Problems and Needs in Track Structure Design and Analysis

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This paper reviews the design aspects of old and new track systems, the research needs in track structure design, and methods of track analysis.

Railroad tracks have been in use since the eighteenth century. Originally many kinds of track systems were built, but during the nineteenth century, two of these—the longitudinal-tie track and the crosstie track—came to predominate. In the longitudinal-tie track, the rails are continuously supported by wooden or metal beams, and the gauge is maintained by cross bars or crossties. In the crosstie track, the rails are discretely supported by closely spaced wooden or metal crossties. Eventually, the crosstie track became the dominant mode of track construction.

Since World War II, concrete has been increasingly used abroad for the production of ties. The ties currently produced are mainly crossties. However, in view of the historic development of the railroad track and the well-established properties of concrete as a structural material and the ease with which it can be formed into various structural shapes, it is not a priori obvious that the crosstie track is technically and economically the most satisfactory solution when concrete is used as the tie material.

In the first part of this paper, the design aspects of various old and new kinds of track systems are discussed to illustrate this point. This is followed by a discussion of various design aspects of the track system used at present in the United States; i.e., the wooden crosstie track.

The second part of this paper presents a discussion of railroad track analyses. Some of the early papers in this field were published in the middle of the nineteenth century, and there have been many since then. A thorough review and discussion of them is beyond the scope of one paper. Thus, this part of the paper is restricted to a brief review of analyses of static stresses and the stability of track structures, which form a large part of the track analyses published.

PROBLEMS AND NEEDS IN TRACK STRUCTURE DESIGN

The steam locomotive was introduced into regular service during the first half of the nineteenth century. The subsequent rapid improvement of the locomotives resulted in continuously increasing train speeds and higher wheel loads, which in turn continuously increased the demand for stronger and better railroad tracks (1).

During the nineteenth century, both the longitudinal-tie track and the crosstie track were used. In the course of time, the use of the longitudinal-tie track decreased and, at present, the crosstie track is the dominant mode of track construction (2,3). Originally, the crossties were made of wood, as shown in Figure 1a. A number of railways also used metal crossties. For the past two decades, prestressed concrete crossties have been increasingly used on many railways abroad. A typical track with prestressed concrete ties is shown in Figure 1b.

In the U.S., because of the heavy wheel loads, the introduction of concrete crossties caused difficulties. Also, the availability of wooden ties at comparable prices, and the thorough familiarity of our railroad engineers and maintenance crews with their installation and maintenance, did not create the sense of urgency for the rapid development of concrete ties that was the case in many countries abroad. However, work on concrete crossties continues (4). A state-of-the-art survey of the development of concrete crossties in the United States is contained in a recent presentation by Weber (5). The design, production, installation, and maintenance of concrete ties are described by Zolotarskii and others (6) and by Shrinivasan (7). It is expected that a prestressed concrete crosstie suitable for United States conditions will be available for use on main lines in the near future.

When the crosstie track was first introduced, wheel loads were small, and tie spacing was relatively large. As the wheel loads progessively increased, the tie cross section increased, and the tie spacing decreased. But, with current track-maintenance practices, the spaces between the ties cannot be reduced beyond a certain limit. Because of this and other mechanical and economical factors (such as the desire to minimize the increase in track maintenance that is caused by the ever-increasing wheel loads and train speeds), attempts are being made to eliminate the tie spaces altogether by using, instead of discrete ties, a continuous reinforced-concrete slab. The rails, discretely or continuously supported, are secured to the slab by fasteners that are anchored in the slab. Sections of such slab tracks have recently been built by a number of railroads abroad (Figure 2a). In the United States, a track of this type was built and successfully used about 50 years ago (8). Descriptions of design details of recently built slab tracks (also referred to as ballast-free tracks) and their performance have been given by Birmann (9), Lucas and others (10), Miyamoto (11), Bramall (12), Eisenmann (13, 14), and elsewhere (15).

