Prediction Model for Unsaturated Hydraulic Conductivity of Highway Soils

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A method of predicting the hydraulic conductivities of subgrades and highway soil materials is discussed. In this method, the relation of soil moisture content and suction head is used to calculate the unsaturated hydraulic conductivity of soil. The value of the hydraulic conductivity at saturation (the soil permeability) is used as a matching factor during the calculations. The soil moisture content and suction head relation and the saturated conductivities of subgrades were determined in the laboratory by using the commercially available Tempe cell. Disturbed and undisturbed samples of Drummer and Fayette-C subgrades, Ottawa sand, and class X concrete sand were used in this study. The comparison between the experimental and the calculated values shows that the method successfully predicts the hydraulic conductivities of highway soil materials.

The engineering problems associated with the behavior of highway subgrade soils and pavement systems in response to moisture changes have been widely studied. The seasonal change in moisture content of subgrade soils and its effects on structural pavement performance are of particular interest to many highway engineers. That the shearing strength of a subgrade soil can be greatly reduced by the influx of moisture during spring thaw or long periods of heavy rainfall is well documented (1). This reduction in strength is generally attributed to an increase in moisture content of the subgrade soil and results in low soil suction (negative pore pressure) between the soil particles and sometimes an associated decrease in soil density. Consequently, the bearing capacity of the subgrade will be significantly reduced, and extensive deflection of the highway pavement may result (2).

It is generally accepted that the shearing strength and the bearing capacity of a fine-grained subgrade soil reach their lowest values at the beginning of the thawing period in the spring when soil suction reaches minimum. During this time frost boils, pumping, and pavement breakup may occur under a moving load (3). After the thawing period low soil suction will gradually increase with time, and as a result the subgrade will gradually gain in strength (4).

Dempsey and Elzeftawy (5) found that numerical models can be used to accurately predict moisture content and its movement in subgrade soils and pavement systems in conditions of constant or variable water-table depths. The mathematics in the numerical method are simple, flexible, and well suited for programming on a digital computer. Several methods of calculating unsaturated hydraulic conductivity by using data on pore-size distribution have been proposed for agricultural soils (6, 7). The calculated values have subsequently been compared with experimentally determined conductivities by many researchers (7). Elzeftawy and Mansell (8) recently concluded that a slightly modified Green and Corey method (7) can be successfully used to predict the hydraulic conductivity and soil moisture diffusivity of a fine sandy soil.

OBJECTIVES

Highway engineers have often questioned whether the moisture properties (parameters) of highway soils can be predicted or calculated for unsaturated or saturated water flow from a simple laboratory test. The purposes of this paper are (a) to show that the moisture properties of highway soils and subgrade can be predicted by using relations between soil moisture content and suction and (b) to propose a simplified laboratory procedure to determine the moisture content-suction relations for highway soils.

BACKGROUND

The transmission of water by soils has widespread relevance to engineering, geologic, and agricultural problems. It is customary to treat flow problems as essays in the solution of Laplace's equation, assuming that the soil-water body obeys Darcy's law, which may be written

$$v = -K \ grad \ \phi \qquad (1)$$

where

> v = flow velocity commonly expressed as cm^3/s/unit area normal to v,
> ϕ = hydraulic potential, and
> K = constant characteristic of the soil, called the permeability of soil to water or hydraulic conductivity.

This law seems to be valid for Reynold numbers less than unity (9).

The one physical property of the soil that enters into a flow problem is the permeability; it must be known if a complete solution is to be obtained. In principle, a measurement of K is a simple matter of measuring the rate of flow of water in a column of the soil between planes of measured separation and hydraulic potential. Permeability has been held to decrease with time because of the percolating water releasing dissolved air into the soil pores, the swelling of colloidal materials, the growth of organisms in the pore spaces, the mechanical blocking by movement of the finest particles of non-cemented material, and the chemical effect of the flowing water on the soil.

It has been more common to describe a granular material in terms of particle-size distribution (mechanical composition) than to regard it as a porous material with a given pore-size distribution. The best known procedure to calculate hydraulic conductivity (K) is perhaps that of Kozeny (10), but somewhat similar expressions have been derived by Terzaghi (11), Zunker (12), and others. The generalized Kozeny equation can be written as follows:

$$K = (g\rho/k\eta)(1/A^2)f^3/(1 - f)^2 \qquad (2)$$

where

g = gravitational constant;
ρ and η = water density and viscosity respectively;
k = arbitrarily determined pore-shape factor, commonly in the range from 2.0 to 2.5;
A = specific surface area of soil, namely, the total surface of the solid part divided by the volume of that solid part; and
f = soil porosity.

