Table 4. Parameter magnitudes for socioeconomically disaggregated version of model.

Variable	Income Group[a] Low (less than Rs 500)	Middle (Rs 500 to 1499)	High (more than Rs 1500)
Behavioral parameters			
α	0.112	0.105	0.120
β	0.120	0.140	0.120
Trip generation rates			
Work to home/employee	0.33	0.62	0.84
Home to service/household	0.29	1.02	2.31
Modal split probabilities[b]			
Work			
Motor vehicle	0.03	0.21	0.69
Mass transit	0.33	0.52	0.27
Bicycle	0.64	0.27	0.04
Service			
Motor vehicle	0.04	0.23	0.71
Mass transit	0.61	0.64	0.21
Bicycle	0.35	0.13	0.08

[a]In 1969, Rs7.49 = $1.00. [b]Entries do not include pedestrian trips.

and transportation model to issues encountered in regional strategic development planning. Analysis models used for this type of planning must have modest data requirements, be adaptable to a variety of issues, and have quick computer turn-around time.

The first application of the model was to aid in the formulation of a consistent and desirable set of public development policies. In this application, the behavioral parameters of the model were disaggregated spatially, but the residential submodel was aggregated over all socioeconomic groups and the service submodel was aggregated over all service sectors.

The second application involved detailed analyses of the distribution of service employment by sector within one subregion. In this application the service employment submodel was disaggregated into a number of service employment sectors.

The final application involved the development of a version of the model disaggregated by socioeconomic group for Delhi, India. A disaggregated version is required because of the tremendous range of socioeconomic groups in Indian cities and the very different spatial distributions of employment and housing opportunities available to these groups. This version of the model has been used to explore the impacts of alternative land development policies on the corridor level volumes of road and public transportation trips.

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Impact of Transportation on Urban Density Functions

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A method is proposed for analyzing the variable impact of transportation on urban structure. The varying coefficient model, which uses the negative exponential density function as a theoretical base, provides a means for systematically incorporating hypothesized effects of current and past levels of transportation while holding constant population, income, and other factors identified with current urban spatial structure. The aspects discussed include the following: the theoretical basis for the hypothesized effects of the conditioning variables to be investigated, the development of the model in relation to changing density functions, the estimation of model parameters by use of available cross-sectional data, the application of the model to the generalized problem of the urban density function, and simulated forecasts and analyses of transportation-related changes in urban structure for selected cities.

The relation between density and distance—or, more generally, the density gradient—has been used in recent years to explain urban spatial structure. The standard functional form assumed for the density gradient is the negative exponential; i.e.,

$$D(u) = D_o \exp(-ru) \quad (1)$$

where

$D(u)$ = density u distance from the urban center;
D_o = density at the urban center; and
r = density gradient, the percentage by which $D(u)$ decreases as distance increases.

Previous models of urban economies have focused on explaining the intensity of land use and employment by distance from the urban center, incorporating modifications to include transportation cost, income, past development, and other selected socioeconomic factors. This paper proposes an alternative method for analyzing the variable impact of transportation on urban structure. The varying coefficient model (VCM), which uses the negative exponential density function as a theoretical base, provides a means for systematically incorporating hypothesized effects of current and past levels of transportation while holding constant population, income, and other factors identified with current urban spatial structures. The VCM thus generalizes the simple exponential density function to accommodate more realistic hypotheses about the impact of transportation on urban structure. Because transportation exhibits high secondary relations with time, the VCM also represents a basis for sharpening existing forecasting tools. In addition, because its use requires little additional computation or data collection, it is useful in exploratory statistical analyses of urban structure and other applied economic problems. This study applies the VCM to the estimation of an urban density function conditioned on factors that vary within and among cities.

THEORY AND DATA

The theoretical foundation for the density gradient provided by Muth (6) can be used to determine qualitative effects of transportation on the intercept and the slope of the resulting exponential function. Briefly, housing is produced by using land that surrounds the central business district (CBD). Workers residing in these households are assumed to commute to and from jobs in the CBD. The optimal household location for a cost-minimizing worker employed in a CBD occurs when

$$-\partial p/\partial u(q) = \partial T/\partial u \quad (2)$$

where

p and q = price and quantity of housing services respectively and
T = transportation cost.

