

should not be disregarded. It would be difficult to defend cut-off points or subdivisions of the list based solely on ranks, particularly when it turns out that the cut-off point discriminates between actions that differ very little in terms of the numerical measure that forms the basis of the ranking. Moreover, the actual distribution of numerical values can provide support for the selection of cut-off points.

In this regard one should recognize the mode or modes of the distribution; their presence and location might assist the analyst in selecting the number of different treatments or types of actions, and the troughs between modes might prove convenient and defensible cut-off points for assigning treatments.

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Optimum Fleet Sizing in the Northeast Corridor

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The Regional Railroad Reorganization Act of 1973 mandated the U.S. Department of Transportation to undertake engineering and planning studies for improved passenger rail service in the Northeast Corridor. In order to obtain fleet estimates and to analyze the effects of management strategies a calculation of the optimum number of cars required for a design day service in the Northeast Corridor was undertaken. A linear programming model that determines fleet requirements for several different formulations of the objective function was formulated. Minimum fleet size was then calculated from a demand forecast based on the service standards prescribed in the Railroad Revitalization and Regulatory Reform Act of 1976. Minimum car-kilometers per day and maximum load factor were also found. The analysis indicated that the most heavily traveled portion of the corridor, Philadelphia to New York, might be better served by adding trains between these two cities.

In 1973, Congress passed the Regional Railroad Reorganization (3R) Act. This complex piece of legislation dealt with passenger as well as freight operations and called for the U.S. Department of Transportation (DOT) to improve passenger rail service in the Northeast Corridor (NEC) as recommended in the 1971 Northeast Corridor Report. The NEC is defined as the rail line extending from Boston to Washington. It is 734 km (456 miles) long and crosses eight states and the District of Columbia. Included in the corridor are four major

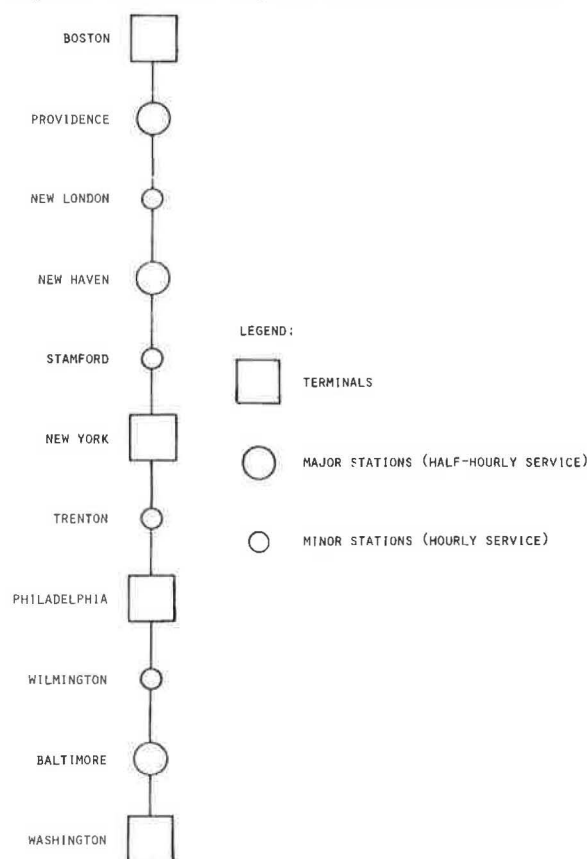
metropolitan areas: Washington, Philadelphia, New York, and Boston.

With the mandate of Congress, the Federal Railroad Administration (FRA) undertook several major studies to examine in detail

1. Ridership that might be expected with high-speed service,
2. Investment required to achieve high-speed service, and
3. Financial viability of the improvement project.

The Transportation Systems Center, supporting the Office of Northeast Corridor Development in FRA, provided the major analytical effort in the areas of financial analysis and demand forecasting. The results of these efforts, as well as those of the engineering studies, provided the necessary background for passage of the Railroad Revitalization and Regulatory Reform (4R) Act that was signed into law in February 1976. This legislation provides \$1.9 billion for improving rail service between Boston and Washington and requires the following trip times by February 1981.

Figure 1. Terminals and major and minor stations in the NEC.



