Aspects of Spiral Transition Curve Design

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Some aspects of the design of spiral transition curves on highways are discussed, and a model for the relation between the design speed and the rate of change of centrifugal acceleration is presented. The model is based on the principle that higher speeds require higher comfort. An identity between the spiral length given by the model and the length of the superelevation runoff is assumed. A modified criterion for the maximum relative slope of the centerline and the edges of a two-lane pavement is proposed. The values for the rate of change of centrifugal acceleration and the maximum relative slope suggested are shown to be reasonable.

When the alignment of a highway changes directly from a tangent to a circular curve, the driver of an automobile on the highway is subjected to a sudden centrifugal force. The use of a spiral transition curve helps to avoid the sudden impact of this force as this curve follows the actual path of the vehicle more closely and improves the visual quality of the highway.

The mathematical expression for the minimum length of a spiral curve was developed by Shortt (6) and is given by

$$L_s = 3.15V^3/R_cC \qquad (1)$$

where

L_c = minimum length of spiral curve (feet),
V = design speed (miles per hour),
R_c = radius of curve (feet), and
C = rate of change of centrifugal acceleration for a unit time interval (feet per second per second per second).

[This model was designed for comparison with the American Association of State Highway Officials (AASHO) standards, which are given in U.S. customary units; values in Tables 1 to 3, the text tables on page 3, and Figures 1 and 3 are not given in SI units.] The factor C is an empirical value that indicates the comfort and safety involved. For a given value of V and R_c, this factor determines the length of spiral needed.

The relation between the design speed and the rate of change of centrifugal acceleration and an evaluation of some practical aspects of a model of V versus C are presented in this paper. The model assumes an identity between the spiral length and the length of superrelevation runoff, and a modified criterion for the maximum relative slope of the centerline and the edge of a two-lane pavement is proposed. A criterion for the maximum radius that requires the use of a spiral transition curve in highway design is also proposed. Most of the analysis is based on the AASHO policy (1), but some aspects are derived from European practices, especially German standards (RAL) (5).

CURRENT PRACTICES FOR DETERMINING C-VALUES

There are a number of methods for determining the value of C at different speeds, but most of them give ranges rather than precise values.

The AASHO policy (1) suggests a range of C-values of 0.3 to 0.9 m/s^3 (1 to 3 ft/s^3), and tabulates values of C that vary linearly from 0.75 to 1.2 m/s^3 (2.5 to 4.0 ft/s^3) for speeds from 80 to 32 km/h (50 to 20 mph) respectively.

The values derived from the RAL Standards (5) are similar, i.e., C = 0.5 m/s^3 (1.6 ft/s^3). However, for speeds above 100 km/h (62 mph), the C-values are lower [e.g., 0.302 m/s^3 (0.99 ft/s^3) at 121 km/h (75 mph)] and decrease as the speed increases. Therefore, C depends on the speed.

The Northwestern University Traffic Institute (NUTI) Geometric Design Notes (4) describe C as the factor of comfort and safety in negotiating highway curves and recommend the use of C = 0.3 m/s^3 (1 ft/s^3) as desirable and C = 0.6 m/s^3 (2 ft/s^3) as a minimum. These notes also suggest that the maximum length of the spiral should also be considered and that the equivalent of an 8-s travel interval is appropriate.

SUGGESTED MODEL FOR C-VALUES

A few basic principles served as guides for the development of the model for the relation between C and V. These include the dynamic safety, simplicity, and practical validity. That C decreased with increasing values of V was established on two bases: The first of these is intuitive—C is often called the comfort coefficient, which suggests that, at high speeds, there should be a lower rate of change, i.e., a smaller amount of centrifugal acceleration acting on the driver in a unit time. The second is that calculations of the centrifugal acceleration (V^2/R) for different speeds and the appropriate minimum radii show that this decreases as the speed increases.

To obtain relatively high comfort at high speeds, increasing maneuver times are necessary. Thus, since C is the centrifugal acceleration for a unit time interval, it must decrease as the speed increases.