Thus, $-\partial p/\partial u(q)$ is the reduction in expenditure necessary to purchase a given q that results from moving u distance away from the CBD. The derivative $\partial T/\partial u$ represents the increase in T that is incurred by making such a move. By using Equation 2 and related formulations of the demand for housing, Muth was able to derive qualitative effects for a number of variables on optimal location. Because the model is well-known, this discussion only reviews the qualitative results as specialized for the variables selected for empirical analysis in this study.

The data consist of a random sample of 43 census tract densities measured u distance from the CBD for each of 39 U.S. cities in 1970. Two corresponding sets of additional data were also used. The first consisted of observations for each of the 43 tracts in the various cities—referred to as tract-specific variables. The tract-specific variables are the percentage of commuters who use public transportation (X_1) and income (X_2). The percentage of public transportation commuters is used to reflect the impact of the introduction and continued use of subways or bus systems on urban structures. Relative costs of private versus public transportation are, of course, difficult to determine. Instead of making statements about relative costs that cannot be tested, this study uses observed behavior to establish the importance of the transportation variable. Muth's model shows that an increase in either the fixed or the marginal costs of transportation decreases the equilibrium distance from the CBD for any household.

The relation between optimal household location and income is important because it determines patterns of housing consumption in different parts of the city. For example, consider a general increase in the level of income for the residents of a city. The increase in income would increase housing consumption and, assuming this outweighed effects of increased transportation cost and housing prices, the equilibrium distance from the CBD would increase for all households. On the basis of this reasoning, the density gradient is expected to vary inversely with the level of income.

The second set of concomitant data is citywide and designed to explain differences among cities attributable to variations in past development. Harrison and Kain (2) have demonstrated the importance of past development on current land use. In fact, they have suggested that the principal differences in urban structures among U.S. cities are attributable to differences in the timing of their development. For example, in the Los Angeles metropolitan area, dwelling units constructed between 1950 and 1960 accounted for almost 40 percent of the total in 1960; in Boston they accounted for only 16 percent (2). Two variables were used this study to capture these effects: relative age of the city (X_3) and city population (X_4). The age of the city, which is based on the last significant spurt of growth, pinpoints the timing of significant structural changes that have occurred in the city. Population levels are used to represent overall scale effects caused by past development. Generally, recent spurts of growth and population increases would tend to reduce the density gradient because of the effect on transportation of technological changes such as freeways and the automobile (6).

THE MODEL

In specifying a model that is consistent with the theory and the data discussed above, the approach was to use the exponential density function but to introduce systematic changes in parameters. That is, the parameters of the density function are hypothesized to vary as a result of the interplay of city- and tract-specific variables. As indicated above, the a priori basis for relating parameters of the exponential density function to city- and tract-specific variables is somewhat limited. Generally, the theory only yields conclusions for signs of anticipated parameter changes.

Because of limited prior information, the VCM proposed is one with a polynomial as the structure for possible changes in parameters. Because the specification locally approximates more complex relations, it is useful for exploratory work. To implement the polynomial specification, let

$$\ln D_o = \ln D_o(X_1, X_2, X_3, X_4) = \sum_{n_1=o}^{q_{o1}} \sum_{n_2=o}^{q_{o2}} \sum_{n_3=o}^{q_{o3}} \sum_{n_4=o}^{q_{o4}} \beta^o_{n_1,n_2,n_3,n_4} X_1^{n1} X_2^{n2} X_3^{n3} X_4^{n4} \qquad (3)$$

and, similarly, for the slope coefficient (r) in Equation 1, let

$$r = r(X_1, X_2, X_3, X_4) = \sum_{n_1=o}^{q_{11}} \sum_{n_2=o}^{q_{12}} \sum_{n_3=o}^{q_{13}} \sum_{n_4=o}^{q_{14}} \beta^1_{n_1,n_2,n_3,n_4} X_1^{n1} X_2^{n2} X_3^{n3} X_4^{n4} \qquad (4)$$

The parameters $\ln D_o$ and r are thus polynomials of orders q_o and q_1 respectively in X_1, X_2, X_3, and X_4 (the four city- and tract-specific variables). Application of this revised specification to the data is straightforward. $\beta^o_{n1,n2,n3,n4}$ and $\beta^1_{n1,n2,n3,n4}$, as well as values for city- and tract-specific variables that correspond to the data points, determine the exponential density function. The special case $n_1 = n_2 = n_3 = n_4 = 0$ is the constant coefficient, log linear density function.

