# Fatigue Behavior of Cement-Treated Materials

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**Fatigue-failure criteria based on the Griffith failure theory have been developed to describe the behavior of cement-treated pavement materials subjected to repeated multiaxial stress applications. These criteria are represented by two relationships, one for curing periods of 4 weeks and the other for curing periods of 10 weeks, that show the variation of the maximum stress level as a function of the number of stress applications required to cause failure. The stress level is defined in terms of the applied principal stresses and the initial tensile strength. For a given set of applied stress pulses, there will be a maximum value of stress level. Fatigue failure occurs when the tensile strength decreases from its initial value to the maximum value of the stress level. The number of stress applications to cause failure can thus be expressed as a function of the stress factor and the tensile strength. This relationship is independent of the duration and frequency of the applied stress pulses. The proposed criteria agree well with fatigue data from a number of investigations, which indicates their general validity.**

In recent years, there have been a number of laboratory studies to determine the fatigue response of cement-stabilized materials. The data developed from these studies have been used to establish design criteria for improved use of cement-treated materials in pavement structures for both highway and airfield pavements. Table 1 summarizes the criteria developed from a number of investigations using repeated-load tests on (a) simply supported beams in flexure [e.g., Pretorius (1), Otté (2), Irwin (3), Scott (4), and Mitchell and Shen (18)], (b) beams resting on an elastic foundation [e.g., Larsen and others (5)], and (c) specimens in direct uniaxial tension or compression [e.g., Bofinger (6)].

These criteria, however, have the following limitations:

1. They have been developed by using the results of simple flexure and direct tension tests in which the critical state of stress at the point at which cracking initiates is either uniaxial or biaxial rather than triaxial as is an actual pavement structure, and therefore a different layer thickness might be selected by using a maximum strain criterion than by using a maximum stress criterion (7);
2. The effects of variations in the frequency, shape, and duration of the stress or strain pulse have not been considered;
3. The additional repetitions required to propagate a crack through the cement-stabilized base after it initiates (8) have not been accounted for in a number of these criteria; and
4. The fatigue behavior of cement-treated materials subjected to compound loading has not been investigated, although the linear summation of cycle ratios has been proposed as a reasonable hypothesis for cumulative damage.

In this paper, the Griffith failure theory (9) is used to develop an analytical model of the fatigue behavior of cement-treated materials subjected to triaxial stress pulses. The results of laboratory tests (8) are used to define fatigue criteria suggested by this analytical model.

Predictions using these criteria are compared with published fatigue data. Verification of these criteria is important because the model from which the criteria were developed can be used to establish an analytical approach for the design of cement-stabilized layers in pavement sections that accounts for both crack initiation and propagation in the layers.

## FATIGUE-FAILURE CRITERIA

### Failure Under Static Loading

Griffith (9) derived a criterion for failure under a two-dimensional state of stress by assuming that fracture is caused by stress concentrations at the tips of minute Griffith cracks or starter flaws that are presumed to occur in the material and that it is initiated when the maximum stress near the tip of the most favorably oriented crack reaches a value characteristic of the material. The Griffith criterion for failure can be written as

$$(\sigma_1 - \sigma_3)^2/(\sigma_1 + \sigma_3) = 8T_0 = \sigma_c \qquad (\sigma_1 + 3\sigma_3 \geqslant 0) \tag{1}$$

and

$$\sigma_3 = -T_0 = -\sigma_c/8 \qquad (\sigma_1 + 3\sigma_3 \leqslant 0) \tag{2}$$

where

$\sigma_1$ = major principal stress,
$\sigma_3$ = minor principal stress,
$T_0$ = tensile strength, and
$\sigma_c$ = unconfined compressive strength.

(Compressive stresses are positive; tensile stresses are negative.) The relationships given in Equations 1 and 2 were derived by assuming that the starter flaws in the material remained open under the action of the applied stresses, $\sigma_1$ and $\sigma_3$.