Many models for the determination of C have been investigated. The derivation of this model divided it into two parts—one for speeds above and one for speeds below 97 km/h (60 mph)—on the assumption that a more moderate change in C is required at higher speeds. Because a linear model does not differ appreciably from a parabolic one for the ranges of values that are considered appropriate, the linear model was chosen because of its simplicity. The model is given in Equations 2 and 3.

$$C = 2.5 - 0.033(V - 30) \qquad (30 \leq V < 60) \qquad (2)$$

$$C = 1.5 - 0.025(V - 60) \qquad (60 \leq V \leq 80) \qquad (3)$$

The cutoff points were established as follows: C = 0.3 m/s^3 1 ft/s^3) for a design speed of 129 km/h (80 mph), and C = 0.75 m/s^3 (2.5 ft/s^3) for a design speed of 48 km/h (30 mph). The latter value is somewhat lower than the currently accepted one but may give a more appropriate spiral length. The value of C = 0.45 m/s^3 (1.5 ft/s^3) was chosen for the design speed of 97 km/h (60 mph) since it agrees closely with the value of C = 0.5 m/s^3 (1.6 ft/s^3) for a design speed of 100 km/h (62 mph) that is commonly used in Europe. V versus C, based on Equations 2 and 3, is shown graphically in Figure 1.

Figure 1. Rate of change of centrifugal acceleration versus design speed.

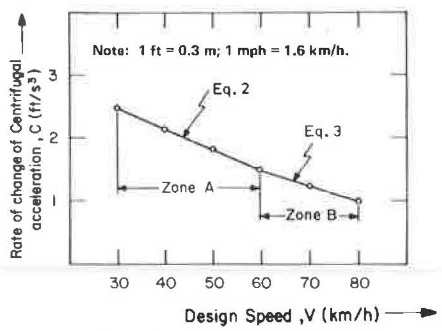


Figure 2. Elements of spiral for a given design speed.

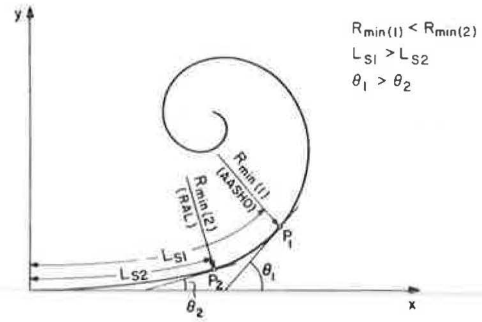


Table 1. L_S, t_M, p, and θ in relation to C at a superelevation of 6 percent.

V (mph)	C (ft/s^3)	R[a] (ft)	L_s (ft)	t_M (s)	p (ft)	θ (°)
30	2.50	273	124.62	2.85	2.37	13.07
40	2.16	508	183.73	3.15	2.77	10.36
50	1.83	833	258.30	3.55	3.34	8.88
60	1.50	1263	359.14	4.11	4.26	8.15
70	1.25	1815	476.23	4.67	5.21	7.51
75	1.13	2206	535.50	4.81	5.42	6.95
80	1.00	2510	642.55	5.51	6.85	7.33

Note: 1 mph = 1.6 km/h; 1 ft = 0.305 m.
[a]Minimum radii suggested by AASHO (1) for a superelevation of 6 percent.

Table 2. L_S, t_M, p, and θ in relation to C at a superelevation of 8 percent.

V (mph)	C (ft/s^3)	R[a] (ft)	L_s (ft)	t_M (s)	p (ft)	θ (°)
30	2.50	250	136.08	3.11	3.09	15.58
40	2.16	464	201.14	3.45	3.63	12.41
50	1.83	758	283.86	3.90	4.43	10.73
60	1.50	1143	396.85	4.50	5.74	9.95
70	1.25	1633	529.31	5.15	7.15	9.28
75	1.13	1974	598.70	5.48	7.56	8.68
80	1.00	2246	718.08	6.16	9.57	9.17

Note: 1 mph = 1.6 km/h; 1 ft = 0.305 m.
[a]Minimum radii suggested by AASHO (1) for a superelevation of 8 percent.