However, it can be shown on theoretical grounds (13) that an expression such as Equation 2 is not applicable to a bundle of capillary tubes if the radii are distributed over a wide range of sizes. Surprisingly, the Kozeny formula does give approximately correct values of conductivity for a variety of industrial powders; in practice, however, it is not possible to vary the porosity over a sufficient range to provide a searching test of the formula in regard to the porosity factor.

Dependence of Hydraulic Conductivity on Moisture Content of Soil

Engineers know that Darcy's law (Equation 1) is valid for a wide texture range of soils saturated with water for Reynold numbers less than unity. The validity of this law when the soil is unsaturated (the degree of saturation is less than 100 percent) has been established during the last 10 years. In its wet state a typical soil contains pores with an upper size limit of about a millimeter, and very little water is lost until the suction exceeds something of the order of a few centimeters of water. Emptying a pore at such suction leaves the solid walls coated with a very thin film of water in which liquid flow takes place slowly (compared with liquid flow when the pore is full). An empty pore thus contributes only negligibly to the total hydraulic conductivity of the soil body. A reduction of the moisture content is thus equivalent to a reduction of effective porosity for the purpose of assessing conductivity and results in a reduction of that conductivity. Because the moisture content is progressively reduced by a progressive increase of suction, the larger pores are emptied in the earlier states of unsaturation and the smaller pores are left full of liquid water. It follows that the earlier stages of moisture reduction are more effective than the later stages in reducing conductivity. It should be emphasized that a pore full of air is not merely ineffective as a conductor but also becomes an obstacle: Liquid water that originally passed through when the pore was full of water is deflected around it when it is dried. In effect the true flow paths become more tortuous and therefore longer; i.e., the drier the soil becomes, the more tortuous are the flow paths.

Model to Predict Hydraulic Conductivity

Childs and Collis-George (13), Millington and Quirk (14), Green and Corey (7), and others have explored the possibility of predicting the hydraulic conductivity of soils and porous materials from data on pore-size distribution. Such predictions are of interest because the hydraulic conductivity-moisture content function is relatively difficult to measure whereas pore-size distribution is easily characterized by the standard measurement of moisture content versus suction.

The conductivities are obtained by dividing the relation of moisture content and suction head [$h(\theta)$] into n equal water-content increments, obtaining the suction (h) at the midpoint of each increment, and calculating the conductivity by using the following equation:

$$K(\theta)_i = (30\gamma^2/\rho g\eta)(\epsilon^p/n^2) \sum_{j=i}^{m} [(2j + 1 - 2i)h_j^2] \quad i = 1, 2, \ldots, m \quad (3)$$

where

$K(\theta)_i$ = calculated conductivity for a specified moisture content corresponding to the ith increment (cm/min);
θ = moisture content (cm^3/cm^3);
γ = surface tension of water (N/cm);
ρ = density of water (g/cm^3);
g = gravitational constant (cm/s^2);
η = kinematic viscosity of water (cm^2/s);
θ_s = saturated moisture content (cm^3/cm^3);
ϵ = water-saturated porosity (cm^3/cm^3), i.e., $\epsilon = \theta_s$;
p = constant whose value depends on the method of calculation (7), equal to 2 in these calculations;
θ_o = lowest moisture content on the experimental $h(\theta)$ curve;
n = total number of pore classes between $\theta = \theta_o$ and θ_s $\{n = m[\theta_s/(\theta_s - \theta_o)]\}$;
i = last moisture-content increment on the wet end (e.g., i = 1 identifies the pore class corresponding to θ_s and i = m identifies the pore class corresponding to θ_o);
h_j = suction (negative pressure) for a given class of moisture-filled pores (centimeters of water head); and
30 = the composite of the constant 1/8 from Poiseuille's equation, 4 from the square of r = $2\gamma/h$, where r is the pore radius and 60 converts from seconds to minutes.