Another noteworthy design is the concrete frame track, which is being developed in the USSR (16, p. 62). For the past several years, this track has been undergoing extensive testing at the Shcherbinka test loop and in a main line in the southern part of the USSR (17). In this system, instead of crossties, 2.50-m long, precast, prestressed concrete frames are placed in the ballast, as shown in Figure 2b. This system has a high lateral rigidity like the slab track, but is lighter and is more accessible, and therefore easier to maintain. In view of the present tamping practice, in which the ballast is compacted only in the vicinity of the rail seats, the frame track is a logical modification of the crosstie track.

The introduction of the slab track, and especially of the frame track, is essentially a return to the longitudinal-tie track, which lost out to the crosstie track some time ago. In this connection, one should note the earlier attempts to introduce the concrete longitudinal-tie track in the U.S. as described elsewhere (18, 19).

According to Winkler (20), one of the main reasons

Figure 1. Crosstie tracks currently in use.

(a)

(b)

for abandoning the longitudinal-tie track in the second half of the nineteenth century was the warping of the long wooden longitudinal ties. Another reason was the difficulty of holding the gauge with the iron rods or wooden crossties that were placed under the longitudinal ties. Also, because the wooden crossties were widely spaced, they contributed to an undesirable periodic disturbance in the ride. This situation is reflected in the railway codes of the period (20, p. 210), which prescribed the use of a crosstie track system when wooden ties were used and the use of a longitudinal-tie track system when metal ties were used.

The slab track and the frame track are free of these limitations, except that the frame track may cause periodic disturbances because of the cross bars in the frames. Whether these disturbances are noticeable will have to be established in tests. Also, when the rails are continuously supported by a slab or frame-track, the height of the rail can be reduced because, as shown by Kerr (21), the slab or frame, when properly designed, will carry part of the load.

These remarks should not be construed as an endorsement of the slab track or the frame track. Their purpose is merely to point out the various new possibilities open to the railroad engineer when concrete is considered as a tie material. In past decades, railroad engineers became used to the crosstie track as the only workable system, and it was natural, when concrete was first considered as tie material, to build concrete crosstie tracks. However, the multitude of other concrete tie systems—e.g., Laval ties and wing ties—that have been proposed recently and the increasing mechanization of track activities mean that the prestressed-concrete crosstie track is not necessarily the most suitable track system for the future.

The development of a proper concrete tie system for U.S. tracks should be a major challenge to railroad engineers. In addition to satisfying the necessary technical criteria, such a track should be easy to install, easy to maintain (this includes replacement of parts), and in general keep costly track maintenance to a minimum.

In the United States at present, the wooden crosstie track is the dominant mode of track construction, and this situation will not change substantially for at least the next several years. Therefore, it is essential to discuss also the design aspects of this track system. This system was developed about 100 years ago. It has performed well for many decades. However, with increasing wheel loads and train speeds some limitations are becoming apparent.

One weak element in the present system is the cut-spike fastener (Figure 3a). As is well known, on main-line tracks that have been subjected to traffic, many cut spikes can be pulled out by hand. One should not wonder why rail turnover occurs sometimes, but rather why it does not occur more often. A recent review of rail turnover has been given by Zarembski (22).

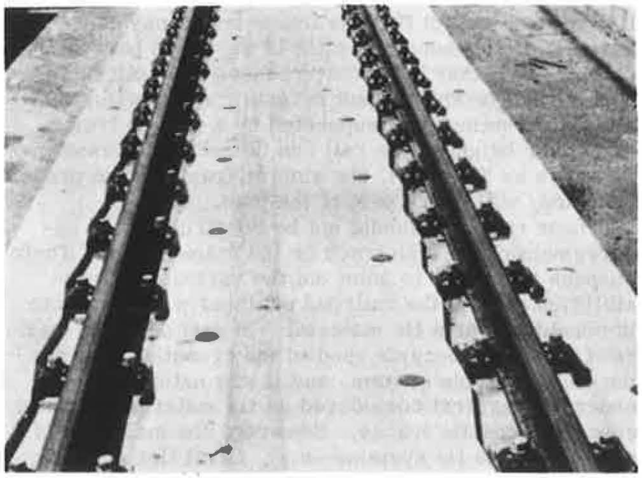
To prevent rail turnovers, a number of European railroads at the turn of the century replaced the cut spikes by screw spikes (Figure 3b) and developed special base plates for use on curves (Figure 3c). Later, the more rigid and elaborate K-type fasteners (Figure 3d) were introduced, and more recently, a variety of spring type fasteners (Figures 3e and 3f), have been developed. A discussion of a large variety of fasteners has been given by Schramm (23).

