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Viscoelastic Deformations in a Two-Layered Paving System Predicted From Laboratory Creep Results

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A new viscoelastic method developed to calculate deformations in a two-layered flexible pavement system subjected to both single and repeated moving loads is described. On the basis of laboratory creep experiments the rheologic behavior of all asphalt-bound materials can be represented in viscoelastic equations by a simple analytical expression for creep compliance. The mathematical analysis is performed by (a) calculating the viscoelastic deformations (vertical, transverse, and longitudinal strains) around the moving load and (b) predicting the repeated load effect, considering the effects of velocity and waiting time between two consecutive loads. The theory is applied to the asphalt pavement design for a steel slab bridge on an Italian motorway. Calculated deformations are compared with actual deformations, measured by means of strain gauges incorporated into the asphalt layers. Temperature for tests and calculations was 40°C (104°F). Both the theoretical predictions and the experimental results show that recovery is considerably delayed for the vertical and the transverse elongational deformations only. No delay was observed for the longitudinal deformation. The contribution of delayed viscoelastic deformations to total deformations was found to be relevant under some actual traffic conditions.

Elastic methods in pavement design can be used to predict the fatigue behavior of flexible pavements and generally to determine maximum deformation from the passage of a single load. These methods provide no useful information regarding the onset of permanent deformation that may be largely due to viscoelastic effects associated with the deformation from repeated loads. However, adequate methods for calculating viscoelastic deformations in layered systems are not currently available.

Some authors have provided interesting solutions to viscoelastic problems (1, 2), and even for multilayered systems (3), but these consider repeated application of static loads. This simplification may lead to a substantial underestimation of deformability in the lowest layer of a paving system, since the diffusion of stresses gives rise to an effective load time that increases with depth if the load moves on a viscoelastic system. Experimental results (4) confirm that load time relates to the load velocity and to the depth of the point under examination. For this reason viscoelastic models dealing with repeated application of a static load underestimate the deformation at greater depths. Therefore, a theoretical method was developed for predicting the viscoelastic behavior of a two-layered system subjected to repeated moving loads (5).

A two-layered, incompressible asphalt pavement system subjected to a sequence of moving loads is examined. On the basis of laboratory experiments (6), asphalt-bound materials can be characterized rheologically in terms of Equation 1, where J(t) is the viscoelastic creep compliance function.

$$J(t) = \alpha_1 J_1 \tau_0^{\alpha_1} \gamma(\alpha_1, t/\tau_0) \quad (1)$$

When a single or only a few load passages are being considered, Equation 1 reduces to the simpler formula

$$J(t) = J_1 t^{\alpha_1} \quad (2)$$

for short time periods, generally <10 s ($t << \tau_0$).

In Equation 1 $\gamma(\alpha_1, t/\tau_0)$ is the incomplete gamma function of α_1 and t/τ_0:

$$\gamma(\alpha_1, t/\tau_0) = \int_0^{t/\tau_0} u^{\alpha_1 - 1} e^{-u} du \quad (3)$$

Equation 2 takes slope α_1 as a straight line on a log-log plot. We assume that characteristics of the subgrade can also be described by Equations 1 or 2, provided we choose suitable values for the parameters. We observed, also, that putting $\alpha = 0$ in Equation 2 gives us the elastic behavior.

Creep tests carried out in the laboratory on a large class of asphalt concretes show that the order of magnitude of τ_0 is nearly 1 000 to 10 000 s at a temperature of 20°C (68°F) and nearly 10 to 100 s at a temperature of 40°C (104°F).

The circular load function has been expressed, according to general Hankel transform methods, as a superposition of Bessel functions of order zero having a continuous spectrum of "wavelengths." The problem for a "Bessel-like" load moving on the surface of a viscoelastic double layer has been solved first.

The viscoelastic problem is then reduced to an equivalent elastic problem by means of a Fourier transformation over the time variable.

