OPTIMAL URBAN BUS SIZE

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London's city transit has been shown by Webster and Oldfield to be operating at almost optimal occupancies of 38 and 18 passengers during the peak and off-peak respectively. The mathematical model developed duplicates these results as well as observations of conventional commuter routes and a long established crosstown line. The relationship combines the complex interaction of vehicle, patron and street into one general equation. The equation may be manipulated to give "mathematically optimal", passenger productivities, vehicle occupancy and fleet size.

London's famous double-decker buses have an average peak hour passenger occupancy of 38 and 18 in the off-peak. Optimal occupancies calculated by Webster and Oldfield (1) are 36 and 23 respectively. Many years of operating experience in North America has developed a balance between the transit service and demand, which in terms of average system productivity, is roughly 60 passengers per hour within cities such as Vancouver and Toronto. There is always the nagging thought that there ought to be more passengers in the vehicle to increase its productivity.

One of public transit's roles is to transport the most people with the fewest vehicles in a way that considers the unique combination of demand, street traffic, vehicle operation and vehicle occupancy. The variables included in this vehicle-patron-street system are listed and defined in Table 1. The transit operator directly controls the passengers carried by a bus P, through vehicle selection, total boarding-alighting time d and the acceleration a, deceleration b. Stop spacing is also at the operator's discretion and therefore the number boarding-alighting at any stop. The city traffic department sets the maximum travel speed V, and to a limited degree the amount of congestion. Finally the vageries of urban travel and city structure determine the number of travellers H and their average transit trip length.

The following develops a simple theoretical model to determine optimal bus size and vehicle productivity. The data also demonstrates that London's and indeed other large cities' buses do operate within the range of the optimal productivity.

Intuition suggests that if too few passengers are transported in each vehicle then many vehicles are needed. Similarly, if too many are carried travel times become extremely slow and again a large fleet is required.

Table 1. Bus-Patron-Road Variables

| Variable | Units | Symbol | Purpose |
|---|---|---|---|
| Demand | $\frac{\text{trips}}{\text{h}}$ | H | Maximum usage |
| Trip Length | km | L | Transit network & multiple rides |
| Dwell time | sec | d | Ease of boarding |
| Boardings | $\frac{\text{patron}}{\text{stop}}$ | n | Stop collection efficiency |
| Accelerations | $\frac{\text{m}}{\text{sec}^2}$ | a,b | Vehicle responsiveness |
| Passengers | $\frac{\text{patrons}}{\text{bus h}}$ | P | Vehicle's income potential |
| Occupancy | $\frac{\text{patrons}}{\text{bus}}$ | R | Vehicle's seat capacity |
| Congestion | % of h | C | Street efficiency |
| Speed | km/h | V | Street processing efficiency |
| Fleet Size | bus/h | N | Operator's cost |

The number of vehicles required along a very long one-way transit route, to maintain an inter-vehicle time headway of h, is the total time to traverse the route, T, divided by the headway or

$$N = \frac{T}{h} \tag{1}$$

If no delays exist along this very long route, then the minimum vehicle requirement is the driving time $T_D$ divided by vehicle headway,

$$N_D = T_D/_h \tag{2}$$

Equating the headways, and rearranging terms gives the estimate of the number of vehicles as,

$$N = \frac{T}{T_D} N_D \tag{3}$$

Assume that the demand for service over any segment of the long route is Ht person bus hours of travel and each bus may provide Pt person bus hours of travel. The minimum number of vehicles is then given by assuming that the number of vehicles required by the patrons demand for service (H/P) is the same as that given by no delays to travel. This idealized service permits no time to be lost due to boardings, traffic signals or road congestion.

The number of vehicles needed along a route, considering the road and passenger delays is then:

$$N = \left(\frac{T}{T-t}\right) \frac{H}{P} \tag{4}$$

where t is the time spent not moving and estimated by:

$$t = dP + \left(\frac{V}{a} + \frac{V}{b}\right) \frac{mP}{n} + C \tag{5}$$

where m = 1 if, at each stop the same number get on as get off and,
m = 2 if all ons and offs occur at different stops.