The strength of cement-treated materials has been studied under triaxial loading conditions (i.e., with $\sigma_2$ equal to $\sigma_3$) by, e.g., Pretorius (1), Nash and others (10), and Abboud (11), and under biaxial loading conditions ($\sigma_2$ equal to 0) by Bresler and Pister (12). Figure 1 presents a plot of this data in normalized form (i.e., $\sigma_1/\sigma_c$ versus $\sigma_3/\sigma_c$, where $\sigma_c$ = unconfined compressive strength). [The data reported by Bresler and Pister (12) are for a concrete material that can be considered similar to soil-cement; they have been included because they are the only known data available in this range of loading conditions.] The information shown in Figure 1 indicates that the Griffith criterion is applicable for values of $\sigma_3/\sigma_c < 0.10$, but that for $\sigma_3/\sigma_c > 0.10$, a modified criterion developed by McLintock and Walsh (15) that assumes that the starter flaws in the material are closed could be used in the form of $\sigma_1 = 5\sigma_3 + \sigma_c$.

As will be seen, the Griffith crack approach appears applicable to the description of cement-treated soils under the various combinations of $\sigma_1$ and $\sigma_3$ that are likely to be encountered in a treated layer. It also offers the advantage of requiring knowledge of only the unconfined compressive strength for its application.

**Table 1. Design criteria for soil-cement bases using present analytical approaches.**

| | | Suggested Criteria | | | | |
|---|---|---|---|---|---|---|
| | | | Number of Repetitions | | | |
| Material | Dynamic Test | Parameter | N | $10^6$ | Unlimited | Reference |
| Soil-cement<br>Silty clay | Direct tension | Maximum tensile stress | – | – | 18 lbf/in$^2$[a] | Bofinger (6) |
| Soil-cement<br>Eliot sand mixture<br>Vicksburg silty clay | Flexural beam | Maximum flexural stress or strain level[b] | – | – | 0.50 | Mitchell and Shen (18) |
| Soil-cement<br>Granular soil<br>A-1, A-2-4, A-2-5, A-3<br>Fine-grained soil<br>A-2-6, A-2-7, A-4, A-5 | Flexural beam on elastic foundation | Radius of curvature (R) | R[c] | – | – | Larsen, Nussbaum, and Colley (5) |
| Cement-treated | Flexural beam | Maximum flexural-strain level[b] | – | 0.33 | 0.25 | Otté (2) |
| Soil-cement<br>A-1-b, A-2-4, A-3, A-4 | Flexural beam | Maximum flexural-stress level[b] | S[d] | 0.67 | – | Scott (4) |
| Cement-treated<br>Gravelly sand<br>Vicksburg lean clay | Flexural beam | Energy density (lb in/ft$^3$)[a, e] | – | 0.0018 (gravelly sand)<br>0.0030 (lean clay) | – | Irwin (3) |
| Cement-stabilized | Flexural beam | Maximum flexural strain ($\epsilon_1$ in/in $\times 10^{-6}$)[a] | N[f] | 50 in/in $\times 10^{-6}$ | – | Mitchell and others (7) |

[a] These criteria were designed for U.S. customary units only; therefore SI units are not given.

[b] Ratio of applied stress or strain to value at failure in static testing.

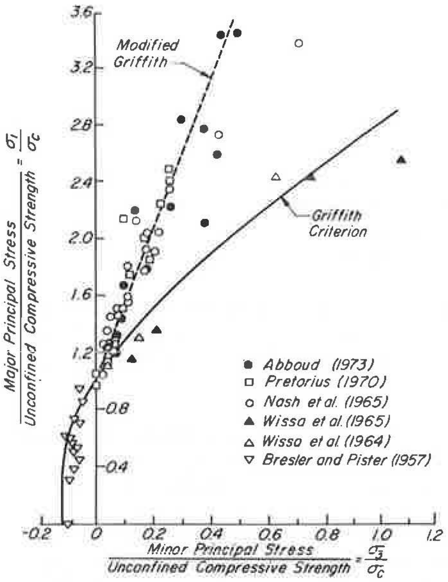
[c] $R = R_c N^a h^{3/4}/(2.1h - 1)$ where R = radius of curvature under a given wheel load (in), h = thickness of base (in), N = number of repetitions, a = 0.025 for granular soil or 0.05 for fine-grained soil, and $R_c$ = critical radius of curvature for a base of thickness h.

[d] $S = 94.4 - 4.71 \log N$.