Elzeftawy and Mansell (8) and Green and Corey (7) conclude that Equation 3 yields reasonable values of the hydraulic conductivities for a range of soil types if a matching factor (usually the ratio of the measured to the calculated saturated conductivity) is used. Elzeftawy and Mansell (8) state that matching at water saturation has a distinct advantage over matching at desaturated moisture contents because inaccuracies in calculated $K(\theta)$ can be more easily tolerated at lower moisture contents than at high moisture contents when calculated results are to be used in subsequent prediction of the movement of water in field soils. They also mention that determinations of water-saturated conductivities are much simpler and quicker to evaluate experimentally than those of unsaturated conductivities. Equation 3 can be written then by using the matching factor (K_s/K_{sc}) in the following form:

$$K(\theta)_i = (K_s/K_{sc})(30\gamma^2/\rho g\eta)(\epsilon^p/n^2) \sum_{j=i}^{m} [(2j + 1 - 2i)h_j^2] \quad (4)$$

where K_s is the measured saturated hydraulic conductivity and K_{sc} is the calculated saturated conductivity. Equation 4 is being used in calculations of $K(\theta)$ for highway and subgrade soils in this study. The only necessary laboratory test is the determination of K_s and the moisture content versus suction for each highway soil [details on the derivation of Equations 3 and 4 are given by Childs and Collis-George (13) and Millington and Quirk (14)].

MATERIALS AND METHODS

Soils

Disturbed and undisturbed soil core samples 5.4 cm (2

in) in diameter and 3 cm (1.2 in) in height were used to determine relations between soil moisture content and suction characteristics of several highway soils. The undisturbed core samples were collected from three profile depths of Drummer subgrade soil at a pavement test site in Piatt County, Illinois. The disturbed core samples were prepared in the laboratory by using Illinoian till, Fayette-C, two grades of Ottawa sand (2 to 0.84 mm and 0.50 to 0.05 mm), and a class X concrete sand. The engineering properties of all soil samples used in this study are given in Table 1.

Equipment

For many years soil physicists removed moisture from soil by creating a pressure difference, generally by suction across a porous ceramic material that served as a link between soil moisture and outside water. Pressure membrane and pressure plate extractors are a modification of this principle. If pressure is applied inside the chamber of the apparatus, a pressure difference is maintained across a porous plate or membrane, the bottom of which is at atmospheric pressure.

For the low range of pressure, 0 to 101.3 kPa (0 to 1 atm), the soil cores were placed in Tempe pressure cells saturated with water and then allowed to drain following sequential subjection to air pressures. Volumetric moisture content was determined from the weight of the cell corresponding to each static equilibrium pressure and the oven-dry weight of the soil core. Before drying, the same cores were resaturated for determination of water-saturated hydraulic conductivity (K_s). A 1519.8-kPa (15-bar) ceramic plate extractor was used to determine the moisture content for all soil samples for the high range of air pressure (101.3, 506.6, and 1519.8 kPa) (1, 5, and 15 atm), as reported by Richards (15).

RESULTS AND DISCUSSION

Data from some agricultural soils were selected from the literature on the basis of having $K(\theta)$ values over a wide range of moisture contents and a detailed moisture content-suction head relation. Soils selected were the Lakeland fine sand of Elzeftawy and Mansell (8), the 1 to 0.5-mm sand of Childs and Collis-George (13), the Botany sand of Watson (16), the Guelph loam of Elrick and Bowman (17), and the Dana loam of Elzeftawy. These data were obtained in the laboratory under conditions that allowed the hydraulic conductivity, moisture content, and suction head to be measured on the same soil sample.

Figures 1, 2, and 3 show the relations of hydraulic conductivity and moisture content for agricultural soils, Lakeland fine sand, 1 to 0.5-mm sand and Botany sand, and Guelph and Dana loam respectively. The lines represent calculated values using the saturated hydraulic conductivities (K_s) (permeability) as matching factors. The symbols represent experimental data. The dry densities and moisture-saturated hydraulic conductivities for the three depths of Lakeland fine sand (AASHO classification A-3) are given in Table 2 (8). The variation in dry density with depth of the field soil profile was almost negligible; however, the hydraulic conductivity (K_s) of the bottom layer is much higher than that of the surface layer. From the data shown in Figures 1 to 3 it can be concluded that measured and calculated values of hydraulic conductivities were in good agreement for the agricultural soils. However, there is some pronounced deviation between calculated and measured conductivities, especially for lower values of volumetric moisture content (θ). This observation supports the suggestion by

Elzeftawy and Mansell (8) that multiple matching factors are needed somewhat below the bubbling pressure to calculate hydraulic conductivities for dry soils that are found particularly in arid and semiarid regions.