The Fourier transform of the relaxation modulus of the materials is easily obtained in an analytic form from Equations 1 and 3. Fortunately, the time-dependent load condition corresponding to a moving Bessel-like load has a Fourier transform that can also be calculated analytically.

As a consequence, general results for stresses and deformations around a moving load can be expressed via a double integral (Fourier antitransformation and integration over the wavelength spectrum of the applied load). Although we have considered a two-layered system, this conclusion applies to systems containing any number of layers.

If we are interested in calculating the viscoelastic deformation still present long after the passage of the load, the Fourier antitransformation integral can be asymptotically evaluated in a closed form, and we are left with the single integration over the Hankel wavelength spectrum. Our calculations show that appreciable accumulation effects from multiple loads are to be expected in connection with the deformation components ϵ_z, ϵ_y, and γ_{yz}. For one load passage, our viscoelastic calculations show that elastic methods predict deformations with an accuracy that is satisfactory for the first layer only. If the response of the second (subgrade) layer is not competely elastic, the methods underestimate the deformability of the lower layer by a factor that increases with the depth.

For accumulated, recoverable viscoelastic deformation from repeated loads, we show that these deformations can be as significant as those of single loads under some traffic conditions. Furthermore, when the subgrade behavior is either elastic or characterized by a viscoelastic response time substantially shorter than that of the asphalt concrete surface layer, the deformation accumulated in the surface layer is independent of the subgrade stiffness.

To predict the onset of permanent deformations from high total deformations (single load + accumulation), some experimental distress criteria are still required.

However, even if the material behaves linearly, permanent deformations may occur after each load passage if one of the rheologic models representing the mechanical characteristics of the two layers contains a Maxwell element in series. In this case only can we correlate permanent deformation with the number of load passages. On the other hand, permanent deformations in asphalt-bound layers may occur if there is a limiting deformation value above which the material undergoes irreversible viscoelastic yielding.

LABORATORY TESTING TECHNIQUES

The viscoelastic properties of asphalt-bound material can be adequately investigated by unconfined creep tests, since it has been shown (7) that the rheologic behavior of asphalt concretes is linear within a range of deformations, e.g., <1200 $\mu\epsilon$ for axial compressive strain in unconfined tests at 21°C (70°F) and <600 $\mu\epsilon$ at 38°C (100°F).

Consequently, the viscoelastic function is independent of test type if the deformations are not too great. For this reason, we think that at the present time linear isotropic viscoelasticity is the only practical approach for characterizing asphalt materials. The viscoelastic parameters of creep compliance can be measured by creep tensile tests at the required temperature. Laboratory creep tests are done with an electrohydraulic system. A step load-controlled function is applied to a cylindrical specimen. The axial strain can be measured 0.1 s after load application with two strain gauges cemented on diametrically opposite sides. A carrier frequency amplifier and a photographic galvanometer recorder are used for strain output measurements.

Specimens are prepared to ensure uniform bulk density. Samples are compacted by the application of static loading at the two ends of a cylindrical mold by means of two free, opposing plungers (8). The distribution of the bulk density inside the cylindrical specimen is checked by means of gamma ray absorption equipment (9).

THEORETICAL RESULTS FOR A TYPICAL TWO-LAYERED SYSTEM

For a sequence of circular loads moving on the surface of a double viscoelastic layer, the following model parameters were chosen:

- A = thickness of first layer of asphalt concrete = 30 cm (12 in);
- R = load radius = 12.16 cm (4.8 in);
- P = load value corresponding to a uniform pressure of 686.4 kPa (99.5 lbf/in^2) = 3250 kg (7165 lb);
- α_1 = parameter of creep compliance for first layer at 20°C (68°F) = 0.3;
- J_1 = parameter of creep compliance for first layer at 20°C (68°F) = 6.8 mm^2/N·s$^{-\alpha}$ (0.047 in^2/lbf·s$^{-\alpha}$);
- τ_o = parameter of creep compliance for first layer at 20°C (68°F) = 10 000 s;
- α_2 = parameter of creep compliance for second layer = 0;
- J_2 = parameter of creep compliance for second layer = 33.6 mm^2/N·s$^{\alpha}$ (0.23 in^2/lbf·s$^{\alpha}$);
- τ_o = parameter of creep compliance for second layer = 0;
- v = load velocity = 10 km/h (6.2 mph); and
- t_o = waiting time between two consecutive loads = 2 s.