[e] Energy density for a linear elastic material under a given state of triaxial stresses is given by $U_d = (1/2E)(\sigma_1^2 + \sigma_2^2) + \sigma_3^2 - (\mu/E)(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)$, where $\sigma_1$, $\sigma_2$, and $\sigma_3$ are the principal stresses; $\mu$ = Poisson's ratio; and E = modulus of elasticity.

[f] $N = 142^{20.3}/\epsilon$.

**Figure 1. Failure envelope for cement-treated soils.**



## Analytical Fatigue Model

Confining pressures ($\sigma_3$) in cement-treated bases are generally small because such base materials support traffic loads in flexure, and the overburden pressure is low. Thus, Griffith's failure theory rather than the modified Griffith theory is applicable [i.e., $\sigma_3/\sigma_1 \leq 1/13$ (8)].

At any point in the base, the stress state generated by a moving wheel load can be represented by the principal stresses ($\sigma_1$, $\sigma_2$, and $\sigma_3$). Because these stresses are repeatedly applied under the action of traffic loads, the strength of the cement-treated material will decrease until failure occurs. It may be hypothesized that the decrease in strength is caused by a reduction in bond strength between the soil, aggregate, and cement.

Mathematically, this strength reduction can be described as follows. Let F be a stress factor defined as

$$F = (\sigma_1 - \sigma_3)^2/8(\sigma_1 + \sigma_3) \qquad (\sigma_1 + 3\sigma_3 \geq 0)$$
$$F = -\sigma_3 \qquad (\sigma_1 + 3\sigma_3 \leq 0) \qquad (3)$$

Failure will take place when the tensile strength of the material decreases from an initial value ($T_i$) to a value ($T'$) equal to the maximum value of the stress factor ($F_{max}$), which is defined by the $\sigma_1$ and $\sigma_3$ pulses as shown in Figure 2.

The decrease in tensile strength for a clayey, gravel soil-cement is a function of $F_{max}$ and is independent of the frequency, shape, and duration of the applied principal-stress pulses. The variation of tensile strength with the number of repetitions of stress applications for a given $F_{max}$ is shown schematically in Figure 3.

The strength decrease illustrated in Figure 3 can be defined in terms of a rate (a) given by

$$\begin{aligned} a &= (\log T_i - \log T')/(\log N_f - \log 1) \\ &= (\log T_i - \log F_{max})/\log N_f \\ &= \log [1/(F_{max}/T_i)]/\log N_f \end{aligned} \qquad (4)$$

where $N_f$ = number of repetitions to failure.

The parameter a can be determined by measuring the slope of the function that represents the decrease in tensile strength with number of stress repetitions. For a given $F_{max}$, there corresponds a given value of a. The variation of a as a function of the maximum stress level (i.e., $F_{max}/T_i$) for a clayey, gravel soil-cement is shown in Figure 4.

These relationships were determined from repeated-load triaxial tests on a soil-cement consisting of a well graded gravel [19-mm (¾-in) maximum particle size] combined in a 5 to 1 ratio with silty clay (1, 8). A water content of 7.5 percent and 5.5 percent cement were used to prepare samples that were then cured in a humid room for varying periods of time. The magnitude, frequency, and duration of the applied $\sigma_1$ and $\sigma_3$-stress pulses were varied, and the decrease of tensile strength was found to depend only on $F_{max}/T_i$.

From the definition of a, the number of stress repetitions to failure ($N_f$) can be written as

$$\log N_f = \log [1/(F_{max}/T_i)]/a \qquad (5)$$

and therefore, the number of stress repetitions to failure can be determined for a given a and $F_{max}/T_i$. The relation that describes the variation of $F_{max}/T_i$ with $N_f$ defines the fatigue-failure criterion for a cement-treated material. The values for a and $F_{max}/T_i$ are obtained from Figure 4 and used in Equation 5 to establish a relationship between $F_{max}/T_i$ and $N_f$ as shown in the following table and in Figure 5. The relationships shown in Figure 5 were also examined relative to their applicability to various other cement-treated soils.

Figure 2. Variation of F for given $\sigma_1$ and $\sigma_3$ pulses.

