Abridgment

Dynamic Properties of Beam-Slab Highway Bridges

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Determination of the dynamic response of bridges has been of interest and concern to engineers for more than three-quarters of a century (4) primarily because of the need to define the amplification of the static response of bridges caused by moving vehicles. This has traditionally been accomplished by oversimplifying the bridge superstructure into an equivalent beam and thus ignoring the high internal indeterminacy of the superstructure. Equivalent-beam modeling encouraged the use of the impact factor or the dynamic load factor in obtaining dynamic response.

Refined analytical studies on simple-span beam-slab highway bridges with prestressed concrete I-beams and without skew have indicated that the true dynamic behavior of the bridge superstructure can be simulated by finite-element modeling of the total superstructure (3). The studies have also indicated that one of the major benchmarks in the differentiation of the dynamic responses of various bridges is the correct determination and comparison of the predominant natural periods of vibration of the superstructure (3, 4). Full-scale field studies carried out on a limited number of bridges have resulted in experimental determination of the first natural period of vibration of bridge superstructures (5). Bridge engineers have been able to make only limited use of the test results because of the limited variation in the major design dimensions of the field-tested bridges: Another bridge with different design dimensions could have a substantially different period of vibration. Before the start of the parametric study reported here, the natural vibration periods obtained in field test were recomputed by means of the finite-element method, and good correlation was observed (3).

DESIGN AND ANALYSIS OF BRIDGES

Eighteen simple-span beam-slab bridges with prestressed concrete I-beams and without skew were designed to represent bridges of this type encountered in the field (6). The design dimensions of the test bridges are given in Table 1. The beams used in the design process correspond to the standard beams used in the state of Pennsylvania. Table 2 gives the stiffness properties of the beams. Although the study used only 18 bridges, previous investigations (2) have indicated that this is a sufficient number if the sample structures are carefully chosen so that they closely approximate the dimensions of existing bridges. The results presented here can be applied to bridges of other dimensions through interpolation and engineering judgment (2).

In defining the dynamic characteristics of the bridges, the superstructures were simulated by using the finite-element method. The bridge superstructure was considered an assemblage of plate bending and beam elements. The analysis was performed by using the SAP IV program (1). Only the first three natural periods of vibration of the superstructures are presented here, partly because of space limitations but also because three periods are sufficient to illustrate the dynamic properties of the bridges. It should be noted that the periods reported here are the natural periods of the bridge; that is, no vehicle is assumed to be on the bridge. The periods of bridges loaded with vehicles will not, however, be substantially different from those given here. The vibrating mass of vehicles is small compared to the mass of the bridge superstructure. These periods can thus be used with good reliability to approximate loaded periods for long-span bridges and with decreasing reliability as span lengths get shorter.

RESULTS

Natural periods of vibration for the three predominant mode shapes are given (in seconds) below. In this and the following table, T_1, T_2, and T_3 denote the first, second, and third natural periods respectively.

Bridge	Period (s) T_1	T_2	T_3
1	0.066	0.059	0.048
2	0.077	0.064	0.052
3	0.075	0.069	0.063
4	0.082	0.073	0.063
5	0.075	0.068	0.064
6	0.082	0.077	0.070
7	0.134	0.114	0.076
8	0.109	0.095	0.071
9	0.136	0.126	0.106
10	0.116	0.106	0.093
11	0.138	0.132	0.120
12	0.120	0.114	0.105
13	0.181	0.157	0.098
14	0.160	0.139	0.095
15	0.177	0.166	0.140
16	0.165	0.152	0.131
17	0.180	0.172	0.158
18	0.168	0.159	0.145

Although there is a substantial change in the design dimensions of the bridges, the variation in the first natural period of the bridge superstructures is not highly sensitive to the major changes in the design dimensions. For a given span length, the first natural period of vibration tends to remain relatively constant. It can thus be concluded that the first mode of vibration and the corresponding period are primarily a function of the span length of the bridge superstructure.

The equivalent-beam approach uses the following formula to predict the natural periods of vibration of the bridge superstructure:

$$T_i = (2\pi/A_i)\sqrt{ML^4/EI} \quad (1)$$

where

M = beam mass per unit length,
L = beam length,
E = beam modulus of elasticity, and
I = beam moment of inertia.

The coefficient A_i is used to obtain different natural periods (4). The values for the first three periods are A_1 = 9.87, A_2 = 39.5, and A_3 = 88.9.

In using the equivalent-beam approach for beam-slab bridges, it is assumed that the beams are the predominant load-carrying system. In computing mass and inertia properties, only the beams are considered. Using

Table 1. Design dimensions of test bridges.