The advantages of the VCM provided by Equations 3 and 4 combined with the hypothesis for the log linear density function should be apparent. The VCM generates city- and tract-specific results but within the context of a functional form that has theoretical and empirical support. Moreover, the flexibility of the VCM would appear to make the exponential density function more useful in policy analysis and prediction. Since the selected city- and tract-specific characteristics may be subject to control by policy action or may be comparatively easily projected on the basis of time, the model can be used for both forecasting and policy analysis even though it is estimated from cross-sectional data. This feature is not without statistical limitations, but it should prove especially useful given the data bases available for studying density patterns in urban economies.

METHODS OF ESTIMATION

The estimation procedure follows from error assumptions and additional information that restricts the numbers of parameters for the model as expressed in Equations 3 and 4. First, the polynomials that relate $\ln D_o$ and r to the X_1, X_2, X_3, and X_4 variables are assumed to be second order. Even if this is assumed, application of the standard formula for permutations shows that there are 1320 parameters for each of the hypothesized conditioning structures on the $\ln D_o$ and r coefficients. Although the data are extensive in comparison with those used in some other studies, they obviously cannot support this ambitious specification. As a result, the number of parameters required to determine the variable coefficients of the log linear density model was further limited.

The approach used to obtain these restrictions is based on the intended uses of the model and on preliminary tests in the sample data. Although there are some obvious statistical problems with this method (8), no alternative was possible. First, four versions of the model of the density function were estimated; in each version the coefficients were functions of only one conditioning variable. For example, in the case of the X_1 tract-specific variable—the percentage of commuters who use public transportation—the assumption was $q_{o_2} = q_{o_3} = q_{o_4} = o$ and $q_{12} = q_{13} = q_{14} = o$, which implies structures for the VCM that are determined on the basis of six parameter estimates. If i denotes the city and j the tract for this special case, the model given in Equations 3 and 4 can be expressed as

$$\ln D(u)_{ij} = \ln D_{o_{ij}} - r_{ij} u + \epsilon_{ij} \qquad (5)$$

for the 43 × 39 observations in the sample. An additive error term (ϵ_{ij}) with a subsequently specified structure has also been included. Applying the special assumptions to Equations 3 and 4 yields

$$\ln D_o(X_1, X_2, X_3, X_4) = \ln D_o(X_1) = \ln D_{o_{ij}} = \sum_{n_1=o}^{q_{o1}} \beta^o_{n_1} X_{ij}^{n1} \qquad (6)$$

and

$$r(X_1, X_2, X_3, X_4) = r(X_1) = \ln r_{ij} = \sum_{n_1=o}^{q_{11}} \beta^1_{n_1} X_{ij}^{n1} \qquad (7)$$

where subscripts for β^o and β^1 that correspond to the excluded conditioning variables have been omitted for convenience.

The model specified in Equations 5, 6, and 7 includes coefficient restrictions across tracts and cities. It is clear, therefore, that pooling of the data on tracts and cities is necessary to estimate the required parameters. In addition, plausible assumptions for the distribution of the structural disturbance ϵ_{ij} point to the advantages of pooling the data (1, 9, 10).

Based on the results from the four simplified VCMs and prior information to be subsequently discussed, a model was specified that incorporates the effects of all the coefficient conditioning variables. In terms of Equations 3 and 4, the structure of coefficient variation for the density function for this final model is

$$\ln D_o = \beta^o_{oooo} + \beta^o_{looo} X_1 + \beta^o_{oloo} X_2 + \beta^o_{oolo} X_3 + \beta^o_{oool} X_4 \qquad (8)$$

and

$$r = \beta^1_{oooo} + \beta^1_{looo} X_1 + \beta^1_{2ooo} X_1^2 + \beta^1_{oloo} X_2 + \beta^1_{o2oo} X_2^2 + \beta^1_{oolo} X_3 + \beta^1_{oo2o} X_3^2 + \beta^1_{oool} X_4 + \beta^1_{ooo2} X_4^2 \qquad (9)$$

It should be apparent that final specification concentrates on variation in the density gradient (r). By using an argument analogous to that made for Equation 5, this structure can be substituted to reparameterize the model of the exponential density function, and generalized least squares methods can be applied to obtain estimates with desirable asymptotic properties. In addition, based on the procedures just described, the central and noncentral F-statistics can be used to test the null hypothesis (the density function model with constant coefficients) for appropriateness given the sample data.