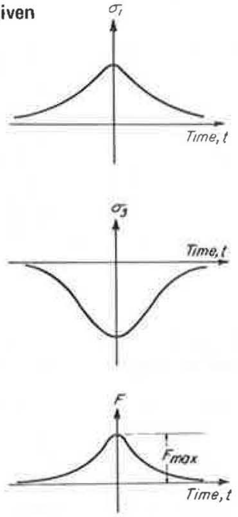


Figure 3. Variation of tensile strength with number of stress applications for a given $F_{max}$.

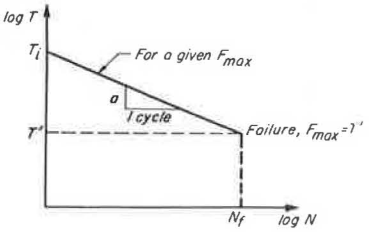
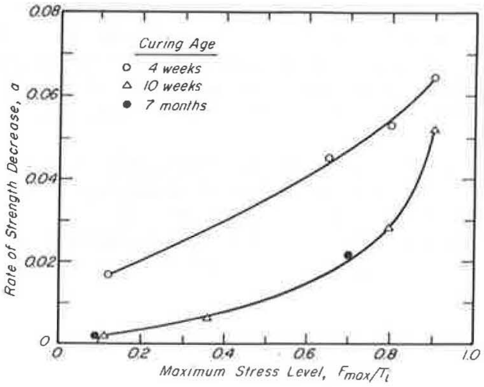


Figure 4. Variation of a with $(F_{max})/T_i$.



|  | $F_{max}/T_i$ |  |
|---|---|---|
| $N_f$ | Samples Cured 4 Weeks | Samples Cured 10 Weeks |
| $10^{35}$ | 0.20 | 0.47 |
| $10^{7}$ | 0.55 | 0.71 |
| $10^{6}$ | 0.58 | 0.73 |
| $10^{5}$ | 0.62 | 0.75 |
| $10^{4}$ | 0.67 | 0.78 |
| $10^{2}$ | 0.78 | 0.84 |
| 1 | 1.00 | 1.00 |

## COMPARISON OF EXPERIMENTAL FATIGUE DATA AND THE FATIGUE-FAILURE CRITERIA

The fatigue data used to evaluate the criteria are given in Table 2 and include the following variables: (a) type of stabilized material, (b) cement content, (c) frequency of applied stress pulses, and (d) curing age.

Comparisons of the experimental and analytical results are shown in Figures 6 to 10. The ratio of the applied stress to the initial strength (i.e., the maximum stress level) is plotted versus the number of repetitions to failure. As noted in Table 2, different tests were used to obtain the fatigue data, including (a) rotating cantilever beam (13), (b) simply supported beam (1, 3, 4, 6), and (c) repeated compression (14). To use Symons' data, which is presented as applied stress expressed as a percentage of static strength versus number of repetitions to failure, $F_{max}$ was taken equal to the applied tensile stress, and $T_i$ was assumed to be the applied stress at $N_f$ equal to one repetition. Analysis of the data shown in Figures 6 to 10 leads to the following conclusions.

1. The fatigue data obtained by testing at different frequencies (200 to 500 and 2800 revolutions/min) of applied stress pulses agree reasonably well with the fatigue-failure criteria in materials ranging from well graded sand to silty clay (Figures 6 to 8). Thus, the fatigue-failure criteria are appropriate for the prediction of the fatigue failure of cement-treated soils independent of the loading frequency.
2. The fatigue criteria for a curing age of 4 weeks agree well with the fatigue data for cement-treated materials that have a curing age of up to 4 weeks (Figures 6 to 9), and the criteria corresponding to a curing age of 10 weeks agree with the data for a curing age of 10 or more weeks (Figure 10).
3. There is reasonable agreement between the experimental values and those that correspond to the suggested analytical criteria, independent of the type of stabilized soil and the cement content (Figures 6 to 10).

Figure 5. Suggested fatigue-failure criteria for cement-treated soils.

