Vehicles per hour in the other direction ranged from 200 to 1200.

Similar trends between simulated and computed delays can be distinguished in the three sets of curves. The computed delays are lower than simulated delays at low to moderate volumes (v/c ratios from 0.44 to 0.74) by an average difference of approximately 10 percent and become equal when the total intersection volume is equal to 1350 vehicles/h and v/c ratio is 0.75. After this point, the computed delays become slightly higher than the simulated delays. The average difference is 1 percent for the 400-vehicles/h curves and 8 percent for the 600-vehicles/h curve.

SUMMARY AND CONCLUSIONS

In this paper a macroscopic model for estimating delays at vehicle-actuated signals was proposed. The model was tested by simulation and has given satisfactory results for a wide range of applications. The model is a simple, yet adequate, model that requires little computational effort even for a complex, multiphase signal operation. Based on the same principles as the most widely accepted model for estimating delay at pretimed signals, this macroscopic model is offered as a useful tool for a quantitative comparison of the two basic types of signal control.

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Abridgment

Cost-Effectiveness of RUNCOST Evaluation Procedure

Peter S. Parsonson, Georgia Institute of Technology

Currently there is emphasis on low-capital programs of transportation-system management (TSM). Regulations issued in 1975 by the Urban Mass Transportation Administration and the Federal Highway Administration require each urbanized area to develop a plan containing a TSM element and a transportation improvement program (TIP). The programs are designed to meet the short-range needs of urban areas through the efficient use of existing facilities. The goal is to reduce traffic congestion and to facilitate the flow of traffic. According to the regulations (1), one of the major categories of TSM action concerns the "efficient use of existing road space