There were also many attempts in the United States to improve the cut-spike fastener. This is evident from the many fasteners that have been registered with the U.S. Patent Office since about 1860, as shown recently by Posner (24). However, the use of the cut-spike fastener has prevailed until the present.

Because of the increasing wheel loads and train speeds and the problems encountered with cut-spike fasteners, there is a need for the development of an improved fastener for U.S. conditions that will be technically sufficient and economically feasible. Although the K-type fastener may prove to be uneconomical for our railroads, simpler systems (e.g., the types shown in Figures 3b and 3f) may satisfy the necessary criteria.

The technologies of producing and preserving wooden crossties are well established. At present, an average

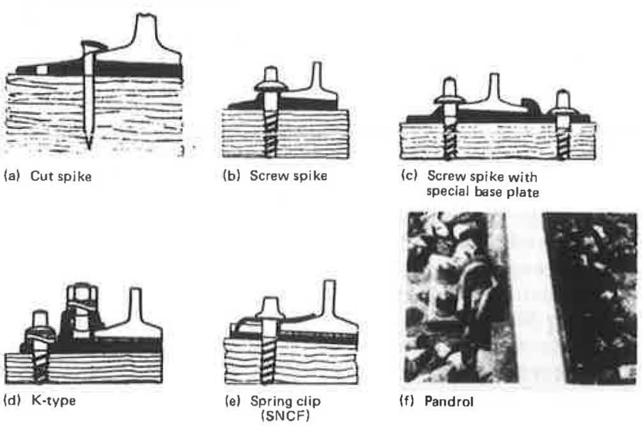
Figure 2. Nonconventional tracks.



(a)

(b)

Figure 3. Rail-tie fasteners.



wooden tie stays in main-line service for about 25 years. To extend this period (often on secondary lines), some railroads restore some of the damaged ties. Extensive studies of this subject are given by Lysyuk (25) and Bondarev and Zhuravskii-Skalov (26).

Ballast material has also been the subject of many studies. Recent results have been given by Raymond and others (27), Klugar (28), Shenton (29), and Knutson and others (30). These studies should contribute to a better understanding of the response of ballast to static and dynamic loads, establish useful criteria for choosing ballast material, and lead to economical methods for its maintenance while in line service.

From a design point of view, the ballast section, as used in the United States at present for continuously welded rails, may require modification. According to tests performed abroad and analytical studies conducted recently, wider shoulders than those used by our railroads [15 cm (6 in) on tangents and 31 cm (12 in) on curves] are needed to prevent track buckling. For example, the DB and the railroads of the USSR use ballast shoulders of 35 and 45 cm (14 and 18 in) on tangents and curves respectively.

The observed movement of ballast particles away from the rail-seat region, which is caused by the passage of trains, suggests that ballast shoulders wider than 15 cm may also reduce track degradation and thus track maintenance.

These facts point toward the need for establishing optimal ballast shoulder widths for U.S. conditions, to prevent track buckling and to reduce track maintenance.

There are many other problems in track design—that should be discussed such as clarification of the need for placing expansion joints in continuously welded tracks (especially on bridges), and the proper design of tracks at the approaches to bridges or in the vicinity of track-highway intersection—but these are beyond the scope of this paper.

PROBLEMS IN TRACK STRUCTURE ANALYSIS

Although the development of the railroad track was mainly intuitive, based on a trial and error approach, since the second half of the nineteenth century, railroad engineers have been attempting to analyze the track and its components.