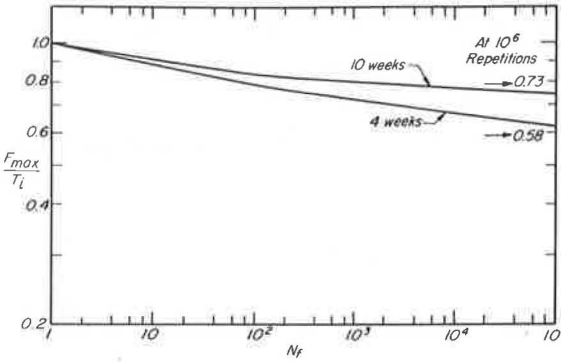


Table 2. Fatigue data for cement-treated soils.

| Material | Cement Content (%) | Water Content[a] (%) | Curing Age (d) | Dry Density (Mg/m$^3$) | Repeated Load Test | Reference |
|---|---|---|---|---|---|---|
| Well graded sand | 6 to 14 | 9 to 11 | 7 to 28 | 1.99 | Rotating cantilever | Symons |
| Uniformly graded sand | 12 to 16 | 13 | 7 | 1.65 | Rotating cantilever | Symons |
| Silty clay | 6 to 14 | 19 | 7 | 1.68 | Rotating cantilever | Symons |
| Crushed rock | 4 to 8 | 8 | 7 | 2.03 | Rotating cantilever | Symons |
| A-2-4, A-4, A-3, A-1-6 | 6.5 to 8.5 | 7 to 11 | 365 | 1.78 to 2.18 | Flexural beam | Scott |
| A-2-4, A-4 | 6.7 to 8.4 | 10.7 to 12.4 | 28 | 1.78 to 1.89 | Flexural beam | Scott |
| Lean clay | 10 | 15.7 | 14 to 28 | 1.73 | Flexural beam | Irwin |
| Gravelly sand | 6 | 6.2 | 21 | 2.15 | Flexural beam | Irwin |
| Clayey gravel | 5.5 | 7.5 | 90 | 2.19 | Flexural beam | Pretorius |
| Silty clay | 8 to 16 | 38 | 14 | 1.39 | Flexural beam | Bofinger |
| Uniform sand | 6 | 9 | 2 to 14 | 1.84 | Direct compression | Gregg |

Note: 1 Mg/m$^3$ = 62.4 lb/ft$^3$.

[a] Mixing content.

Figure 6. Comparison of fatigue data for cement-treated (6 to 14 percent cement) well graded sand (7 and 28 d curing) at different loading frequencies with suggested fatigue-failure criterion.

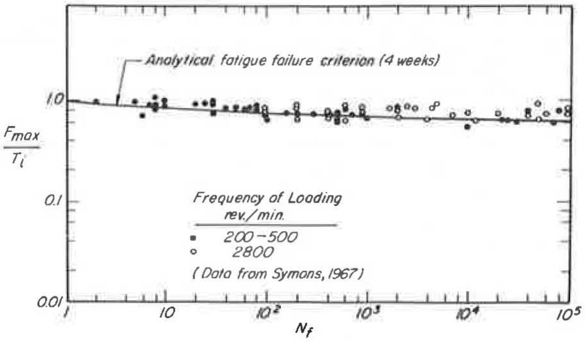


Figure 7. Comparison of fatigue data for cement-treated (12 to 16 percent cement) uniformly graded sand (7 d curing) at different loading frequencies with suggested fatigue-failure criterion.

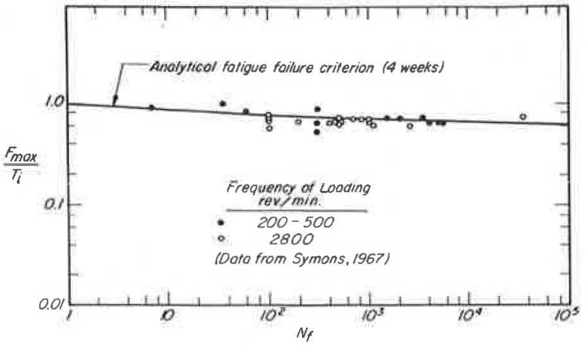


Figure 8. Comparison of fatigue data for cement-treated (6 to 14 percent cement) silty clay (7 d curing) at different loading frequencies with suggested fatigue-failure criterion.