Time	Trip
1 h	Philadelphia to New York
2 h 40 min	Washington to New York
3 h 40 min	Boston to New York

In order to meet these deadlines, extensive improvement of the right-of-way will be undertaken. Track will be realigned and upgraded; bridge and tunnel structures will be modified. The New Haven to Boston segment of the right-of-way will be electrified, and the electrification system in the remainder of the corridor will be improved. Fifteen stations will be extensively renovated, and new equipment service facilities will be constructed. Finally, new rolling stock will be required for corridor service.

Financial analysis of the improved service required an estimate of the fleet size for each year of operation. It was assumed, for purposes of the financial analysis, that the required fleet was

$$\text{Number of cars in fleet} = \frac{(\text{annual passenger-kilometers}) / [(\text{seats per car}) (\text{load factor}) (\text{annual car utilization in kilometers})]}{(1)}$$

There were two major shortcomings to this approach. It required an estimate of system load factor, and it did not consider the variation in patronage at different hours of the day, or on different days. It could therefore not determine if the proposed system would have enough equipment to meet demand peaks, nor was there a way to determine if certain trains would be filled to capacity, causing travelers to be turned away. The need for a more careful analysis of scheduling and fleet sizing led to the work described in this paper.

FLEET MANAGEMENT STRATEGIES

Proper fleet management results in reduced fleet size, lower operating costs, and increased ridership and deals with scheduling both of trains and of the units that make up these trains.

At present only conventional diesel trains operate on the nonelectrified portion of the corridor north of New Haven, while two types of service operate south of New Haven on the electrified portion: conventional trains and Metroliners. However, since conventional trains and Metroliners offer very different levels of service, the users are deprived of the benefits from the actual frequency of the trains over this northernmost portion of the corridor. Also, because the north is not electrified, travel between the southern and northern portions of the corridor often requires a time-consuming transfer at New York.

When electrification of the entire corridor is completed, a more integrated schedule will be possible, and more options will be available for fleet management. The following are some possibilities:

1. All cities can be treated alike, for example, the current situation in which conventional trains stop at all the cities along their routes;
2. More frequent service can be given to major cities, for example, half-hourly service to Washington, Baltimore, Philadelphia, New York, New Haven, Providence, and Boston and hourly service to Wilmington, Trenton, Stamford, and New London; and
3. Express and feeder systems can be established whereby local trains would stop at all stations along a segment of the corridor and transfer passengers to an express train at the first major corridor city.

All three strategies can be subdivided into those requiring a constant train length and those permitting cars to be added and deleted at one or more intermediate stops.

For the purposes of this study it was decided to begin by modeling the fleet management strategy that provides more frequent service to major cities and allows for modification of the train length at selected stops.

SYSTEM DESCRIPTION

Eleven cities along the NEC rail line are assumed to have improved service. Seven of these cities receive half-hourly service and the remaining four, hourly service. Figure 1 identifies the 11 cities and the level of service at each. In addition, it is assumed that train length can be modified at Philadelphia and New York as well as at the two end points, Washington and Boston. The switching points are referred to as terminals. The trip times required by the 4R Act include intermediate stopping times. Because the time gained by not stopping at a station is negligible (estimated at 1.25 min by the Engineering Division of the NEC Project Office), it is assumed that the skip-stop service has the same running time as the local service trains. There is a 20-min time requirement for reversing the direction of a car, which can be done at any of the four terminals.

The assumed uniform fleet with an average car capacity of 75 passengers corresponds to Amfleet equipment and allows one snackbar car for every four cars. Parlor car service is not considered.

All equipment is locomotive hauled with a maximum train length of 14 cars, not including the locomotive. Maximum train length is determined by the platform lengths planned for the improved system. If more than 14 cars are required to satisfy the projected demand, a

second section will be added to the schedule. All dead-heading is accommodated through the existing schedule.

MODEL FORMULATION

The system described above can be modeled by a transshipment network whose unit of flow is one car. In its simplest form, the network has one node for each potential arrival or departure time at each city. These nodes are connected by two types of directed arcs, storage arcs and train arcs.

Storage arcs connect each time node for each city to the immediately following time node; flow along one of these arcs represents storage of cars at a city during the interval between two times. Train arcs connect a time node in one city to a subsequent time node in a different city; flows along these arcs represent movement of cars in scheduled trains from one city to another.

Network flows must satisfy constraints of several sorts: flow must be conserved at every node (cars do not enter or leave the system); flows along train arcs must be great enough to meet demands; and all flows must be integer and nonnegative.