EMPIRICAL RESULTS

Results from an application of the density function model with constant coefficients on a city-by-city basis are given in Table 1. These estimates provide a source of comparison for estimates derived from the alternative VCMs. The results in Table 1 demonstrate a concern about the appropriateness of the exponential density hypothesis with constant coefficients. Both $\ln D_o$ and r are statistically significant for most of the 39 cities. But there are important differences in their magnitudes, especially for r. In addition, the estimated density function for the pooled data did not explain a high proportion of the variation that was observed in the de-

Table 1. Ordinary least squares estimates of coefficients for exponential density function for 39 cities and pooled city data.

City	$\ln D_o$	r Estimate	r t-Statistic	R^2	City	$\ln D_o$	r Estimate	r t-Statistic	R^2
Akron	9.273	-0.202	-2.86	0.167	Pittsburgh	9.689	-0.121	-2.14	0.100
Baltimore	9.767	-0.186	-12.37	0.783	Portland	9.193	-0.139	-4.75	0.355
Birmingham	9.017	-0.190	-6.38	0.498	Providence	9.090	-0.135	-4.54	0.335
Chicago	9.745	-0.039	-1.60	0.059	Richmond	8.716	-0.221	-6.71	0.523
Cincinnati	9.669	-0.162	-4.78	0.358	Rochester	9.845	-0.327	-10.32	0.722
Dayton	9.245	-0.179	-4.62	0.342	Salt Lake City	8.883	-0.128	-4.17	0.298
Denver	9.624	-0.206	-5.37	0.413	San Antonio	9.300	-0.212	-6.44	0.503
Detroit	9.714	-0.075	-3.86	0.281	San Diego	9.141	-0.065	-2.79	0.159
Flint	9.482	-0.386	-6.82	0.532	San Jose	8.990	-0.085	-2.12	0.099
Fort Worth	8.399	-0.059	-2.38	0.121	Seattle	9.220	-0.140	-6.02	0.469
Houston	9.209	-0.153	-5.17	0.395	St. Louis	10.029	-0.170	-7.48	0.577
Jacksonville	9.205	-0.343	-10.34	0.723	Spokane	8.762	-0.256	-5.24	0.404
Louisville	8.619	-0.139	-6.12	0.478	Syracuse	9.938	-0.487	-15.62	0.856
Memphis	9.463	-0.173	-5.79	0.450	Tacoma	9.078	-0.177	-4.20	0.284
Milwaukee	10.013	-0.207	-6.53	0.509	Toledo	9.835	-0.317	-7.12	0.553
Nashville	9.078	-0.269	-8.42	0.634	Tucson	8.459	-0.146	-2.88	0.169
New Haven	9.791	-0.510	-10.75	0.738	Utica	9.421	-0.374	-5.78	0.449
Omaha	8.845	-0.114	-2.41	0.124	Washington, DC	9.980	-0.136	-3.96	0.277
Philadelphia	10.612	-0.195	-6.05	0.471	Wichita	9.000	-0.227	-4.63	0.343
Phoenix	9.089	-0.134	-4.54	0.335					

Note: Estimated coefficients for the pooled city data are $\ln D_o = 8.41$, $r = -0.010$, and $R^2 = 0.010$.

pendent variable. In all cases, a greater proportion of variation could be explained for city-by-city estimates of the density function than for the model that used pooled data. Although the results emphasize the limitations of empirical generalizations that are based on the density function hypothesis with constant coefficients, they are typical of other results obtained by using data from U.S. cities (3, 6). The null hypothesis—that the constant coefficient density function is appropriate for all cities—is rejected at the 1 percent level in both cases. Obviously, more elaborate hypotheses are required to explain population density within and across cities.