Early workers attempted to determine the bending stresses in the rails and ties. In 1867, Winkler (31) analyzed the stresses in the rails of a longitudinal-tie track by considering the rails as continuously supported beams. The differential equation for the bending of an elastic beam is

$$EI(d^4w/dx^4) + p(x) = q(x) \qquad (1)$$

where

$w(x)$ = vertical deflection of point x on track axis,
EI = flexural rigidity of rail and tie,
$p(x)$ = continuous contact pressure between tie and base, and
$q(x)$ = distributed vertical load,

all as shown in Figure 4.

For the response of the base, Winkler proposed the relation

$$p(x) = k_r w(x) \qquad (2)$$

where k_r = base parameter for one rail and tie. This is the origin of the well-known Winkler foundation model. The resulting track equation

$$EI(d^4w/dx^4) + k_r w = q \qquad (3)$$

is a fourth-order ordinary differential equation and represents the response of a beam that is attached to a spring base (Figure 5).

In 1882, Schwedler (32), in discussing the bending stresses in the rails of a longitudinal-tie track, gave the following solution of Equation 3, for the case when

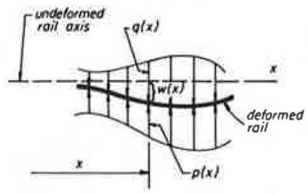


Figure 4. Equilibrium position of deformed rail.

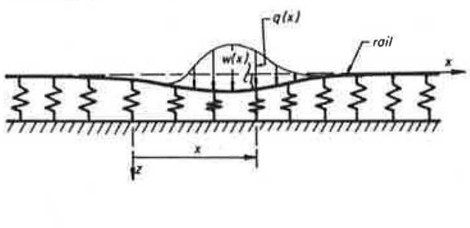


Figure 5. Continuously supported rail subjected to a load q(x).

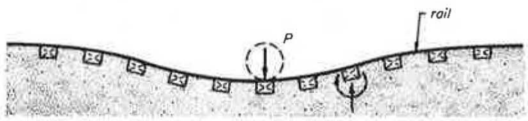


Figure 6. Deflected rail-tie structure.

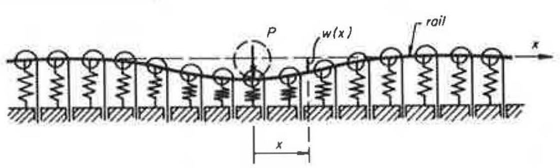


Figure 7. Continuously supported rail with rotational resistance of ties.

a very long track is subjected to a concentrated force (P)

$$w(x) = (P\kappa/2k_r)\, e^{-\kappa x}\, [\cos(\kappa x) + \sin(\kappa x)] \tag{4}$$

and the corresponding expression for the bending moments

$$M(x) = (P/4\kappa)\, e^{-\kappa x}\, [\cos(\kappa x) - \sin(\kappa x)] \tag{5}$$

where $\kappa = (k_r/4EI)^{1/4}$. Schwedler used these expressions as influence functions to determine the effect of several wheel loads.

In 1888, Zimmermann (33) published a book that contained solutions of Equation 3 for many special cases of interest in the analysis of railroad track. Like Schwedler, Zimmermann used the solutions he had obtained to analyze the longitudinal-tie track, but he also analyzed the ties of the crosstie track. He, like Schwedler, compared the analytically obtained and the measured deflection curves of a longitudinal-tie track caused by two loads of 6.3 Mg (7 tons) each. The close agreement found between the measured and the calculated deflections pointed to the conclusion that the bending theory for a beam on a linear Winkler base was sufficient for the analysis of the longitudinal-tie track. More details of the analysis of longitudinal-tie tracks have been given by Kerr (21).

The development of analyses for the rails of a crosstie track was more involved. The rail was first considered as a beam resting on discrete rigid supports, then as one resting on discrete elastic supports, and then as one resting on a continuously supported beam. A critical survey of these rather turbulent developments has been given by Kerr.