Characteristic curves for soil moisture content versus suction, obtained by stepwise drainage, are shown in Figure 4 for Drummer subgrade [0 to 30-cm (0 to 12-in) depth], Illinoian till, and class X concrete sand highway soils. It is obvious that the soil-moisture characteristic curve is strongly affected by soil texture. The greater the soil-fines content (silt and clay) is, the greater is the water content at any particular suction and the more gradual is the slope of the curve. In sandy soil most of the pores are relatively large; once these large pores are emptied at a given suction, only a small amount of water remains. In a clayey soil the pore-size distribution is more uniform and more of the water is absorbed so that increasing the suction causes a more gradual decrease in water content.

The calculated and experimental hydraulic conductivities for the three profile depths of Drummer subgrade soil (AASHO A-7-6)—0 to 30, 30 to 75, and 75 to 90 cm (0 to 12, 12 to 30, and 30 to 36 in)—are shown in Figure 5. The experimental data were obtained by a method that uses the steady-state flow concept and is similar to that of Elzeftawy and Mansell (8).

Figure 6 shows the hydraulic conductivity [$K(\theta)$] of Illinoian till subgrade as a function of the subgrade soil moisture content. The experimental values were obtained from a compacted laboratory soil column (18). This Illinoian till was compacted at an optimum moisture content of 11 percent and a density value of 1.72 g/cm^3 (107.1 lb/ft^3). Notice that an increase in the soil moisture content (θ) from 0.28 to 0.42 cm^3/cm^3 (in^3/in^3) has increased the hydraulic conductivity from 3.1 nm/h to 1.9 mm/h (0.12 × 10^{-6} to 0.07 in/h) respectively.

The hydraulic conductivities of Ottawa sand and class X concrete sand are shown in Figure 7 and Fayette-C subgrade and Beer Farm agricultural soil in Figure 8. These soils were compacted air-dry in the Tempe pressure cells to determine the soil moisture content versus suction relation [$h(\theta)$] for each soil. The saturated hydraulic conductivities [soil permeability (K_s)] for these highway soils were determined by using the same soil core samples. The solid lines in the figures represent the calculated values of $K(\theta)$ and the circles, squares, and triangles represent the experimental data. These highway soils were selected to cover a wide AASHO classification range—from A-3 to A-7. They also represent a wide range of pore-size distributions on which the calculations of hydraulic conductivities are based. The saturated conductivities (permeability) ranged from 0.22 cm/h (0.019 in/h) for Fayette-C to 16.7 cm/h (6.6 in/h) for the class X concrete sand. It is very clear that the calculations of $K(\theta)$ are in excellent agreement with the experimental data.

Just as the flow of heat can be expressed in the form of a diffusion equation familiar to engineers, in which the diffusivity is expressed in terms of thermal conductivity, density, and specific heat of the material, so Darcy's law (Equation 1) may be put into a diffusionlike form in which moisture diffusivity is given by

$$D(\theta) = K(\theta)/C(\theta) \qquad (5)$$

where $C(\theta) = \partial\theta/\partial h$ is the specific water capacity of the soil. Since the soil moisture content-suction function [$h(\theta)$] is hysteretic (θ depends on drying or wetting of the soil system), it follows that soil moisture diffusivity is also a hysteretic function. In calculating $D(\theta)$ of highway soils and subgrades from the available $K(\theta)$ data, attention should be paid to the drying or wetting pro-

Table 1. Engineering properties of soils studied.