Estimates for the pooled data in which the parameters vary according to the scheme given in Equations 6 and 7 are given in Table 2. Recall that the conditioning variables are use of public or private transportation (X_1), income (X_2), age (X_3), and population (X_4). The specification is that the coefficients for the density gradient are quadratic functions of these conditioning variables. Examination of the significance levels of the parameters on the linear and quadratic terms for the specifications given in Table 2 indicates that each of the conditioning variables is important in shifting the density from city to city and between tracts. This general observation is confirmed by comparing the R^2's in Table 2 with those given in Table 1 for the constant coefficient model as applied to pooled data.

On a more specific basis, results obtained by using the public-private transportation variable to condition the coefficients of the density function show that its major effect is on the coefficient of distance (r). For the constant term, the estimated parameter on the linear term is not statistically significant and the parameter estimate for the quadratic is only marginally so. Estimates on the constant, linear, and quadratic terms for the coefficient of distance are -0.0867, 0.456, and -0.0517 respectively, and all are statistically significant. The estimates show that the public-private transportation variable first increases and then density decreases. What the result shows is that, in cities and tracts for which the value of the public-private transportation variable is low, the density gradient is lower than it is in cities for which the transportation variable has a high value. Thus, if other things are equal, cities that have below-average levels of public and private transportation and contemplate policy measures designed to improve it should expect a decrease in the absolute value of density.

Parameter estimates for the density function, specified with coefficients conditioned as hypothesized in Equations 8 and 9, are given in Table 3. The table is similar in construction to Table 2 except that estimates in the constant columns are repeated for reference. The table gives all parameters as statistically significant, and the R^2 for the pooled data is improved to 0.49. The parameter estimates are generally interpreted as were those given in Table 2.

For the constant coefficient $\ln D_o$, the estimated linear parameters show that densities in the CBD increase with increased public and private transportation, income, and age and decrease with population. The significant estimates of the linear and quadratic parame-

Table 2. Estimation of exponential density function with coefficients jointly conditioned on selected variables (variation in parameters according to Equations 6 and 7).

Conditioning Variable	Parameter	$\ln D_o$ Coefficient	$\ln D_o$ t	r Coefficient	r t	R^2
X_1	Constant	8.545	86.08	-0.0867	13.92	0.32
	Linear (X_k)[a]	-0.179	0.87	0.456	12.76	
	Quadratic (X_k^2)[a]	0.222	1.91	-0.0517	2.14	
X_2	Constant	8.895	87.68	6.59 E-3	2.81	0.26
	Linear (X_k)[a]	5.91 E-5	2.72	-2.17 E-5	12.54	
	Quadratic (X_k^2)[a]	-4.714 E-9	4.36	8.75 E-10	7.33	
X_3	Constant	8.689	85.88	-0.120	1.42	0.21
	Linear (X_k)[a]	0.015	4.81	-1.19 E-3	2.50	
	Quadratic (X_k^2)[a]	-9.96 E-5	5.86	1.23 E-5	5.03	
X_4	Constant	9.367	119.59	-2.90	22.18	0.28
	Linear (X_k)[a]	-1.33 E-6	8.55	4.20 E-7	20.92	
	Quadratic (X_k^2)[a]	5.10 E-13	8.51	-1.14 E-13	16.05	

Note: E term indicates decimal movement.

[a] The index k takes on values 1, 2, 3, and 4 to indicate each of the four conditioning variables.

Table 3. Estimation of exponential density function with coefficients jointly conditioned on selected variables (variation in parameters according to Equations 8 and 9).

Conditioning Variable	Parameter	$\ln D_o$ Coefficient	$\ln D_o$ t	r Coefficient	r t	R^2
X_1	Constant	8.8746	91.88	-2.0146 E-1	11.53	0.49
	Linear	2.3102 E-1	1.71	2.4260 E-1	6.42	
	Quadratic	—	—	-3.7874 E-3	-11.21	
X_2	Constant	8.8746	91.81	-2.0146 E-1	-11.53	0.49
	Linear	1.2194 E-5	1.42	-9.6841 E-6	-7.78	
	Quadratic	—	—	1.2931 E-10	3.30	
X_3	Constant	8.8746	91.88	-2.0146 E-1	-11.53	0.49
	Linear (X_k)[a]	5.8489 E-4	-0.80	9.3994 E-4	3.56	
	Quadratic (X_k^2)[a]	—	—	-6.3914 E-6	-4.67	
X_4	Constant	8.8746	91.88	-2.0146 E-1	-11.53	0.49
	Linear (X_k)[a]	-3.2657 E-8	-0.41	2.5331 E-7	15.20	
	Quadratic (X_k^2)[a]	—	—	-6.6665 E-14	-14.54	

Note: E term indicates decimal movement.