EVALUATION OF MODEL

To evaluate the suggested model, the values of the minimum length of spiral (L_s), the maneuver time (t_M), the offset from the initial tangent to the shifted circle (p), and the spiral angle (θ) (2) were calculated for the minimum values of radii suggested by AASHO (1) as appropriate for certain speeds at superelevations of 0.06 and 0.08. The results are summarized in Tables 1 and 2 respectively.

The following results are apparent.

1. The spiral length increases with increasing speed, but does not become unreasonably long.
2. The offset from the initial tangent to the shifted circle increases as the speed increases.
3. The spiral angle decreases as the speed increases [except between 121 and 129 km/h (75 and 80 mph)].
4. The spiral length and spiral angle, which are shown in Figure 2, depend only on the radius of the circular curve for a given design speed since the design speed determines C, which is actually the only factor defining the spiral; i.e., the spiral is defined by C (for a given speed), and its endpoint is determined by the radius.
5. The maneuver time increases continuously. (It can be proven that the maneuver time increases in a hyperbolic manner as V increases.) At speeds above 80 km/h (50 mph), this time is within the range set by the NUTI notes (4).

Table 3. Evaluation of spiral elements using different minimum radii.

Superelevation (%)	V (mph)	C (ft/s^3)	R (ft)	L_s (ft)	t_M (s)	p (ft)	θ (°)
6	50	1.83	1147[a]	187.50	2.56	1.27	4.68
6	75	1.13	3278[a]	358.66	3.26	1.63	3.13
8	50	1.83	852[b]	247.20	3.37	2.98	8.30
8	75	1.13	2459[b]	476.10	4.33	3.84	5.55

Note: 1 mph = 1.6 km/h; 1 ft = 0.305 m.
[a]Minimum radii suggested by RAL (5). [b]Minimum radii suggested by Craus (3).

A further analysis of the spiral elements was carried out by using the suggested model for C and different minimum radii. (Since the same model for C is being used, the same spiral is being investigated, but its endpoints will be different.) Some results using the minimum radii recommended by RAL (5) for a superelevation of 0.06 and those recommended by Craus and others (3) for a superelevation of 0.08 are given in Table 3. The values of the spiral lengths and the maneuver times are reasonable and within appropriate boundaries. Since the radii recommended by AASHO (1) are smaller than those recommended by RAL (5) and by Craus (3), the values of L_s and θ given in Table 3 are smaller than the corresponding values given in Tables 1 and 2.

INTERCHANGEABILITY BETWEEN SUPERELEVATION RUNOFF AND SPIRAL LENGTH

The current policy of many design agencies is to use the whole length of the spiral curve to make the desired change in the cross slope. This common practice simplifies the construction and the calculations. The AASHO policy (1) assumes that, for the most part, the calculated values for the lengths of spiral and superelevation runoff do not differ very much. The consistency of this approach was verified by substituting various specific lengths that are suggested for the superelevation runoff of a two-lane pavement (as given in Table 3-2 of the AASHO policy. The values of L_s in Equation 1. The C-values derived in this way are given in the table below for three rates of superelevation and different speeds (1 mph = 1.6 km/h and 1 ft/s^3 = 0.280 m/s^3).

Design Speed (mph)	C (ft/s^3) e = 6%	e = 8%	e = 10%
30	2.79	2.30	2.00
40	3.11	2.52	2.30
50	3.21	2.69	2.33
60	3.31	2.72	2.36
65	3.41	2.79	2.43
70	3.34	2.79	2.50
75	3.11	2.59	2.30
80	3.25	2.75	2.50

At a constant superelevation rate the C-values increase with the speed, which contradicts the basic principle of the model. The use of higher C-values at high speeds in itself also leads to some discrepancies related to the safety and comfort concept of highway design.