Soil Material	Sand (%)	Silt (%)	Clay (%)	Liquid Limit (%)	Plastic Limit (%)	Compacted Dry Density (kg/cm³)	Optimum Moisture Content[a] (%)	Saturated Hydraulic Conductivity (μm/s)
Drummer soil[b]								
0 to 30 cm	6	77.2	16.8	42.5	26.8	1520	21.2	0.0586
30 to 75 cm	6.3	80.5	13.2	54.7	29.2	1300	21.8	0.1
75 to 90 cm	6.9	82.6	10.5	48.9	32	1430	22.5	0.0694
Illinoian till[c]	62	20	18	22.2	14.7	1720	11.7	0.861
Lakeland fine sand[d]	98	2	0	NP	NP	1560	0.65[e]	41.1
Ottawa sand[d]								
0.50 to 0.05 mm	100	0	0	NP	NP	1700	0.50[e]	28.6
2 to 0.84 mm	100	0	0	NP	NP	1650	0.43[e]	37.2
Fayette-C subgrade[c]	7	75	18	32	23	1250	17.2	0.597
Concrete sand[d]	98.8	1.2	0	NP	NP	1640	0.61[e]	46.4
Beer Farm soil[f]	12.3	37.45	50.25	54.9	29.4	1020	24.3	2.48

Notes: 1 cm = 0.4 in; 1 kg = 2.2 lb; 1 cm³ = 0.06 in³.
      NP indicates nonplastic.

[a] Gravimetric moisture content.   [d] AASHO A-3.
[b] AASHO A-7-6.   [e] Air-dry gravimetric moisture content.
[c] AASHO A-4.   [f] AASHO A-7.

Figure 1. Experimental and calculated hydraulic conductivities of Lakeland fine sand.

Figure 3. Experimental and calculated hydraulic conductivities of Botany sand.

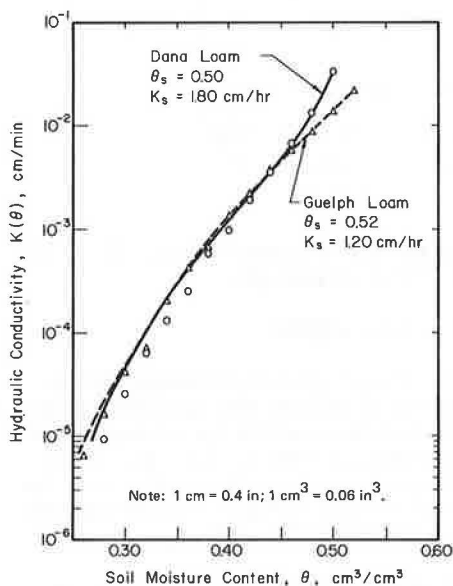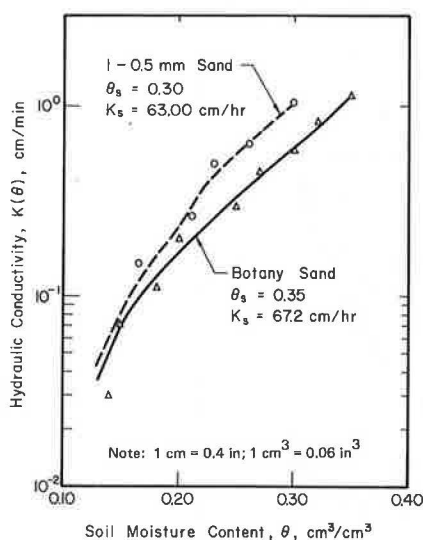Figure 2. Experimental and calculated hydraulic conductivities of Dana and Guelph loam.

Table 2. Dry density and saturated hydraulic conductivity for three depths of Lakeland fine sand.

Soil Depth (cm)	Dry Density, ρ_s (g/cm³)	t	Saturated Hydraulic Conductivity, K_s (cm/h)	t
0 to 15	1.56	0.06	14.8	1.12
30 to 45	1.57	0.03	13	0.93
60 to 90	1.57	0.05	17.1	1.09

Notes: 1 cm = 0.4 in; 1 g/cm³ = 0.036 lb/in³.
      t-distribution at 95 percent confidence level.

cesses of these soils.