The method that ultimately prevailed for the analysis of the bending stresses in the rails is based on the assumption that for a crosstie track also, the rails respond as a continuously supported beam. Thus, Equation 3 is valid for this case also, provided the coefficients EI and k are properly chosen. Early investigators who used this approach were Flamache (34) in 1904, Timoshenko (35) in 1915, and the ASCE-AREA Special Committee on Stresses in the Railroad Track (36) in 1917. The trend toward steadily increasing wheel loads, which has been countered by a steady decrease in the crosstie spacings, increased the justification of the continuity assumption. The analytical solutions based on Equation 3 were compared with corresponding measured results by the ASCE-AREA Special Committee and by Wasiutynski (37). The relatively close agreement found indicated that Equation 3 is also suitable for the analysis of the rails of a crosstie track (21).

Once the continuity of rail support is accepted, the determination of the vertical force the rail exerts on a tie is as follows: It is the contact pressure $p = k_r w$ given in Equation 2 integrated from half span to half span, or approximately the pressure ordinate at the rail seat multiplied by the center-to-center tie spacing. The determined largest force (F_{max}) that each rail can exert on a crosstie caused by the anticipated wheel loads of a moving train is then used for the stress analysis of the crossties.

One deficiency of the track analyses based entirely on Equation 3 is suggested by the observation that, for example, in front of a locomotive, over a certain interval, the track lifts off the ballast. In this liftoff region, Equation 3 is not valid, because $k_r = 0$. Problems of this type have recently been solved by Weitsman (38).

Another deficiency of Equation 3 for the analysis of the bending stresses in the rails has been discussed by Hanker (39, 40), who pointed out that when the rails deflect in the vertical plane, the crossties rotate about their axes and, because the ballast resists these rotations, the ties also exert moments on the rails (Figure 6). By assuming that discrete rotational resistances act at each tie, Hanker proposed an approximate method to remedy this deficiency. In 1974, Kerr (1) suggested that the effect of these reaction moments can easily be taken into consideration by assuming that the reaction moments are also continuously distributed along the beam. By assuming that the distributed reaction moment at point x is proportional to the angle of rotation at x, Kerr obtained the following equation:

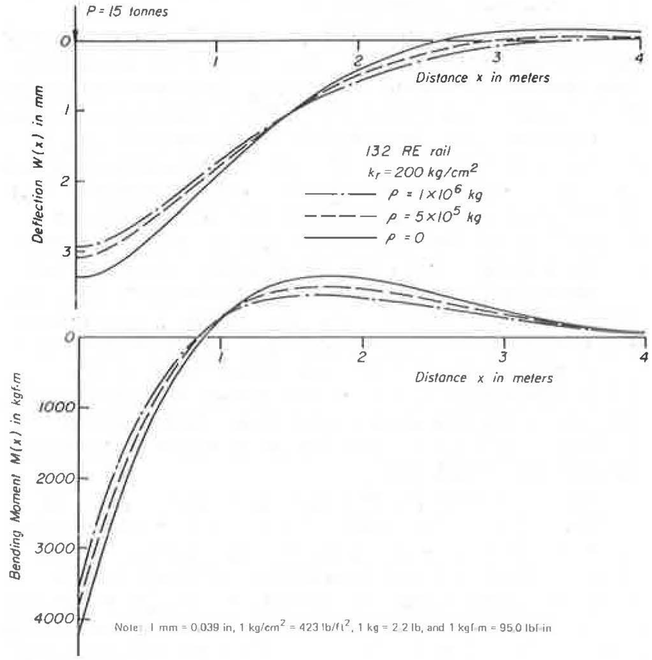
$$EI(d^4w/dx^4) - \rho(d^2w/dx^2) + k_r w = q \tag{6}$$

where ρ = proportionality constant. Equation 6 is a fourth-order ordinary differential equation with constant coefficients. It is identical to the equation of a stretched beam that is attached to a Winkler base. Thus, in the framework of a linear formulation, the effect of the continuous reaction moment is the same as the effect of an axial tension force in the rails of intensity ρ.