This network system may be transformed to an equivalent set of linear programming (LP) constraints. (LP is an efficient technique for computing an optimum solution.) In more precise terms, this is done as follows. First, define the relevant sets as

$$\begin{aligned} C &= \text{set of cities;} \\ T &= \{0, \dots, \tau - 1\} = \text{set of time intervals into} \\ &\quad \text{which the day (or other sched-} \\ &\quad \text{ule period) is divided; and} \\ S &\subset \{(c, t, c', t') \in \\ &\quad C \times T \times C \times T \mid c \neq c'\} = \text{schedule for which each ele-} \\ &\quad \text{ment } (\epsilon \text{ is "element of") rep-} \\ &\quad \text{resents a train that leaves} \\ &\quad \text{city } c \text{ at time } t \text{ and arrives} \\ &\quad \text{at city } c' \text{ at time } t'. \end{aligned}$$

Then represent the demands by

$$d_{cc'}[t, t'] > 0 = \text{smallest (integral) number of cars required to meet demand for train } (c, t, c', t') \in S.$$

Express the nodes of the network as

$$A_c[t] \quad \text{for all } c \in C, t \in T.$$

The directed arcs representing storage of unused cars (U) are then

$$U_c[t]: A_c[t] \rightarrow A_c[(t+1) \bmod \tau] \quad \text{for all } c \in C, t \in T.$$

The arcs representing movement of cars in trains (X) are

$$X_{cc'}[t, t']: A_c[t] \rightarrow A_{c'}[t'] \quad \text{for all } (c, t, c', t') \in S.$$

The LP structural variables corresponding to each arc represent the flow over the arc as

$$\begin{aligned} u_c[t] &\text{ flow over } U_c[t] && \text{for all } c \in C, t \in T, \text{ and} \\ x_{cc'}[t, t'] &\text{ flow over } X_{cc'}[t, t'] && \text{for all } (c, c', t, t') \in S. \end{aligned}$$

The constraints on network flow are then expressed as in the following table.

Constraint	Expression
Conservation of flow	$u_c[(t-1) \bmod \tau] + \sum_{\{(c_1, t_1, c, t) \in S\}} x_{c_1 c}[t_1, t] = u_c[t] + \sum_{\{(c, t, c_2, t_2) \in S\}} x_{cc_2}[t, t_2]$ <p>for all $c \in C, t \in T$;</p>
Satisfaction of demand	$x_{cc'}[t, t'] \geq d_{cc'}[t, t']$ <p>for all $(c, t, c', t') \in S$</p>
Nonnegativity	$u_c[t] \geq 0$ <p>for all $c \in C, t \in T$</p>
Integrality	$u_c[t] \text{ integer}$ <p>for all $c \in C, t \in T$</p> $x_{cc'}[t, t'] \text{ integer}$ <p>for all $(c, t, c', t') \in S$</p>

Satisfaction of demand ensures nonnegativity of the x variables.

Given that all $d_{cc'}[t, t']$ are integers, a fundamental property of transshipment problems guarantees that every basic solution to the above LP is an integral solution. Consequently, a feasible solution to the above problem—and hence a feasible allocation of cars to trains—may be determined directly by application of the (phase 1) simplex method. Given any linear objective function, the simplex method will also find the most feasible allocation. Objectives of special interest follow.

Capital Cost

The daily cost of amortizing the passenger-car fleet, here referred to as the capital cost, may be considered proportional to the number of cars in the fleet. Hence, minimizing fleet size is equivalent to minimizing capital cost. A linear expression for this objective is

$$Z_{CAR} = \sum_{c \in C} u_c[\tau - 1] + \sum_{\substack{(c, t, c', t') \in S \\ t' < t}} x_{cc'}[t, t'] \quad (2)$$

This expression counts the number of cars in the system during the last interval of the day. The first sum represents the number of cars in storage during the interval, while the second represents the number in trains that are running at that time.

Operating Cost

Cost proportional to the number of car-kilometers run in a day, here called operating cost, is another logical candidate for minimization. Letting the distance from c to c' be $m_{cc'}$, total car-kilometers per day equal the linear form

$$Z_{KM} = \sum_{(c, t, c', t') \in S} m_{cc'} x_{cc'}[t, t'] \quad (3)$$

Load Factor

Given fixed demands, it is reasonable to try to maximize system load factor in order to minimize the cost of providing service. By definition, system load factor is $Z_{LF} = (\text{passenger-kilometers/day}) / (\text{seat-kilometers/day}) = [(\text{passenger-kilometers/day}) / (\text{seats/car})] / (\text{car-kilometers/day})$. Since both passenger-kilometers per day and seats per car are fixed by the problem, Z_{LF} is inversely proportional to car-kilometers per day equals Z_{KM} . Hence, minimizing operating costs is equivalent to maximizing the system load factor.

