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Abridgment

Postoptimality Analysis Methodology for Freeway On-Ramp Control

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Postoptimality analysis is concerned with changes in an optimum decision value caused by changes in the parameters (input data) of a decision model. It is one way of approaching issues of uncertainty when using deterministic techniques such as linear programing (LP). We applied the techniques to a northbound section of the Eastshore Freeway (I-80) in the San Francisco Bay Area. The LP technique bases its calculations on point estimates rather than on a range of values. Postoptimality analysis assists in determining the importance and effects of deviations from such estimates.

The superiority of postoptimality analysis associated with LP over other mathematical programing techniques lies in its simplicity and systematic procedures. Postoptimality analysis allows us to obtain from the final (optimum) LP tableau (in addition to the optimum solution) a wealth of information on a wide range of operations in the neighborhood of the optimum.

Previous studies have focused on the potential applications of postoptimality analysis (1, 2). However, no such analysis has been attempted in recent applications of LP to freeway on-ramp control (3, 4).

It should be clear that one way to analyze postoptimality is to formulate and resolve a modified problem. This modification could, for example, be a slight change in one of the model parameters. Still, to investigate the effects of this slight change, the analyst must put this change into the model and rerun the program. Such a procedure is clearly inefficient and time consuming. Substantial economy of time and analysis is often possible if the information available in the optimum solution to the original problem is fully utilized instead. We shall demonstrate this.

THE LP CONTROL MODEL

The LP control model used here is similar to Wattleworth's original formulation and can be regarded as a resource allocation model. The resources—freeway subsection capacities—are allocated to competing input-station demands in order to maximize a certain objective function (for example, total allowable input rate) that is subject to the constraint of no congestion on the freeway and other operational constraints. The allowable flow rates at each input station are our decision variables, which are typically characterized by the upper and lower bounds imposed on them. Eldor has presented the mathematical details of the model (5, 6).

POSTOPTIMALITY ANALYSIS METHODOLOGY

The type of postoptimality analysis that can be performed on variations of a parameter depends upon its role in the optimization problem. The LP technique allows the effects of some variations to be examined quite easily.

Table 1. Capacity constraints: expected flows and slack and dual variables.

Subsection No.	No. of Lanes	Capacity	Expected Flow	Excess Capacity	Value of Dual 1[a]
1	3	5750	5404	346	0
2	3	5806	5806	0	0.227
3	3	5728	5263	465	0
4	3	5806	5671	135	0
5	3	5520	5295	225	0
6	3	5950	5539	411	0
7	3	5806	5083	723	0
8	3	5880	5803	77	0
9	3	5950	5803	147	0
10	3	5950	5577	373	0
11	3	5728	5146	582	0
12	4	6850	6188	662	0
13	3	5800	5800	0	1.069
14	3	5806	4642	1164	0
15	3	5800	4950	850	0
16	3	5049	4431	618	0
17	3	4746	4431	315	0
18	3	4700	4651	49	0

[a] Dual 1 (I) equals the value of the dual variable associated with the I th capacity constraint; the value of the objective function equals 8748 vehicles/h.

A detailed discussion regarding analysis of model parameters is given by Eldor (5, 6), who was concerned with the sensitivity of an optimum decision to possible variations on the right and upper-bounding vectors only.

Changes in the Right Vector

The right vector (b_k) represents the capacities of freeway subsections. Because these capacities are the true resources in the optimization process, it is often interesting to investigate the effects of their variations on the objective function (or the measure of effectiveness).

The dual variable associated with the kth capacity constraint indicates the rate of change in the objective function due to a unit change in the capacity of the kth subsection. In addition, one should investigate the range (of capacity) of this dual variable (or shadow price) when all else remains constant. Such an investigation is of practical importance to the analyst because it will assist him or her in determining the importance of deviating from his or her initial (point) estimate of capacity that was used by the LP model.

Let b_k' be a new right vector defined as

$$b_k' = \begin{cases} b_k + \delta , & k = h \\ b_k , & k \neq h \end{cases} \tag{1}$$

Thus, δ represents a change in only one of the right vector entries (δ can be either positive or negative).

The specific questions often asked in this type of analysis are what the range of δ is for which the optimum basic sequence is still optimum or the optimum solution to the dual problem is still optimum, and what the corresponding change in the value of the objective function would be. We then formulated answers by using Equation 1 as a starting point and the information automatically generated by the LP algorithm (in particular the final working tableau). The measures were developed specifically for the upper-bounding version of the simplex method.

If both the immediate shadow price and the range of applicability of changes in a given capacity constraint are known, the analyst has a great deal of information about the value of changing a single capacity estimate. This we shall demonstrate with a real-life application.

Changes in the Upper-Bounding Vector

The LP decision model we used incorporates two types of upper bounds on the decision variables: the demand at an input station and the maximum metering rate limit. Because the larger of these parameters is clearly redundant for the optimization process, the analyst may combine the two into one constraint. Consequently, the redundant constraint (or quantity) does not enter explicitly the problem. An investigation of the variations in the upper-bounding vector entries and their effects on the value of the objective function thus deals with either of these two parameters (demand or maximum metering rate limit) at each input station. We followed procedures similar to the ones discussed above to develop and computerize (6) our information and measures.