[a] The index k takes on values 1, 2, 3, and 4 to indicate each of the four conditioning variables.

ters for the coefficient of distance show that r increases at higher levels of public and private transportation use and higher income levels and decreases with increasing city age and population. The first effect would indicate a flatter density gradient in cities with higher average income and greater use of public transportation.

Perhaps the best way to assess the implications of this final version of the VCM is to evaluate the density function for each of the cities included in the sample. This has been done for the city-specific conditioning variables at within-city sample means (Table 4). Such information makes it possible to do specialized analyses for particular cities by using the estimates given in Table 3. More generally, it is apparent from a comparison of Tables 1 and 4 that the VCM produces reasonable estimates for the density function. The advantage of the VCM is thus the improved fit and increased reliability of parameter estimates and, most importantly, the increased possibility of functional analysis of population density based on commonly advanced arguments of socioeconomic conditioning.

USE OF EMPIRICAL RESULTS

Two examples demonstrate how the empirical results can be used in the context of policy making and forecasting. One example involves a representative city obtained by setting the conditioning variables of the density function coefficients at mean sample values. The second example used is that of Washington, D.C.

The analysis of the impacts of changes in public transportation, income, age, and population is made on a partial basis; that is, the value for one of the conditioning variables is changed while others are held at current levels for the two example cities. First, three levels are considered for each of the variables that are assumed to condition the coefficients of the density function: the current level and 50 and 100 percent increases in it. The results obtained by using these assumptions are given in Table 5. These results show, for example, that in the typical city setting public and private transportation at the current level increases $\ln D_o$ by 0.0494 and r by 0.0517. By contrast, increasing the public and private transportation variable by 100 percent raises the value of the constant by 0.0989 and the gradient by 0.103 04. Similar interpretations of the results apply for the Washington, D.C., example and for the other conditioning variables.

What the results in Table 5 show is that the major impact of the conditioning variables is on the density gradient. This is not surprising since the specification of the structure for the varying coefficients featured possible changes in the gradient. What is encouraging is that the results are reasonable for the changes considered even though some of the results are for values of the conditioning variables far from the sample means. This indicates that the surface being approximated by the polynomial is sufficiently stable so that projections or forecasts based on assumed values of the conditioning variables can be viewed with some confidence.

Impacts of changes in the transportation variable on the density gradient (r) and representative structural shifts in the density function are plotted in Figure 1. The interpretation for the shifted density functions is that they are cross sectional and thus refer to levels of equilibrium. Thus, shifts that result from changes in the conditioning variables represent density relations to which the cities would gravitate as a result of policy changes or other possible exogenous effects. Finally, the similarity in the shifting density gradients shown in Figure 2 indicates that the VCM can be consistent with cities and tracts that have different characteristics

Table 4. Estimates of density function coefficients based on varying coefficient model.

City	$\ln D_o$	r	City	$\ln D_o$	r
Akron	9.0223	-0.175 4	Pittsburgh	9.1589	0.030 81
Baltimore	9.1219	-0.086 5	Portland	9.0365	-0.131 39
Birmingham	9.0055	-0.141 01	Providence	9.0587	-0.191 57
Chicago	9.1495	0.078 5	Richmond	9.0846	-0.129 26
Cincinnati	9.0504	-0.083 39	Rochester	9.0772	-0.151 85
Dayton	9.0676	-0.168 36	Salt Lake City	9.0096	-0.215 1
Denver	9.0399	-0.119 55	San Antonio	9.0016	-0.101 21
Detroit	9.0373	-0.018 7	San Diego	8.9008	-0.113 02
Flint	9.4459	-0.297 51	San Jose	9.0112	-0.191 52
Fort Worth	9.0097	-0.165 0	Seattle	9.0423	-0.127 74
Houston	8.9848	-0.044 85	St. Louis	9.0911	0.080 84
Jacksonville	9.0128	-0.107 18	Spokane	8.9996	-0.206 7
Louisville	9.081	-0.125 008	Syracuse	9.0781	-0.164 27
Memphis	9.0308	-0.044 47	Tacoma	9.0094	-0.209 05
Milwaukee	9.0907	-0.056 99	Toledo	9.0388	-0.156 14
Nashville	9.024	-0.109 8	Tucson	8.9777	-0.207 1
New Haven	9.1092	-0.199 26	Utica	9.0841	-0.234 5
Omaha	9.0436	-0.131 25	Washington, DC	9.1478	-0.000 955
Philadelphia	9.1784	0.075 24	Wichita	9.000	-0.203 38
Phoenix	8.9984	-0.151 33			