Different maximum relative slopes (Δ) between the profiles of the edges and the centerline of a two-lane pavement should be used to make the lengths of the superelevation runoff satisfactory for use as lengths of spiral transitions. These values can be calculated by

$$\Delta = be/L_s = beR_cC/3.15V^3 \qquad (4)$$

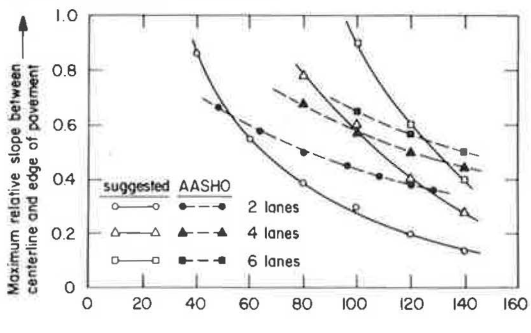
where

R_c = minimum radius suggested by AASHO for a given superelevation and design speed (feet),
C = value defined by Equations 2 and 3, and
b = lane width [12 ft (3.7 m)].

The results for three rates of superelevation are given in the following table (1 mph = 1.6 km).

V (mph)	Δ for e = 6% (%)	Δ for e = 8% (%)	Δ for e = 10% (%)
30	0.58	0.71	0.81
40	0.39	0.48	0.55
50	0.28	0.34	0.39
60	0.20	0.24	0.28
70	0.15	0.18	0.21
75	0.13	0.15	0.17
80	0.11	0.13	0.15

Figure 3. Suggested values of maximum relative slope between centerline and edge of pavement as a function of design speed (e = 8 percent).



Note: 1 km/h = 1.6 mph.

The Δ-values suggested by AASHO are a function of the design speed only, but those given in the above table are a function of both the design speed and the superelevation rate. These values are lower than the AASHO values, except at the low design speed of 30 mph (48 km/h) for the superelevations of 8 and 10 percent.

The maximum relative slope suggested by AASHO for a four-lane pavement is greater than the suggested value for a two-lane pavement by a factor of 1.33. However, since the length of the spiral should be determined mainly by dynamic and comfort considerations, rather than by the number of lanes, the Δ-value for four-lane highways should be doubled.

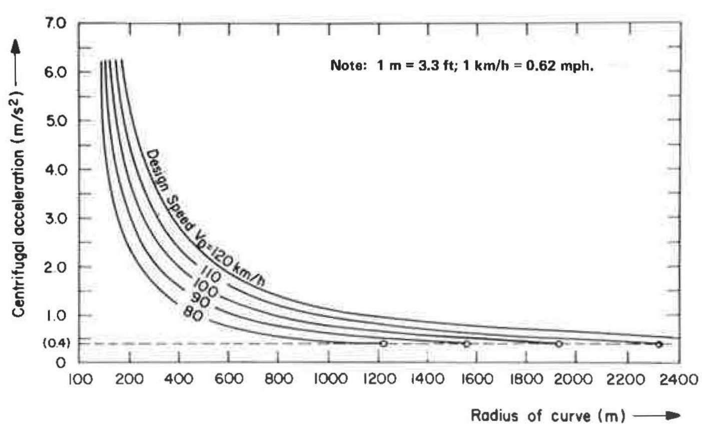
A further analysis that was based on the minimum radii suggested by Craus and others (3) for a superelevation of 8 percent and was carried out in SI units is given in Table 4. The maximum relative slope between the profiles of the centerline and the edges of four and six-lane pavements is doubled and tripled accordingly.

Table 4. Calculated values of Δ for minimum radii given by Craus for superelevation of 8 percent.

V (km/h)	C (ft/s^3)	C (m/s^3)	R (m)	Δ for Two-Lane Highways (%)	Δ for Four-Lane Highways (%)	Δ for Six-Lane Highways (%)
40	2.67	0.81	50	0.86	–	–
60	2.26	0.69	125	0.55	–	–
80	1.83	0.56	260	0.39	0.78	–
100	1.43	0.44	500	0.30	0.60	0.90
120	1.13	0.34	750	0.20	0.40	0.60
140	0.83	0.25	1080	0.14	0.28	0.40

Note: 1 km/h = 0.62 mph; 1 m = 3.28 ft.