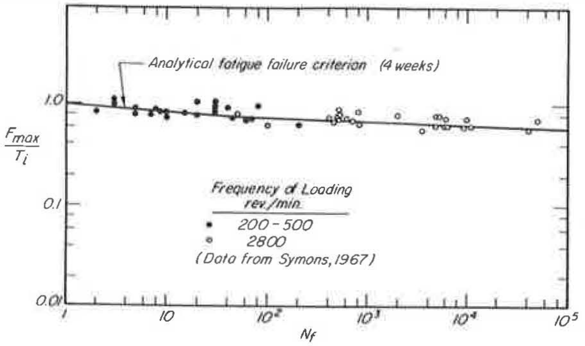


Figure 9. Comparison of fatigue data for cement-treated soils (<4 weeks curing) with suggested fatigue-failure criterion.

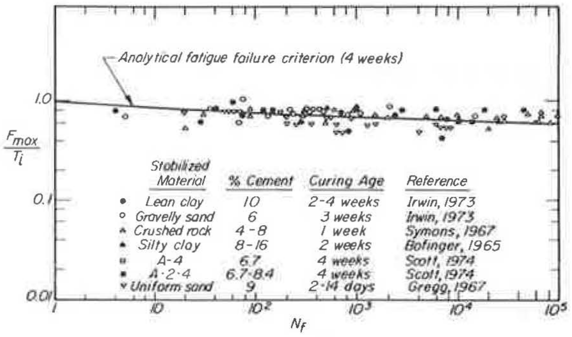
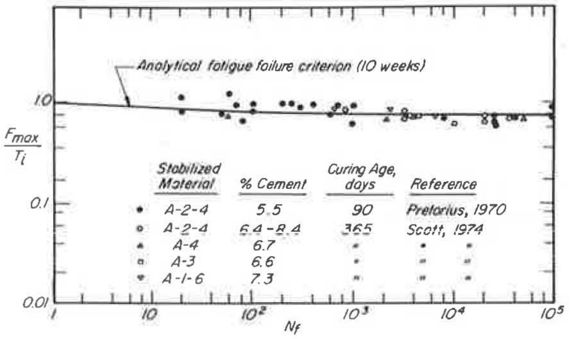


Figure 10. Comparison of fatigue data for cement-treated soils (>10 weeks curing) with suggested fatigue-failure criterion.



## SUMMARY

Fatigue-failure criteria corresponding to curing ages of 4 and 10 weeks have been developed for a clayey, gravel soil-cement by using an analytical fatigue model based on the Griffith failure theory. In these criteria, the number of repetitions to failure is expressed in terms of the maximum stress level applied, and the resulting relationship is independent of pulse duration, frequency, and shape.

The suggested criteria fit the available fatigue data for cement-treated soils, for different curing ages, independent of type of material, cement content, and frequency of applied loads, which indicates the possible applicability of these criteria for cement-treated soils in general.

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**Mr. Raad was at the Department of Civil Engineering, University of California, Berkeley, when this research was performed.*

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# Stabilization of Expansive Shale Clay by Moisture-Density Control

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**Stabilization of expansive Pierre shale has been a continuing problem in South Dakota for many years. A procedure has been developed to produce a roadbed with very few differentials that uses special undercutting to a depth of 1.83 m (6 ft), replacement of subgrade by selected materials, rigid control of moisture to achieve low density at a high moisture content, and lime treatment in the upper 15.2 cm (6 in) of subgrade. A density of 92 percent of the maximum American Association of State Highway and Transportation Officials T 99 test value and a moisture content of 3 percent above the optimum were set as targets. The high degree of stability of roads constructed by this procedure is shown by the very good roughness-index ratings. The average roughness index, based on a 0 to 5 rating system, is 4.34 for the full length of 209.2 km (130 miles) of surfacing. The projects observed and tested for this study have been in service from 5 to 8 years.**

A good road is one that is stable from the subgrade depth to the top of the surfacing. To achieve this type of stability, it is necessary to design each component so as to use the best materials and techniques available. Stability, especially in the subgrade, is dependent on the environment in which the material is located. All of the components of the road structure must be able to resist the deteriorating effects of climatic cycles and traffic loads. A road structure is usually divided into four component parts—subgrade, subbase, base, and bituminous mat or wearing course. In a rigid design, it is composed of a concrete surfacing and a base. The subgrade is the native material below the imported materials.