For the sake of simplicity the second layer was assumed to be elastic (α_2 = 0), and all materials were assumed to be incompressible (Poisson's ratio = 0.5).

Figure 1 shows depth versus the total vertical strain localized along the axis of the Nth moving load, for N = 1, 100, and 1000.

THEORETICAL RESULTS OF VISCOELASTIC THEORY ON A ONE-LAYERED SYSTEM WITH RIGID SUPPORT

Our viscoelastic theory was applied to the design of an asphalt pavement for a steel slab bridge on an Italian motorway.

The thickness of the asphalt-bound material was 10 cm (3.9 in). We assumed that there were no friction interface conditions between steel and asphalt layer, because of the presence of a thin [2 mm (0.08 in)] layer of asphalt waterproofing. The steel was assumed to behave rigidly.

The creep compliance of asphalt concrete was measured in our laboratory at 20 and 40°C (68 and 104°F) (Figure 2). The aggregate gradation for the asphalt concrete composition is given below, where 1 μm = 39 μin.

Figure 1. Accumulation of vertical deformation versus depth along center of mass of load system.

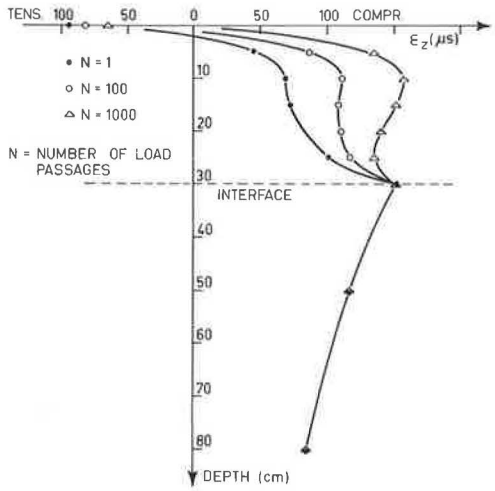


Figure 2. Creep compliance of asphalt concrete measured in laboratory.

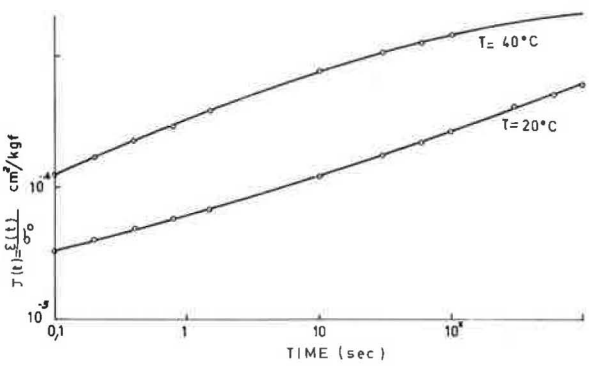
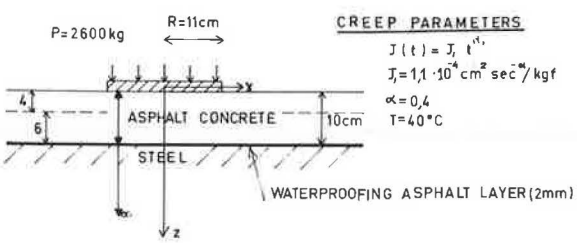


Figure 3. Experimental load parameters.