Figure 4. Recommended criteria of maximum radii that require use of spiral transition curves.



Since it is logical to assume that a four-lane highway will have higher design speeds than will a two-lane highway, the Δ-values for a four-lane highway are given for speeds equal to or greater than 80 km/h (50 mph) and, similarly, the Δ-values for six-lane highways are given for design speeds equal to or greater than 100 km/h (62 mph). This analysis leads to the following conclusions.

1. The Δ-values between the profiles of the edges and the centerline of a two-lane pavement are lower than the values suggested by AASHO (1). The exact relation is shown in Figure 3. The use of lower Δ-values gives a more gradual superelevation runoff, and the identity in length with that of the spiral may result in a rather simplified design.
2. For multilane highways, the Δ-values suggested here are higher than the AASHO values at lower speeds but not at speeds above 100 km/h (62 mph) on four-lane highways and 120 km/h (75 mph) on six-lane highways.

MAXIMUM RADIUS FOR NECESSARY USE OF SPIRALS

The need for transition curves is most pronounced on sharper curves. On curves having larger radii there is less need for the use of spirals.

Several criteria have been suggested for the use of spirals. One method designates a single degree of curve that is applicable to all design speeds. Another method suggests the use of spiral curves when p, computed by Equation 1 with C = 0.6 m/s^3 (2 ft/s^3), is greater than 0.3 m (1 ft). The method given in the NUTI Geometric Design Notes (4) suggests that the spiral be used on curves that require a superelevation rate of 0.03 or more

The following assumptions suggest another criterion for the introduction of spiral curves. A gently curving alignment that requires little centrifugal-acceleration resistance should not require spirals.

The minimum amount of centrifugal acceleration for the introduction of spiral transition curves is 0.4 m/s^2 (1.3 ft/s^2). The criterion for the maximum radii that will require use of a spiral is

$$V^2/R_c = 0.4 \tag{5}$$

where R_c = maximum radius for necessary use of spiral transition (meters) and V = design speed (kilometers per hour). The values calculated by Equation 5 can be read directly from Figure 4, which shows that the centrifugal force varies hyperbolically with speed and radii.

The centrifigual-force criterion has two advantages. First, this criterion is based on the actual force that is imposed on the traveling vehicle. Second, this criterion agrees with the assumption, given by Spindler (7), that the safest and most comfortable situation is that in which the side-friction factor and the superelevation equally resist the centrifugal acceleration; i.e., the ratio of e to (e + f) should be 0.5.

CONCLUSIONS

A model for the relation between the rate of change of centrifugal acceleration on a spiral transition curve and the design speed is presented. The model has two regions and decreases linearly. The resulting spirals and their properties are discussed. The practical reasons of safety and uniformity were used as a guideline to the suggestion that the maximum relative slope between the edges of the pavement and the centerline should be greater than that recommended by AASHO. An identity between the superelevation runoff and the spiral lengths is assumed. The criterion of the maximum radius for the use of spiral transition curves is also discussed.

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New Concepts in Design-Speed Application

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The design-speed concept, as presently applied, does not preclude inconsistencies in highway alignment. The basic problem, particularly in the range of design speeds below 90 km/h (55 mph), is the tendency on the part of the driver to continually accelerate and decelerate. A secondary problem is the speed differential between automobiles and trucks. To overcome these weaknesses in current practice, a new concept in the definition and application of design speed is presented. The overall object is to meet driver expectations and to comply with his or her inherent characteristics to achieve operational consistency and improve driving comfort and safety. The principle used in the updated design-speed approach is the 15-km/h (10-mph) rule, which during periods of free-flow conditions, entails three considerations: (a) A reduction in design speed