Many desirable extensions and refinements of this model are presented in full detail in Fourer (1). Variations on the network permit the number of nodes to be greatly reduced and make possible a distinction between northbound and southbound trains. Techniques for opti-

mizing two or more objectives sequentially or in combination are also developed.

BASE RUN DATA

A hypothetical case representing service on a busy day in 1982, the first full year of improved service, was chosen for analysis. Annual patronage for 1982 was calculated by using a computer-based model (2) developed by Peat, Marwick, Mitchell and Company (PMM). The input data were those derived from PMM's base assumptions, with the exception of trip times, which were increased to reflect trip times required by the 1976 4R Act.

PMM's model estimated annual two-way patronage for individual station pairs in the NEC. Annual one-way patronage was computed by halving the two-way figures. A few possible station pairs were omitted, either because they could not be separated from other pairs or because competitive commuter service is available for their travelers. All of these excluded pairs are short in distance and are deemed relatively insignificant to corridor service.

The base run modeled patronage for a design day calculated as 1/270 of the annual amount. This concept of design day, representing approximately the tenth busiest day of the year, has been employed before in engineering studies of the NEC. Note that the fleet size determined by the model represents only those vehicles required for scheduled service. Additional units will be needed to accommodate maintenance requirements.

To derive the patterns of demand between station pairs over a day, the base run employed a set of cumulative demand functions. Following a PMM study method (3, pp. C.7-C.14), demand for service from a larger station to a smaller one was taken to be departure based (that is, dependent on the time of departure), while demand for service from a smaller to a larger station was arrival based (dependent on time of arrival). Demand between cities of comparable size was determined by averaging arrival-based and departure-based distribution functions. The demand distributions employed in the base run were bimodal Gaussian-like probability distributions fit to actual arrival and departure counts for Tuesday, May 21, 1974.

SOLUTION

The base run formulation was solved and analyzed by using the SESAME interactive linear programming system and supporting computer routines. The values of the objectives at their optimums for the base data were found to be $\min Z_{CAR} = 164$ cars, $\min Z_{KM} = 211\ 400$ car-km/day, and $\max Z_U = 74.15$ percent.

The next step was to minimize total operating and capital cost of the base model, expressed as

$$P_{CAR}Z_{CAR} + P_{KM}Z_{KM} \quad (4)$$

where

- P_{CAR} = capital cost per car/day,
- Z_{CAR} = number of cars in the system,
- P_{KM} = operating cost/car-km, and
- Z_{KM} = car-km/day.

The properties of an optimum solution depend on the value of P_{CAR}/P_{KM} , the ratio of capital cost per day to operating cost per kilometer. For the base data, there are three significantly different regions into which this ratio may fall.

1. Capital cost per day ≥ 724 (operating cost per kilometer). Here capital cost dominates; in any optimum solution the number of cars is at its absolute minimum, 164. The minimum number of car-kilometers per day, given 164 cars, is 218 800; the system load factor (which is inversely proportional to total car-kilometers) is 71.65 percent.

2. 724 (operating cost per kilometer) \geq capital cost per day ≥ 290 (operating cost per kilometer). At this level the influence of capital cost declines somewhat. The number of cars in an optimum solution increases to 167; car-kilometers per day decline to 216 700 (system load factor is 72.37 percent).

3. Capital cost per day ≤ 290 (operating cost per kilometer). Here operating cost dominates. In an optimum solution, car-kilometers per day is at its absolute minimum, 211 400 (system load factor is 74.15 percent), while the number of cars in the system increases to 185.

The results are shown graphically in Figure 2. Clearly, the biggest jump is at the critical ratio $P_{CAR}/P_{KM} = 290$, the round-trip distance between New York and Philadelphia. At ratios below this point, buying an extra car is economical even if it saves just one New York to Philadelphia run. At higher ratios it pays to buy a smaller fleet, running each car (on the average) more kilometers every day. The magnitude of the jump—about a 10 percent difference in fleet size—is not surprising. Demand is heaviest along the New York to Philadelphia segment and is highly unbalanced: northbound travel peaks in the morning; southbound demand is highest in the afternoon. Consequently, a fair amount of dead-heading can be avoided if a larger fleet is available. An examination of the passenger load on the three links in the network revealed that modifying the schedule to have some trains run only between Philadelphia and New York would accommodate projected demand and eliminate excessive switching at these points.