Combining the preceding equations the total fleet size is:

$$N = \frac{TH}{P\{T- \left[dP + \left(\frac{V}{a} + \frac{V}{b}\right) \frac{mP}{n} + C\right]\}} \tag{6}$$

Differentiated this equation with respect to P yields the "optimal" passengers per bus hour P* and fleet size N* as:

$$P^* = \frac{T-C}{2d + \frac{2m}{n}\left(\frac{V}{a} + \frac{V}{b}\right)} \tag{7}$$

$$N^* = \frac{4TH\left[d + \frac{m}{n}\left(\frac{V}{a} + \frac{V}{b}\right)\right]}{(T-C)^2} \tag{8}$$

The last equation implies the truly detrimental outcome of congestion on fleet size. Congestion represents a real loss to the transport system.

The determination of optimal size is possible for three transit operations. The first is downtown commuting. If all passengers must pass the maximum load point then H assumes this value and P* is the average passenger loading per bus and is the "optimal". The theory is for very long routes and does not consider buses making several trips past the maximum load point within the time T. The theory may be adjusted to accept multiple cycles, by dividing the resulting fleet size by a factor representing the number of times each vehicle passes the maximum load point for example. If P* sets the vehicle size then the operator must control variables such as loading time d, vehicle speed V, number boarding at each stop n, and time lost to other traffic C.

A more interesting bus route is the crosstown. Assume it transports a uniform number of passengers at all times over its entire length which is very long. The average loading, if the boarding and alighting is uniform, is the total driving time $T_D$ proportioned by the actual driving time passengers spend on their journey, multiplied by the number of passengers carried during the time of interest. Mathematically it is:

$$R = \frac{P(L/V)}{T-t} \tag{9}$$

At any instant, for an optimal P*, the optimal uniform occupancy R* is:

$$R^* = \frac{L}{V} \left[\frac{1}{d + \frac{m}{n}\left(\frac{V}{a} + \frac{V}{b}\right)}\right] \tag{10}$$

Dial-a-bus may be considered as a set of vehicles in continuous service over a very long and erratic route. The average vehicle occupancies is the same as that for a crosstown bus.

The optimum equation of N* and P* is shown in Fig. 1 by the dashed line. The parameter characteristics are those for a Canadian city of one million people. The somewhat parabolic shape of the curves demonstrates the consequences of attempting to load too many persons into a vehicle. If the boarding and alighting time per passenger is, for example, 10 seconds, adding 10 more passengers over the maximum increases productivity by 20 percent while increasing the fleet size only slightly. Adding 10 more passengers increases the productivity by 15 percent but the fleet size by somewhat more than 12 percent. Any further increase in passenger productivity gives an even more rapidly increasing fleet size.

The most productive urban transit vehicles reported in Canada are those within the cities of Toronto, Montreal, and Vancouver where on an average they transport 60 passengers per hour. If the average total dwell time per passenger is 6 to 8 seconds and thirty percent of a vehicle's route service time is lost, from Fig. 1, then a value between 55 and 65 would appear optimal. Winnipeg, Edmonton and Regina have a passenger productivity of 50 and Calgary is down at 40. These values would reflect the vehicle size if all patrons behave as knowledgeable commuters and travel past a maximum load point. A lower limit on vehicle size is that of a crosstown bus and for the preceding example R* is 20 people in the vehicle.

Data from three very different transit operations are summarized in Table 2 together with estimates of the average vehicle occupancy and fleet size. The crosstown route in Vancouver and the assumed crosstown route in the City of London have an estimated occupancy, R*, well within ten percent of the observed value. London's "optimal" bus occupancy suggested by Webster and Oldfield is 36 and this is within fifteen percent of the result estimated from Equation 10. The vehicle occupancies for the two dial-a-bus experiments depart by twenty-five percent from the optimal riders. The fleet size in both cases represents the next largest whole number. The estimated fleet size is a very utopian view of transit service neglecting; mechanical breakdown, stocastic flows, and for dial-a-bus lost drivers, cancellations and no-show passengers.