Table 5. Impact of current values and 50 and 100 percent increases in levels of conditioning variables on central densities ($\log D_o$) and density gradient (r) for the typical city and Washington, D.C.

	Conditioning Variable			
Level	X_1	X_2	X_3	X_4
Typical City				
Current				
D_o	0.049 4	0.1187	0.3269	0.0185
r	0.051 7	-0.0823	0.0326	0.1223
50 percent increase				
D_o	0.074 2	0.1781	0.4907	0.0278
r	0.077 5	-0.1138	0.0343	0.1673
100 percent increase				
D_o	0.098 9	0.2374	0.6539	0.0371
r	0.103 14	-0.1395	0.0253	0.2016
Washington, DC				
Current				
D_o	0.100 3	0.1965	0.4094	-0.0247
r	0.104 6	-0.1030	0.0345	0.1535
50 percent increase				
D_o	0.150 4	0.2347	0.6141	-0.0371
r	0.156 3	-0.1385	0.0283	0.2016
100 percent increase				
D_o	0.200 5	0.3130	0.8189	-0.0494
r	0.207 7	-0.1634	0.0063	0.2306

Figure 1. Impact of public transportation on density gradient (r).

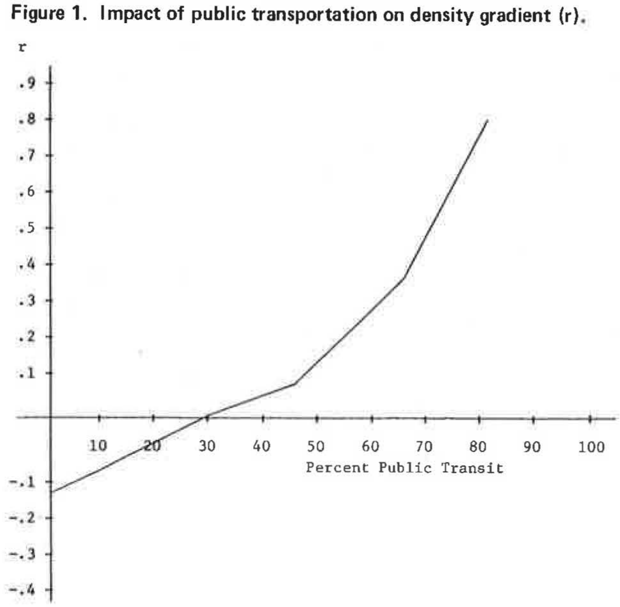


Figure 2. Impact of public transportation on density function for 50, 10, and 0 percent use of public transit by commuters (assuming same percentage of riders at all distances).

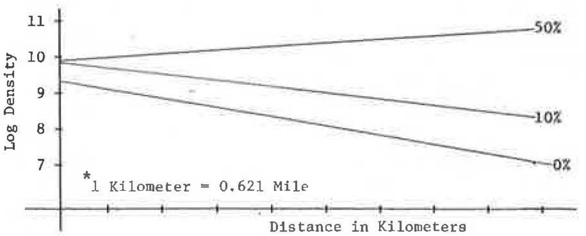
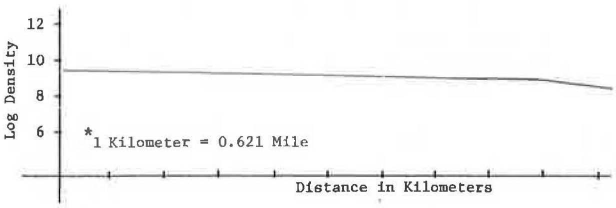


Figure 3. Impact of public transit for a special case [30 percent transit riders the first 5 km (3 miles) from city center and decreasing patronage beyond 5 km].



and can thus explain much of what in a simpler hypothesis would be attributed to spurious variation.