POSTOPTIMALITY ANALYSIS APPLICATION

Northbound I-80 was selected as the test system for demonstrating the postoptimality analysis methodology. Our roadway and traffic data included the capacity profile of the freeway, 15-min origin-destination tables, and metering rate limits. The analysis concerns one 15-min interval of the afternoon peak period (5).

An efficient upper-bounding LP algorithm and the methodology presented above were computerized and integrated into a new software system called freeway responsive control optimization techniques (FRESCOT) (6), which is an efficient ANS FORTRAN traffic-management package for freeway on-ramp control. Eldor (6) gives detailed discussions of FRESCOT and a program listing and user's guide.

Using the computerized package with the input data, we derived a control strategy coupled with postoptimality analysis measures. The strategy and related measures of effectiveness are presented by Eldor (5, 6). Samples of the measures for the capacity constraints are given in Tables 1, 2, and 3. Each computer run also generates similar measures for the upper-bounding vector.

Table 1 includes the value of the dual variables, the slack variables (excess capacity), the expected flow in each subsection, and the capacity values used as input for the control run. Excess capacity is simply the difference between the capacity and the expected flow and is exactly the value of the slack variable (at optimality) introduced into a capacity constraint in the process of converting the LP problem into a standard form (all constraints are converted into equalities). The slack variable measures the rate of capacity underutilization of a subsection. A positive slack means that the constraint is nonbinding. The slack variables are directly associated with the dual variables, which are equal to zero for all the nonbinding constraints.

The optimality ranges for changes in the right vector are given in Table 2. For each subsection, a range of applicability of the capacity estimate is given with the corresponding range of changes in the value of the objective function. For example, consider subsection 2. The initial estimate of capacity for this subsection was 5806 vehicles/h. The range of optimality for this capacity estimate, over which the associated dual variable is unchanged, is 5757 to 5852; the corresponding range for the objective function is 8737 to 8759. The results in Table 2 show the effects of deviating from the initial capacity estimate.

Table 3 provides the associated control strategy with each change (at the bounds) of the capacity vector as given in Table 2. The computer generates a strategy for each binding capacity constraint (6). The strategy for the nonbinding capacity constraints (within the range

Table 2. Ranges of optimality for vector changes.

Subsection No.	Optimality Range of Subsection Capacity: Change of Lower Bound	Change of Upper Bound	Lower-Bound Value	Upper-Bound Value	Optimality Range of Objective Function: Change of Lower Bound	Change of Upper Bound	Lower-Bound Value	Upper-Bound Value
1	-346		5404		0	0	8748	8748
2	-49	46	5757	5852	-11	10	8737	8759
3	-465		5263		0	0	8748	8748
4	-135		5671		0	0	8748	8748
5	-225		5295		0	0	8748	8748
6	-411		5539		0	0	8748	8748
7	-723		5083		0	0	8748	8748
8	-77		5803		0	0	8748	8748
9	-147		5803		0	0	8748	8748
10	-373		5577		0	0	8748	8748
11	-582		5146		0	0	8748	8748
12	-662		6188		0	0	8748	8748
13	-751	35	5049	5835	-802	38	7946	8786
14	-1164		4642		0	0	8748	8748
15	-850		4950		0	0	8748	8748
16	-618		4431		0	0	8748	8748
17	-315		4431		0	0	8748	8748
18	-49		4651		0	0	8748	8748

Table 3. Strategy for ranges of optimality for vector changes.

Origin	Subsection 2: Lower-Bound Capacity	Subsection 2: Upper-Bound Capacity	Subsection 13: Lower-Bound Capacity	Subsection 13: Upper-Bound Capacity
1	5404	5404	5404	5404
2	353	448	402	402
3	408	408	408	408
4	244	244	244	244
5	720	720	720	720
6	1080	1007	240	1080
7	308	308	308	308
8	220	220	220	220

of optimality) remains unchanged. Thus, four additional, meaningful control strategies that are associated with possible changes in the initial capacity estimates of the bottleneck subsections are provided.

SUMMARY AND CONCLUSIONS

We present postoptimality analysis methodology for applying LP to freeway on-ramp control. In addition to the optimum control strategy for the original (or initial) data set, the analyst is provided with much valuable information concerning the deviations from the initial capacity and upper-bound estimates. Not only are ranges of optimality given with their associated changes in the value of the objective function, but the corresponding control strategies are also provided. The expense of generating this information is, practically speaking, negligible. Only simple calculations are required (6), and the computerization of these calculations can be considered as a one-time effort.

The methodology developed in this study is also applicable to priority-entry LP on-ramp control. Expansion of the FRESCOT software system to account for priority entry schemes has been initiated and will be reported at another time.

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