Sieve Size (μm)	Percent Passing by Weight	Sieve Size (μm)	Percent Passing by Weight
12 500	100	850	23.5
9 500	89.3	425	17.0
6 300	70.0	180	12.1
4 000	53.5	75	9.0
2 000	34.5		

The percentage of asphalt by weight of total mix (60 to 70 penetration) was 5.5; the bulk density was 2.29 g/cm^3 (0.082 lb/ft^3); the percentage of voids was 6.3; and the specific gravity of aggregate was 2.66 g/cm^3 (0.096 lb/ft^3).

The viscoelastic deformations were calculated at 40° C for velocities and experimental load parameters shown in Figure 3.

Vertical deformations relative to distance from the load center at a fixed depth of 4 cm (1.6 in) and velocity of 5 km/h (3.1 mph) are shown in Figure 4. Positive distances correspond to instants of time preceding load passage. The lack of symmetry of the curves with respect to time reversal should be noted. The same configuration can be seen for the transverse (Figure 5) and the longitudinal deformations (Figure 6).

The influence of velocity is clearly shown in Figure 7, where the maximum tensile longitudinal strain is reported as a function of velocity. It must be pointed out that the calculations show a considerable delay in recovering from deformation for the vertical and the transverse deformations only, which experience accumulation when the paving system is subjected to repeated moving loads.

Longitudinal and transverse deformations in the asphalt pavement were measured at depths of 4 and 10 cm (1.6 and 3.9 in) by means of strain gauges incorporated into the concrete during the laying operations (Figure 8). Ordinary strain gauges glued between thin kapton foil and HBM-DD 1 strain gauges were used for the measurements. A typical measured longitudinal deformation profile is shown in Figure 9. The transverse deformation is shown in Figure 10. Measurements refer to 40°C (±2°C) (104°F). In conformity with theoretical predictions, no accumulation was observed for the longitudinal strain.

Calculated and measured amounts were correlated for the transverse strain and are shown in Figure 11, and in Figure 7 for the maximum longitudinal strain. The comparison between experimental and theoretical values for longitudinal strain (at a depth of 4 cm) is fairly good with reference to the velocity dependence of deformation. Actual experimental figures are 15 to 20 percent larger than the calculated ones.

Figure 4. Viscoelastic solution for a moving load: vertical deformation.

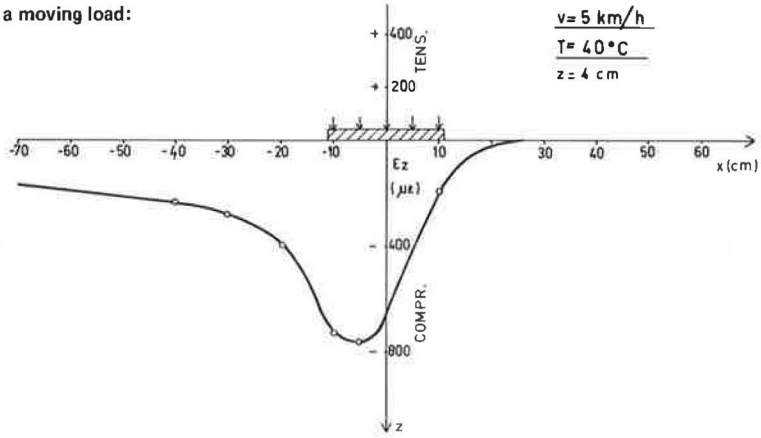


Figure 5. Viscoelastic solution for a moving load: transverse deformation.