Bridge	Span (m)	Width (m)	Slab Thickness (mm)	Beam Spacing (m)	Number of Beams	Beam
1	12.20	7.93	190	1.46	6	I
2	12.20	8.46	203	2.44	4	I
3	12.20	13.57	190	1.60	9	I
4	12.20	14.02	215	2.56	6	I
5	12.20	19.05	190	1.66	12	I
6	12.20	19.51	215	2.61	8	I
7	19.82	7.93	190	1.46	6	II
8	19.82	8.38	203	2.44	4	IV
9	19.82	13.41	190	1.60	9	III
10	19.82	13.95	215	2.56	6	IV
11	19.82	18.90	190	1.66	12	III
12	19.82	19.44	215	2.61	8	IV
13	27.44	7.77	190	1.46	6	V
14	27.44	8.38	203	2.44	4	VII
15	27.44	13.41	190	1.60	9	VI
16	27.44	13.87	203	2.56	6	VII
17	27.44	18.90	190	1.66	12	VI
18	27.44	19.36	203	2.61	8	VII

Note: 1 m = 3.28 ft; 1 mm = 0.0394 in.

Table 2. Stiffness properties of beams.

Beam	Maximum Moment of Inertia (mm^4)	Minimum Moment of Inertia (mm^4)	Area (mm^2)	St. Venant Torsional Stiffness (mm^4)	Notation[a]
I	13.66 (10^9)	2.66 (10^9)	234 (10^3)	3.81 (10^9)	PDT20/30
II	31.66 (10^9)	7.74 (10^9)	397 (10^3)	13.35 (10^9)	PDT24/36
III	34.69 (10^9)	10.36 (10^9)	443 (10^3)	17.65 (10^9)	PDT26/36
IV	71.96 (10^9)	8.37 (10^9)	457 (10^3)	13.04 (10^9)	PDT24/48
V	88.50 (10^9)	8.98 (10^9)	492 (10^3)	14.64 (10^9)	PDT24/51
VI	106.33 (10^9)	9.59 (10^9)	526 (10^3)	16.52 (10^9)	PDT24/54
VII	195.87 (10^9)	15.87 (10^9)	675 (10^3)	23.00 (10^9)	PDT26/63

Note: 1 mm^4 = 2.4 (10^{-6}) in^4; 1 mm^2 = 0.0015 in^2

[a] From Standards for Bridge Design of Pennsylvania Department of Transportation (6).

this concept and the formula given above yields the natural periods given in the table below.

Bridge	Period (s) T_1	T_2	T_3
1	0.163	0.041	0.018
2	0.194	0.049	0.022
3	0.167	0.042	0.019
4	0.202	0.051	0.023
5	0.169	0.042	0.019
6	0.203	0.051	0.023
7	0.337	0.087	0.038
8	0.256	0.064	0.029
9	0.326	0.082	0.037
10	0.263	0.066	0.030
11	0.329	0.082	0.037
12	0.507	0.127	0.057
13	0.398	0.099	0.045
14	0.329	0.083	0.037
15	0.377	0.094	0.042
16	0.333	0.083	0.037
17	0.379	0.095	0.043
18	0.334	0.084	0.037

A comparison of the data in the preceding table, for true periods, and those in the table above, for approximate periods, indicates that substantial discrepancies exist in all values. It can thus be stated that the equivalent beam defined by the above approach will yield incorrect results.

Another approach used to define equivalent-beam properties has been to consider the full slab and the beams in the definition of the appropriate values. This corresponds to the assumption that bridge beams will essentially act like T-beams, the slab width being the spacing of the beams. The use of Equation 1 for this assumption results in a first period of vibration that is close to the true periods; however, unacceptable differences still exist in the higher natural periods. Thus, if it is necessary to find the first period of vibration of the beam-slab bridge superstructure, fully acceptable results can be obtained by using the equivalent-beam approach and the appropriate formula and assuming that both the beams and the bridge deck will fully participate through their contributions to stiffness and mass.

CONCLUSIONS

The dynamic or vibrational characteristics of beam-slab bridge superstructures can be predicted and compared by use of predominant natural periods of vibration. The parametric study indicated that

1. The first natural period of bridges is predominantly dependent on the span length,
2. The dynamic characteristics of bridges vary as bridge configurations change,
3. The first natural period of bridges, regardless of the bridge configuration, does not vary substantially for a given span length,
4. The use of the equivalent-beam approach with the correct formula and the correct stiffness and mass contributions results in an accurate estimate of the first natural period of vibration of the structure but in poor estimates for all other dynamic properties of the structure, and
5. The contribution of the slab should be included in the estimation of the dynamic characteristics of bridge superstructures.

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