The values of R* underestimates the number of seats needed in the vehicle since it represents an "average" occupancy or size and makes no allowance for irregular loading. Webster and Oldfield increased the average occupancy by 50 percent and equated this to the number of vehicle seats to accommodate all the irregularities of passenger loading. Navin (2) has shown that for many busy midday routes the mean occupancy (R*) plus two standard deviations ($\sigma$) of the occupancy can serve almost all routes without anyone being denied service. The observed coefficient of variation ($\sigma$/R*) has a typical value of 0.35 for crosstown type routes. If the optimal size of bus is set at R* + 2$\sigma$, this equals 1.7R* and for Vancouver during the off-peak is 36. During the off-peak most operating agencies have a policy of providing one seat per customer, therefore 36 seat buses are needed in Vancouver.

The peak hour vehicle productivity P* may represent the vehicle occupancy past the maximum load point. The Canadian transit operating practice is to provide seating for P*/1.5 people (3). The commuters in Vancouver travel approximately 7.7 km and for typical commute conditions the optimal productivity is 70 giving 47 seats per vehicle or thirty percent more than during the off peak. Buses in Vancouver have crush loads of 80 and 50 seats, a ratio of 1.6, slightly greater than the operating policy of vehicle loads being 1.5 times greater than seated capacity at the maximum load point. All P* patrons passing the maximum load represents the most extreme loading condition and gives the maximum vehicle size. If not all the patrons pass the maximum load point then the vehicle size may be reduced. The Vancouver experience indicates that sixty to seventy percent of the people do not pass the maximum load point. The average vehicle size may be reduced to serve 52 people, and seats may be provided for most.

Table 2. Observed and Estimated Transit Characteristics

| | London U.K. | Vancouver Canada (3) | Dial-a-Bus (1) | Dial-a-Bus (2) |
|---|---|---|---|---|
| H | - | n/a | 15 | 45 |
| L | 2.7 | 5.0 | 2.5 | 3.0 |
| d | 3 | 7 | 60 | 5 |
| n | 4 | 2 | 1 | 1 |
| V | 40 | 50 | 50 | 40 |
| C | n/a | 20 | 20 | 20 |
| R | 38 | 17.5 | 1.5 | 11 |
| N | n/a | n/a | 3 | 2 |
| R* | 41.7 | 17.0 | 2.0 | 9.8-13 (4) |
| N* | n/a | n/a | 2.2 | 1.8 |

(1) Columbia Md., (2) Bay-Ridges Ontario,
(3) Crosstown route (4) Productivity estimated by P*

The mechanics of transit vehicles travelling along very long fixed routes can be manipulated to develop equations to give a theoretically "optimal" productivity and fleet size. The equations use variables that are both easy to estimate and representative of the observed vehicle-patron-street system. The theory also points out the detrimental impact of overloading vehicles or employing too large a vehicle that may become overloaded given the travel market characteristics and demand.

Comparisons of theory and experience lends credibility to the simple equations. Estimates for the average vehicle occupancy along fixed route transit are withing ten percent of the observations and even for small dial-a-bus services the estimates are within twenty percent. Webster and Oldfield's optimal vehicle occupancy for London buses comes within fifteen percent of those estimated by the simplified procedure presented. The optimal peak hour bus for Vancouver is estimated to have 47 seats and room for a total of 52 passengers. The most severe condition has all the commuters passing the maximum load point which suggests a total vehicle capacity of 70. The equations tend to overestimate as would be expected of theoretically "optimal" values.

"Optimal" productivity represents an idealized objective towards which transit operators may strive. The number of seats and passenger handling characteristics of the vehicle and operation can be designed to help accomplish the goal of transporting the most people in the fewest vehicles.

## References


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Figure 1. Transit bus productivity.