To show the effect of the tie-resistance moments on the behavior of the track, let us analyze a long straight track in which each rail is subjected to a wheel load P (Figure 7), by using equation 6. Since the track response is expected to be symmetric, the origin of the coordinate system is placed at the load P, and the analysis is restricted to $x > 0$. From regularity conditions at $x = \infty$ and the matching conditions at P,

$$(dw/dx)|_0 = 0, \quad (d^3w/dx^3)|_0 = P/2EI \tag{7}$$

Figure 8. Dependence of deflections and bending moments on ρ.



the resulting solution for x > 0 is (41, p. 129)

$$w(x) = (P/2k_r)(\kappa^2/\alpha\beta)\, e^{-\alpha x}(\beta\cos\beta x + \alpha\sin\beta x) \qquad (8)$$

and the corresponding moments are

$$M(x) = (P/4)(1/\alpha\beta)e^{-\alpha x}(\beta\cos\beta x - \alpha\sin\beta x) \qquad (9)$$

where $\alpha = (\kappa^2 + \rho/4EI)^{1/2}$, $\beta = (\kappa^2 - \rho/4EI)^{1/2}$.

To show the effect of the new parameter (ρ), Equations 8 and 9 were numerically evaluated for a track having a base parameter for one rail of k_r = 200 kgf/cm² (84 500 lbf/ft²), various values of ρ, and P = 15 Mg (15.3 tons). The results obtained are shown in Figure 8. For these track parameters, the largest deviation of the deflections and the bending moments is up to 20 percent, a range anticipated by Hanker (40, p. 47).

If, in addition, the base response is improved by replacing the Winkler foundation by a two-dimensional Pasternak foundation (42, 43), then Equation 6 becomes

$$EI(d^4w/dx^4) - (\rho + G)(d^2w/dx^2) + k_r w = q \qquad (10)$$

where G = second foundation modulus. Equation 10 is identical to Equation 6 except for the coefficient of the second term. Thus, the solution stated for Equations 8 and 9 is also valid for this case if ρ is replaced by (ρ + G).

When determining the vertical force a rail exerts on a tie, it should be noted that if Equation 6 is used for the analysis of the rails, then the pressure exerted by one rail is as before—i.e., $p(x) = k_r w(x)$. However, if Equation 10 is used, the pressure is $p(x) = k_r w(x) - G d^2w(x)/dx^2$ (42).

The determination of the base parameter k_r, which enters Equation 3, has been discussed by Kerr (21) who showed that the conducted tests for obtaining k_r, which loaded only one tie, are conceptually incorrect, because k_r depends on the loading area. Because of the simplifying assumptions implicit in Equation 2 and hence also in Equation 3, the determination of k_r should be such that the analytically obtained quantities (such as rail deflections and the stress distribution in the rails) should agree with the corresponding actual quantities as closely as possible.

To achieve this objective, first of all, a test should involve a relatively long section of track. If the actual track is not being tested directly, the size of the ties and their spacing in the test section should be the same as in the actual track. The method of determining k_r from the test data should be such that the analytical and the corresponding test results agree as closely as possible. For example, k_r in Equation 3 can be determined by equating the deflections or stresses at one point only (e.g., at P). Another way is by using a least-squares fit over a specified region. However, this procedure requires measurements at many points along the track, and Kerr (21) has shown that for the determination of k_r, which appears in Equation 3, one measurement at one point is sufficient.

The procedure for determining the two track coefficients (k_r and ρ) that appear in Equation 6 is similar to the one discussed above, except that at least two measurements are necessary. For example, subject each rail of a track to a load P, as shown in Figure 7, measure the resulting (instantaneous) deflections at P and at a point on the track half way between P and the point where the rail deflections approach zero, and equate these two values with the corresponding deflections given by Equation 8. This gives two simultaneous algebraic equations, with k_r and ρ as the only unknowns, whose solution is the two track parameters.