The other jump, at $P_{CAR}/P_{KM} = 724$, represents a point at which the cost of a car equals the cost of running it from New York to Washington and back. This is a fairly insignificant critical ratio, however, as the optimum at ratios below 724 requires only three cars more than the optimum above 724.

Several estimates of the actual P_{CAR}/P_{KM} are plotted against the critical ratios in Figure 3. The estimates suggest that P_{CAR}/P_{KM} probably falls into region 1 and, hence, that capital cost probably predominates. Moreover, if the ratio is not in region 1 it would very likely be in region 2, where the optimum solution is not very different.

OTHER ANALYSES

A number of additional analyses were conducted by using the base model and data. These have been described fully elsewhere (1) and can be summarized as follows.

Sensitivity to Demand

Alternative estimates of demand were derived through scaling the base patronage estimates by a constant factor; nine factors, ranging from 0.7 to 1.3, were chosen. Optimum solutions were calculated for each alternative demand estimate. It was found that both the minimum fleet size and the minimum number of car-kilometers that must be run with a minimum fleet were roughly proportional to total patronage over the range of factors chosen: $\min Z_{CAR} \approx 0.0000103$ (total annual patronage); $\min Z_{KM} | Z_{CAR} \approx 0.0138$ (total annual patronage).

Minimizing Turnaround

Using an expanded version of the model that distinguished northbound and southbound trains, we could minimize turnaround (changing car directions at terminal stations). Analysis of the optimum solution suggested that many cars are needed only for the Philadelphia to New York segment to satisfy peak demand northbound in the morning and southbound in the afternoon. This suggests a revised schedule in which New York to Philadelphia shuttle trains, in addition to the usual through trains, are run at peak hours.

Locomotive Requirements

Operating under simple assumptions, the model may be adapted to analyzing requirements for locomotives as well as for cars. We determined for the base data that a single solution minimized both the number of locomotives (31) required and the number of locomotive-kilometers (54 840) run.

FUTURE WORK

Many more sophisticated sensitivity analyses are conceivable if one allows patronage between different station pairs to vary at different rates. Other parametric studies include changing car capacity, altering turnaround time, and modifying train size limitations. Schedules can also be modified.

In addition, other fleet management strategies (several have been mentioned above) should be investigated in similar fashion and comparisons drawn. The present linear programming formulation is not capable of handling the more sophisticated express-feeder arrangement. It is likely that there is a suitable integer programming formulation that would, however, require different optimization techniques.

Finally, it should be noted that, although the model has been formulated for NEC operations, the same technique could be applied to other portions of the Amtrak system.

Figure 2. Cars and car-kilometers when total cost is minimized.

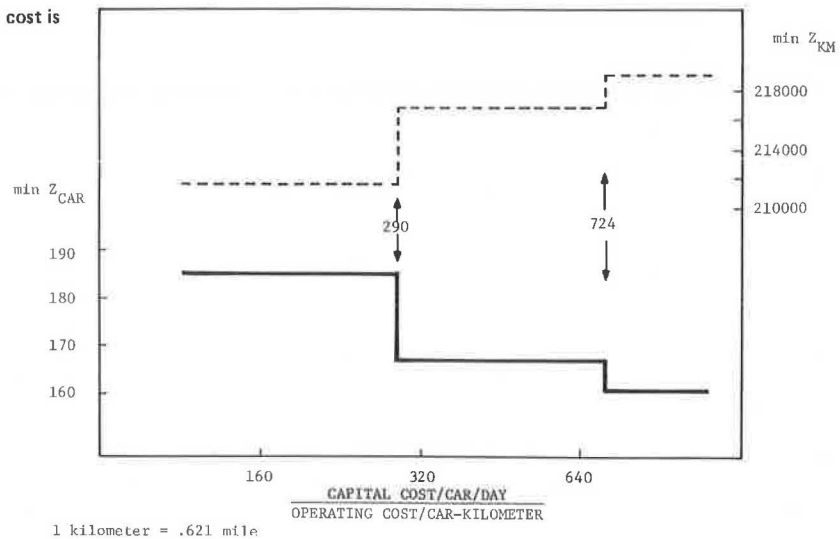
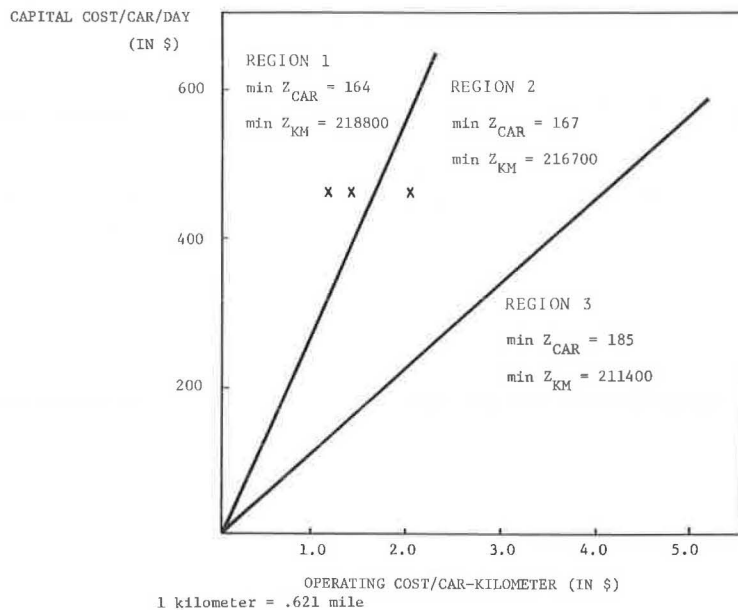