Mills (4), Mohring (5), Muth (6), Pendleton (7), and others have found empirical evidence that improvements in transportation tend to reduce the density gradient. The evidence provided by the VCM indicates that, as the percentage of public transit users increases, r decreases; in fact, as shown in Figure 1, r becomes positive when the number of public transit riders exceeds 30 percent. This occurs in four cities: Chicago, Philadelphia, Pittsburgh, and Washington, D.C. The estimates of r based on city-specific values for the conditioning variables (Table 4) indicate that r was positive in all cases except that of Washington, D.C., in which case it was essentially zero. Thus, the city-specific results based on the VCM (and also the ordinary least squares estimates given in Table 1) corroborate the findings of the more general analysis of the impact of transportation on the density gradient.

Additional information for policy analysis is contained in Figure 3, which assumes that a relatively substantial number of riders consistently use public transit to travel some predetermined distance from the CBD and that eventually, at greater distances, the number of riders decreases. Because the marginal cost of public transportation is mostly time related, this result would apply if identical income groups had a tendency to locate approximately equal distances from the CBD. In general, then, subsidies designed to increase the number of public transit riders would result in decentralization. Because the percentage of public transit use is a tract-specific variable, the VCM approach can measure, within a particular area of a city, changes in density patterns that are caused by a shift in the number of transit riders. For example, the impact of the new mass transit system in Washington, D.C., could be approximated for each specific city tract. This allows for the development of spatial—or, more generally, three-dimensional—density functions.

SUMMARY AND CONCLUSIONS

The varying coefficient model has been proposed as a method for introducing city- and tract-specific variables into the exponential density functions used to study urban structure. A major advantage of the VCM is that it permits the introduction of such variables while retaining an interpretation that can be reconciled with the body of theory that justifies the use of the exponential functional form. As a result, results obtained by applying the VCM can be compared with the massive empirical literature on urban density functions. Most estimated density functions are only special cases of the general VCM with a polynomial structure that relates the coefficients of the density function to socioeconomic conditioning variables.

Application of the VCM to data from 43 randomly selected 1970 census tracts for each of 39 U.S. cities provided a number of interesting results that may help to resolve a problem raised by recent studies in the application of the density function. It has been shown that questions about the appropriateness of the exponential functional form and specification errors associated with the omission of city- and tract-specific variables can be handled in the context of applied density function studies by using the VCM framework. In this study, the explanatory power of the density function and the significance levels of the structural parameters were greatly enhanced by the application of the VCM in studying the 1970 data.

The results also show that the conditioning variables that reflect transportation mode, city age, household income, and population could be used to provide reasonable explanations of apparent structural differences between cities and tracts. Of these results, perhaps the most interesting relates transportation mode to density. Analysis of the polynomial structure relating these tract-specific variables to the density gradient gave results that have a natural interpretation based on the opportunity cost of travel time as income increases. Other results, although perhaps less novel, are consistent with hypotheses that have emerged from more elaborate theories that support the exponential density function.

The most important results of the application of the VCM are those that relate to the use of the urban density function as a tool for policy analysis and projection. Until now, empirical work on urban density functions (including tests of the form of the density function and exploratory analyses of possible additional variables for explaining density patterns) has been largely descriptive. By introducing a method for including possible policy control variables and additional uncontrollable variables directly related to time, this study offers an expanded area of application for the density function hypothesis. The analysis of a typical city and of Washington, D.C., shows that effects of policies designed to influence transportation mode can be directly examined in the context of an estimated density function. When density is a target in urban planning, estimated VCMs of the type presented in this study can assume an important role in the structure of planning models. The relation between city age, population, and time illustrates how the model can be used in forecasting. Because these uncontrollable variables can be accurately projected on the basis of simple expressions in time, the cross sectionally estimated density function can be used for forecasting changes in urban structure. Although such forecasts can yield little information about the adjustment to new levels of equilibrium, they should provide urban economists with a valuable tool. The lack of information on rates of adjustment indicates that this is an area that requires further research.

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