The above analyses assume that the load P is vertical and that it acts centrally through the vertical rail axis. However, in general, a railroad wheel does not act on the rail centrally. Furthermore, the wheels of a moving train also exert lateral forces on the rails. The corresponding stress analyses and test results have been given by Timoshenko (44) and by Timoshenko and Langer (45). More recent discussions of these stresses have been given elsewhere (46, 47).

In the wheel-rail contact region, where very large loads are transmitted from the wheel to the rail over a very small area, the actual stresses deviate considerably from those calculated by the beam theories. The occurence of shelling rail failures has prompted many analytical and experimental studies of this problem. An extensive survey of these analyses and tests has been given by Paul (48) and related papers that discuss rail reliability and rail failures by Steele (49) and Stone (50).

The determination of the stress distribution in the ballast has been discussed by Clarke (51) and more recently by Lundgren, Martin, and Hay (52) and So and others (53).

Another research effort has been an investigation of thermal track buckling. This problem increased in importance and urgency with the introduction of continuously welded rails on main lines shortly after World War II. A thermally buckled track is shown in Figure 9. Since the early 1930s many analyses of track stability and results of track-buckling tests have been published, but in spite of this, there is no generally accepted method available for analyzing this problem.

A critical survey of the analyses of thermal track buckling and related tests has been given by Kerr (54). This survey showed that the majority of the published results are not suitable for analyzing thermal track-buckling problems, because they are based on formulations that do not describe correctly the physical problems under consideration. One of the deficiencies is the omission of consideration of the decrease in the axial force in the rails due to buckling. Those few analyses that are conceptually correct have analytical faults with unknown effects on the final results.

Figure 9. Thermally buckled track.

To eliminate some of these problems, Kerr has given an improved analysis of track buckling in the lateral plane (55, 56). This analysis was subsequently used to determine the validity of a conjecture made by Kerr (57) that many track-test facilities, especially those used by British Railways and the German Federal Railroad were too short. The results obtained (58) show that the data recorded on short test tracks may deviate strongly from those obtained on long tracks in main-line service. These results also provide a guide for choosing the proper track length for thermal-buckling tests and for interpreting the test results obtained on short test tracks.

The mathematical level of Kerr's analyses is relatively high. Therefore, to simplify their use, the results obtained were evaluated numerically for a wide range of the track parameters encountered in the United States and the results presented graphically. Examples that show their use are given in a recent paper that also contains a general discussion of track buckling and measures for preventing it (59).

When a track is very rigid in the lateral plane (such as a slab track) or when a conventional crosstie track is prevented from moving sideways by adjacent structures, then the track can buckle in the vertical plane by lifting off the ballast. This problem has been analyzed recently by Kerr and El-Aini (60), and by El-Aini (61).

In all of the track-buckling analyses published, it is assumed that the track buckles either in the lateral or in the vertical plane. The effects of these restrictions on the results obtained have been studied by Kish (62).

The track-buckling studies are not yet complete. A complete clarification of this problem will require additional analyses, as well as laboratory and field tests, in which heated tracks will be subjected to moving trains, to validate the criteria used and the analytical results obtained.

CONCLUSION

A review of past and present track designs and their performances has shown the need for improvements in the present crosstie track system and for establishing the proper track system for the future, if concrete is used as tie material. Searching for these innovations should take into consideration that they must be economically feasible as well as technically sufficient.

Experimental and analytical investigations of the response of the track and its components to static and dynamic load are being made in many countries. Some areas are by now well understood, but others require more research. Although some areas of investigation are amenable to analytical studies, many less explored areas will at first require extensive experimental research programs.

It is reasonable to expect that in the United States, the recently intensified research activities will contribute in a relatively short time period to the solution of many problems of interest to our railroads. In view of the almost complete absence of railroad-related courses in the curricula of our engineering schools, special attention should be given to the transfer of these new results to the practicing railroad engineer, so that they may be used for improving our railways.