It was felt that choosing an agricultural soil sample representative of Illinois soil might give a better idea of the validity of the model, especially when it is applied to a heavy clay soil. A surface soil sample was collected from the Beer Farm in Illinois, and its h(θ) function and K_s were obtained in the laboratory. The engineering properties of this sample are given in Table 1, and its experimental and calculated K(θ) are shown in Figure 8. The agreement between the model prediction and experimental values of the hydraulic conductivity is excel-

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Figure 4. Soil moisture content-suction relations of Drummer soil, Illinoian till, and class X concrete sand.
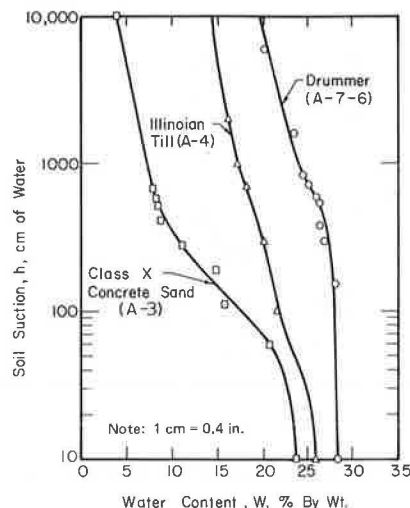

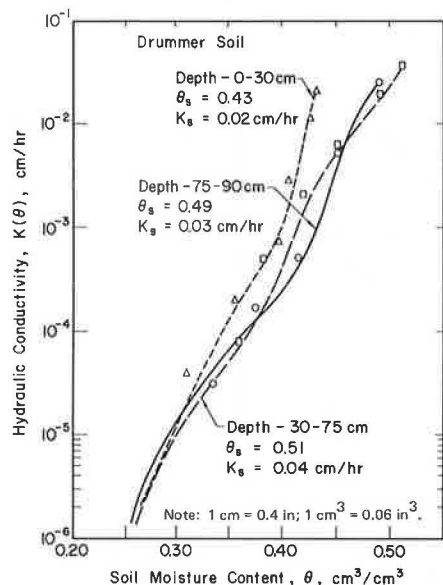Figure 5. Experimental and calculated hydraulic conductivities of Drummer subgrade.

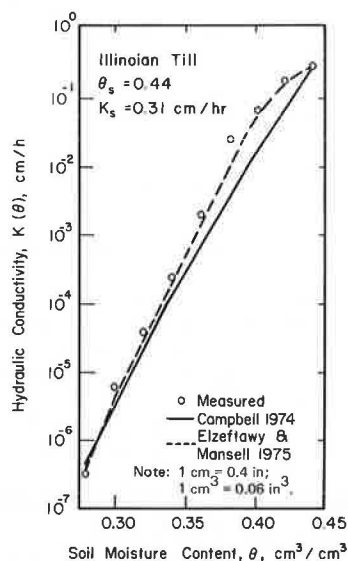Figure 6. Experimental and calculated hydraulic conductivity of Illinoian till subgrade.

Figure 7. Experimental and calculated hydraulic conductivities of Ottawa sand and class X concrete sand.
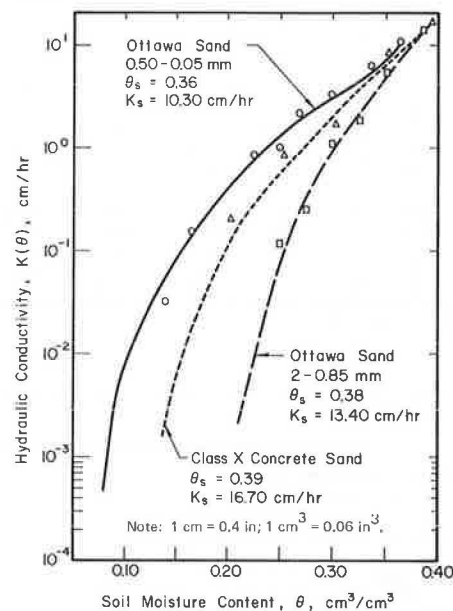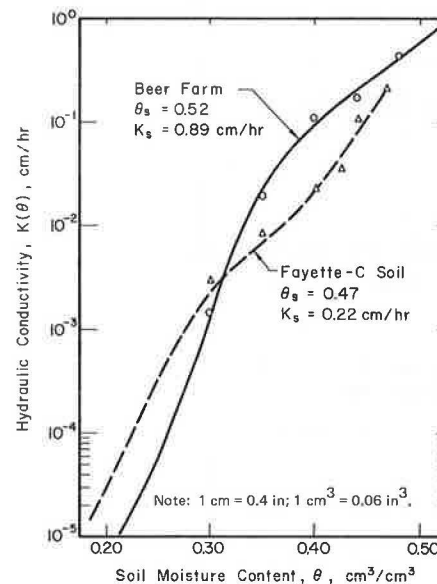

Figure 8. Experimental and calculated hydraulic conductivities of Fayette-C subgrade and Beer's Farm agricultural soil.

lent regardless of the heavy nature of this soil (50.25 percent clay and 37.45 percent silt).