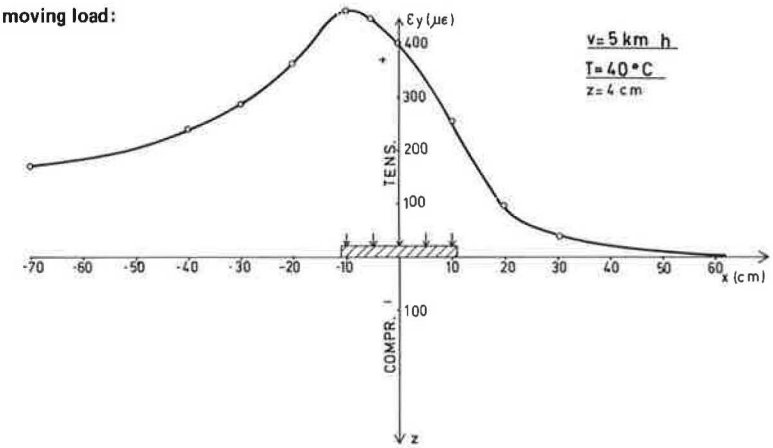


Figure 6. Viscoelastic solution for a moving load: longitudinal deformation.

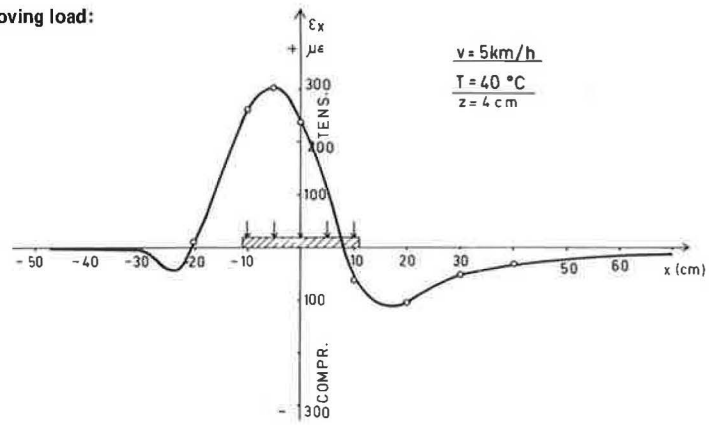


Figure 7. Longitudinal deformation versus load velocity.

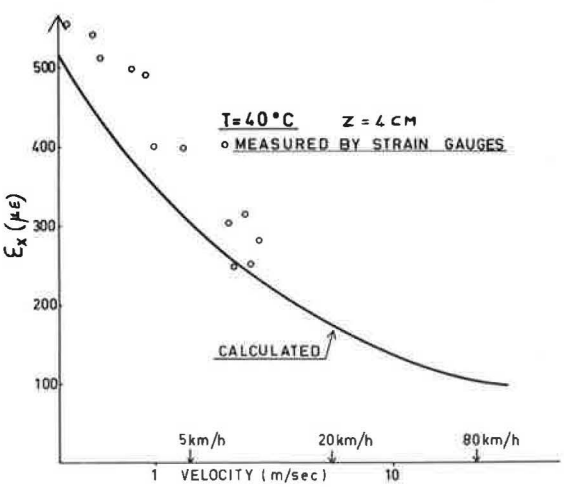


Figure 8. Strain gauges for measuring transverse and longitudinal deformation.

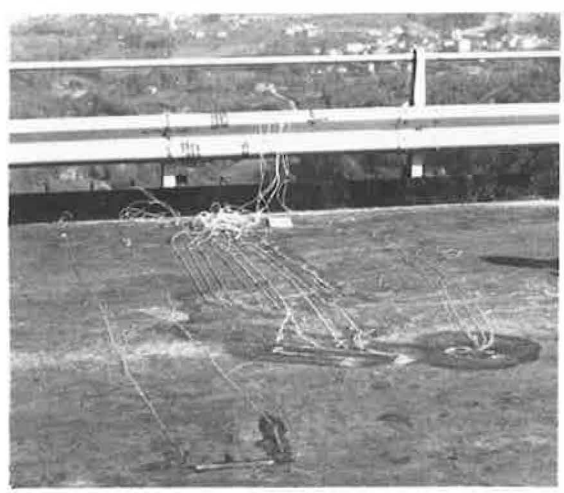


Figure 9. Experimental longitudinal deformation.