Figure 3. Three solution regions plotted as a function of capital cost per day and operating cost per kilometer.



ACKNOWLEDGMENTS

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Computer Methods in Blocking and Train Operations Strategies

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This paper presents a set of computer-aided methods for developing blocking and train operations strategies for railroad networks. These methods are iterative processes in which complex, judgmental decisions are made by experienced railroad operators and extensive, repetitive calculations are performed by a computer. By using these methods, railroad operators can compare the consequences of various blocking and train operations strategies in terms of such measures as car switching, yard loading, block size, car-kilometers, ton kilometers, train-kilometers, and the like, which are calculated by the computer; operators can then develop efficient blocking and train operations strategies.

The blocking and train operations strategies currently used by various railroad companies have taken years of professional experience, judgment, and knowledge to develop. However, because of mergers, railroad networks have become increasingly extended and complex, and network conditions and demand patterns have been changing continuously. Blocking strategies thus tend to lag behind the real-world situation by even a year or two and create a need to be constantly reviewed and revised.

One outstanding example of such a need occurred recently when Congress charged the U.S. Railway Association (USRA) with the responsibility of developing a systemwide operating and management plan for the rail operations of the Consolidated Rail Corporation (Conrail). A key element of USRA's approach to this problem was to develop detailed schemes for blocking railroad cars and forming trains, as well as for routing and scheduling these trains within the network both on rail lines and through the yards.

To get some idea of the magnitude of the problem, consider the following statistics about the Conrail network. It has about 32 200 km (20 000 miles) of track, part of which is double; it handles approximately 40 000 cars per day, including both loaded and empty cars; and it has 500 to 600 distinct origins and destinations (actually many more when considered in detail). With such a

large network and so much activity, it is obviously exceedingly difficult and laborious to analyze and develop blocking and train operations strategies purely manually.

On the other hand, the interrelations among the demand patterns, the car blocking, the train routing, and the constraints on rail tracks and yards are inherently so complex that the logic of forming blocks and trains cannot realistically be stated in sufficiently concrete steps for purely automatic generation of blocking and train operations strategies. Consequently, USRA needed a method by which complex judgmental decisions could be made by experienced railroad operators but the extensive and tedious calculations would be performed by a computer.

The resulting method, the subject of this paper, was developed by a team of researchers from USRA and Stanford Research Institute (SRI) and was used extensively in developing both the preliminary and the final plans for the Conrail system. However, because the method and the computer programs described in this paper are so general, they have also successfully been used to analyze and develop suitable blocking and train operations strategies for other railroad networks.

STATEMENT OF THE PROBLEM

In its basic form, our blocking and train operation problem can be stated as follows: Given a railroad network in terms of the origin-destination (O-D) nodes (yards) and the connecting links (tracks) and given the O-D demand data on railroad cars, we wanted to develop an efficient blocking and operations strategy for the movement of railroad cars.

Unfortunately, no single criterion of efficiency can be realistically defined for comparing various alternatives. However, operators used the following typical attributes of blocking and train operations strategies