SUMMARY AND CONCLUSIONS

The procedure discussed here for predicting the hydraulic conductivities of highway soil materials provides an economical and accurate method for determining the necessary hydraulic parameters for the study of unsaturated and saturated moisture flow in pavement materials and subgrades. The following conclusions can be drawn from the results of the study.

1. The model successfully predicts the hydraulic

conductivity of a wide range of subgrades and highway soils.

2. The simplified laboratory procedure proposed is reliable and can be easily used to determine the soil moisture-suction relations of highway soils.

3. Quicker and more economical evaluation of the unsaturated hydraulic conductivities of subgrade and highway soils can be done by using the proposed method.

ACKNOWLEDGMENTS

REFERENCES

1. G. Cumberledge. A Study of Factors Influencing the Reduction of Highway Subgrade Support During the Spring Thaw Period. West Virginia Univ., Morgantown, MS thesis, 1967.
2. A Guide to the Structural Design of Flexible and Rigid Pavement in Canada. Canadian Good Roads Association, 1965.
3. K. A. Turner and A. R. Jumikis. Subsurface Temperatures and Moisture Contents in Six New Jersey Soils. HRB, Bulletin 135, 1956, pp. 77-108.
4. L. Y. C. Yao and B. B. Broms. Excess Pore Pressures Which Develop During Thawing of Frozen Fine-Grained Subgrade Soils. HRB, Highway Research Record 101, 1965, pp. 39-57.
5. B. J. Dempsey and A. Elzeftawy. A Mathematical Moisture Model for Evaluating the Effect of Water in Multilayered Pavement Systems. TRB, Transportation Research Record 612, 1976, pp. 48-55.
6. R. H. Brooks and A. T. Corey. Hydraulic Properties of Porous Media. Colorado State Univ., Hydrology Papers, Vol. 3, 1964, pp. 1-27.
7. R. E. Green and J. C. Corey. Calculation of Hydraulic Conductivity: A Further Evaluation of Some Predictive Methods. Proc., Soil Science Society of America, Vol. 35, 1971, pp. 3-8.
8. A. Elzeftawy and R. S. Mansell. Hydraulic Conductivity Calculations for Unsaturated Steady-State and Transient-State Flow in Sand. Proc., Soil Science Society of America, Vol. 39, 1975, pp. 599-603.
9. G. H. Fancher, J. A. Lewis, and K. B. Barnes. Pennsylvania State College, Bulletin 12, 1933.
10. J. Kozeny. Über Kapillare Leitung des Wassers im Boden. Ber. Wien Akademia, Vol. 136A, 1927, pp. 271-306.
11. K. Terzaghi. Principles of Soil Mechanics: 3. Determination of Permeability of Clay. Engineering News Record, Vol. 95, 1925, pp. 832-836.
12. F. Zunker. Die Durchlässigkeit des Bodens. Trans., 6th Commonwealth International Society of Soil Science, Vol. B, 1933, pp. 18-43.
13. E. C. Childs and N. Collis-George. The Permeability of Porous Materials. Proc., Royal Society, London, Series A, Vol. 201, 1950, pp. 392-405.
14. R. J. Millington and J. P. Quirk. Permeability of Porous Solids. Trans., Faraday Society, Vol. 57, 1961, pp. 1200-1207.
15. L. A. Richards. Methods of Soil Analysis—Part 1. American Society of Agronomy, Monograph 9, 1965, p. 128.
16. K. K. Watson. An Instantaneous Profile Method for Determining Hydraulic Conductivity of Unsaturated Porous Materials. Water Resources Research, Vol. 2, 1966, pp. 709-715.
17. D. W. Elrick and D. H. Bowman. Note on an Improved Apparatus for Soil Moisture Flow Measurements. Proc., Soil Science Society of America, Vol. 28, 1964, pp. 450-452.
18. A. Elzeftawy and B. J. Dempsey. Unsaturated Transient and Steady State Flow of Moisture in Subgrade Soil. TRB, Transportation Research Record 612, 1976, pp. 56-61.