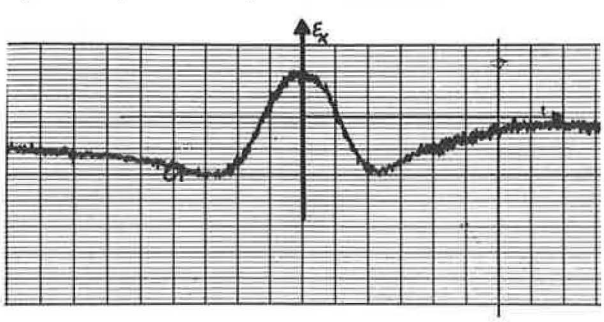


Figure 10. Experimental transverse deformation.

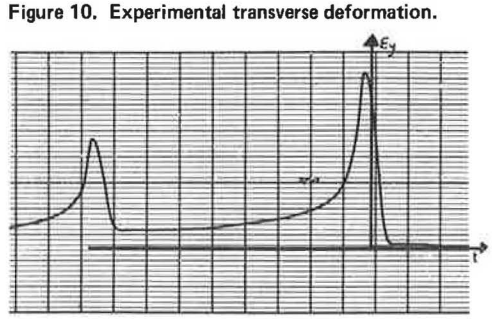


Figure 11. Calculated and experimental transverse deformation.

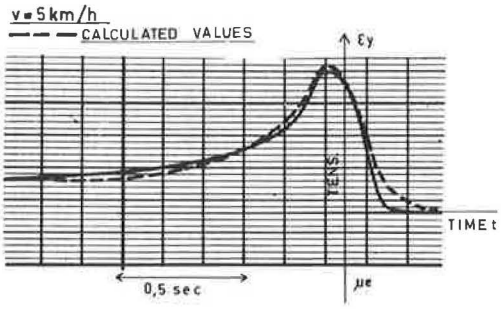
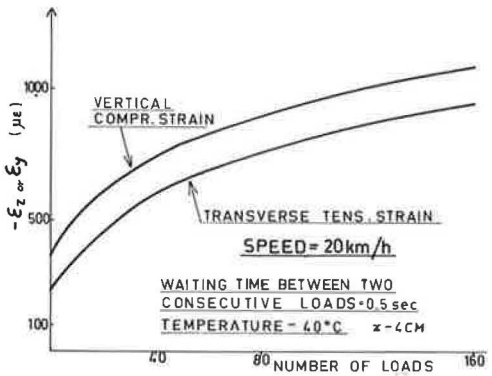


Figure 12. Accumulation of vertical and transverse deformation.



Theoretical calculation of accumulated deformation was performed for vertical and transverse strains; the fixed waiting time between two consecutive loads was 0.5 s (this is adequate when considering a large number of loads) and velocity was 20 km/h (12.4 mph). The strain accumulation is shown according to the number of loads in Figure 12; N = 40 represents five heavy trucks with trailers (under Italian traffic conditions).

The increase in vertical and transverse deformations is significant: After N = 20 passages, the transverse deformation is two times greater than that for a single passage.

Experiments are being conducted to validate our predictions concerning the repeated load effect.

GENERAL CONCLUSIONS

On the basis of the viscoelastic analysis of a two-layered system subjected to moving loads, the following can be said:

1. The calculations show considerable accumulation of deformations for the vertical and the transverse strains, but not for the longitudinal strain;
2. Strain measurements carried out in the asphalt layers on a steel bridge of an Italian motorway confirm the theoretical predictions;
3. Those viscoelastic parameters for the asphalt layers used in our viscoelastic model can be measured by simple uniaxial laboratory creep tests; and
4. The accumulation of deformations in asphalt pavements cannot be adequately investigated by repeated application of static loads, for with moving loads the loading time is related to load velocity and also depends on depth.

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