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Use of Floating-Slab Track Bed for Noise and Vibration Abatement

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Underground rail rapid transit systems can produce ground-borne vibration and noise from trains that creates intrusion in buildings located close to the underground facilities. This intrusion is usually a low-frequency (31.5- to 125-Hz range) noise or rumble transmitted via the intervening ground to the building structure. The use of floating-slab track bed, concrete slabs supported on resilient elements, to isolate the vibration of the rail support from the subway structure has been effective in reducing the transmission of vibration and noise to the surrounding ground and nearby buildings. This paper presents details on two types of lightweight floating-slab track bed; i.e., the continuous and the discontinuous designs. Some sections of continuous floating-slab track bed are in service at the Washington Metropolitan Area Transit Authority Metro System, and measurements of the reduction of the noise and vibration levels are presented.

Rail transit vehicles produce ground-borne vibration and noise that can and do create intrusion in nearby buildings, and this is particularly so for underground transit facilities that may be very near to buildings. This noise and vibration, which originates at the interface of the wheel and the rail, has been a significant problem along some subway corridors. With modern, lightweight vehicles and continuously welded rail, the vibration is seldom of sufficient amplitude to be felt as mechanical vibration or motion, and the only sensation is that of a low-frequency noise or rumble. But with older vehicles and jointed rail, the noise is sometimes accompanied by noticeable vibration.

Ground-borne noise can be reduced by vibration isolation of the track bed to interrupt the transmission path. The use of a floating-slab track bed, which consists of a concrete slab supported on resilient pads, can provide a vibration-isolated inertial base for support of the running rails. This design has been found effective in reducing the transmission of vibration and noise to the surrounding ground and nearby buildings in a manner similar to that of the inertial bases on springs that are used to support stationary machines. The use of floating-slab track bed provides for both reduced intrusion in nearby buildings and the placement of new rail transit subways in closer proximity to buildings.

A number of designs for vibration-isolated track bed have been developed ranging from heavy bridgelike structures with thick rubber support pads and damping applied to the bridge deck to relatively light concrete slabs without damping supported on thin resilient pads. Two basic forms of the relatively light slabs have evolved: (a) continuous slabs that are cast in situ and (b) discontinuous precast slabs. The original lightweight floating-slab design now in use in North America was developed in 1970 for the Washington Metropolitan Area Transit Authority and is of the continuous configuration. Trains have been operating on these slabs since 1975 with excellent performance. No operational information is yet available about the second-generation discontinuous-slab design, which was developed in 1974; the installations using it are not yet operational.

In the design of subway transit facilities, floating slabs are used only in critical areas where it is necessary to reduce ground-borne noise because of the critical proximity of buildings. These track beds add significantly to the cost of the subway structure, and their use is not appropriate except to avoid unacceptable noise intrusion.

DESIGN

The lightweight floating-slab design is based on the concept of the inertial mass-on-spring vibration isolator and uses a simple single-degree-of-freedom analysis for the vertical motion of the floating slab. A maximum deflection of 3 mm (0.125 in) under the static load of the train is generally imposed to limit the rail deflections to acceptable values. To avoid modal interactions and provide adequate control of the motion of the slab along with achieving a significant reduction of the ground-borne noise, the slab mass is made at least equivalent to the train mass and three times the bogie unsprung mass, considering the masses to be distributed over the vehicle length. The vertical fundamental resonance frequency for uniform motion of the slab, loaded with the bogie mass as a dynamic load, must be less than 16 to 18 Hz to provide reduction of the low-frequency audible sound. The design goal is generally 13 to 15 Hz; lower frequencies can be used only if greater rail deflection is allowed or if more space is used, to allow for the greater mass of the slab.

With a loaded resonance of 15 Hz for uniform vertical harmonic motion and a maximum live-load static deflection of 3 mm, the mass of modern rail transit vehicles leads to a design consisting of concrete slabs 275 to 375 mm (11 to 15 in) thick and 3.0 to 3.5 m (10 to 11.5 ft) wide and supported on 75-mm (3-in) thick elastomeric pads. The slabs must be completely isolated from the subway structure and, therefore, the lateral and longi-