DEVELOPMENT OF A SIMPLIFIED METHOD OF LATERAL LOAD DISTRIBUTION FOR BRIDGE SUPERSTRUCTURES

Tarek S. Aziz, Acres Consulting Services Limited
M.S. Cheung, Public Works Canada
Baidar Bakht, Ontario Ministry of Transportation and Communications

A new simplified method for establishing the live load design moments in most common types of bridges is presented. The proposed method was developed by using orthotropic plate theory and was checked by the grillage analogy method. The basis of the method together with the details of development methodology are discussed. Different variables which may affect load distribution in bridges were examined, and the main variables governing the distribution factors were selected for the present study. The study deals mainly with single span right bridges which can be analysed using the orthotropic plate theory or grillage analogy. However the limits of applicability of the distribution factors to continuous and skew bridges and bridges with diaphragms and edge beams are also discussed.

Vehicle loading on a highway bridge is distributed transversely to the main longitudinal girders by the floor system, which consists of deck slab and supporting members. The interaction between the different components of a highway bridge is difficult to determine, thus the complete structural analysis of a bridge is a complex undertaking. For the purpose of designing new bridges or evaluating existing ones, most codes (including American (1) and Canadian (2) bridge codes) provide empirical rules for transverse load distribution. Currently, both AASHTO and CSA-S6 permit each longitudinal girder in the bridge to be designed for some portion of the wheel loads of the standard truck. In 1973, the AASHTO code introduced more realistic empirical rules for load distribution in box girder bridges. The formulae were based on work carried out by Motarjemi and Van Horn (3). Further research on three categories of bridges, mainly beam and slab bridges, box girder bridges and multibeam bridges, was carried out by Sanders and Elleby (4). Based on the results of this research, distribution formulae for multibeam bridges have been incorporated in AASHTO Interim Bridge Specifications 1974 (5).

A new simplified method for establishing the live load design moments in most types of single and multi-span bridges (see Figure 1) is proposed in this paper.

The method is intended to be used in the design of new bridges as well as the evaluation of load-carrying capacity of existing ones. The method allows a bridge engineer to determine the maximum bending moment in any girder due to the worst loading condition without having to carry out a computer analysis (grids, orthotropic plate or finite element analysis). Because the load distribution phenomenon is treated more realistically in this method, some of the unnecessary built-in conservatism of the old standards can be eliminated. At the same time, the simplicity of the current AASHTO approach is maintained in the format of presentation.

The method may be used to derive load distribution factors for any other type of truck configurations. Recently, similar distribution factors have been developed by the authors for the Ontario loading and are now incorporated in a new bridge code for Ontario (6).

At present, distribution factors are used in four sections of the AASHTO code. The number of variables considered in each section is different, and, in each case, distribution factors are based on studies done on a particular type of bridge. It is appropriate at this stage to make several comments on the current AASHTO specifications.

1. In the current AASHTO Code, bridge type and girder spacing are considered to be the two most important parameters affecting the distribution of lateral loads. However, for some bridge types (box girders, composite box girders, and multibeam) distribution formulae also include other parameters.

2. Apart from the Interim Specifications (5) for multibeam bridges, AASHTO formulae do not account for variation of the member stiffness and its effect on structural behaviour. The formulae, in general, appear to consider only the geometrical variables of the bridge.

3. Reduction factors for multi-lane bridges are either included implicitly in the distribution formulae or neglected intentionally. Thus the stochastic nature of the live loading phenomenon may not be accounted for properly.

Variables that Govern Lateral Load Distribution

In general, the following variables may affect load distribution in bridges. Each of them have been studied and their effects on lateral load distribution are accounted for in the development of the simplified load distribution charts proposed in this paper.

The Stiffness Characteristics of the Bridge

These variables affect the behaviour in many ways. For example, if an equivalent orthotropic plate is used to model the bridge, its stiffness parameters (7) D_x, D_y, D_{xy}, D_{yx}, D_1, D_2 appear directly in the governing equation of the orthotropic plate, and their effect is self-evident.

Through a mapping process (7) the above stiffness variables can be reduced into two characteristic nondimensional variables α and Θ where:

$$\alpha = \frac{D_{xy} + D_{yx} + D_1 + D_2}{2\ (D_x D_y)^{0.5}} \tag{1}$$

and

$$\Theta = \frac{W}{2L} \left(\frac{D_x}{D_y}\right)^{0.25} \tag{2}$$

In which D_x, D_y, D_{xy}, D_{yx}, D_1 and D_2 are stiffness parameters, W is the width of the bridge and L is the length of the bridge.

Figure (1) shows the variation of α and Θ for different types of bridges. The practical range of α is between 0 and 2 while the practical range of Θ is between 0.125 and 2.50. The values of α and Θ for some 80 existing bridges are plotted on the α-Θ space. It can be seen from the figure that different types of bridges fall into different locations of the α-Θ space (for example, floor systems incorporating timber beams have an α between 0.0001 and 0.01 with a great variation of Θ, while concrete slab bridges have a constant α of 1.0).

The Width of the Bridge (W)

This variable affects the behaviour of the bridge by determining its maximum number of lanes (design lanes) and also by defining the aspect ratio, W/L. The larger the aspect ratio, the higher the flexural parameter, Θ of the bridge. Bridges with small aspect ratios usually tend to have better load distribution characteristics.

The Number of Lanes (N_L)

In general, as the number of loaded lanes on a given bridge is increased, the girders tend to share the load more equally and thus the moment distribution in the transverse direction becomes more uniform.

The Number of Girders (N_G)

The load per girder decreases as the number of girders increases. In addition, the number of girders affects the values of both α and Θ.

Truck Locations on the Deck

The transverse location of the truck may affect the load distribution factors significantly. The edge distance, defined as the distance between the edge of the bridge and the outer most line of wheel loads, and the distance between trucks could result in very different load distribution factors. Various combinations of truck locations and edge distances have been studied. Critical positions are used to determine the load distribution factors.

Axle Width of the Truck

Axle width of the truck affects the load distribution factors significantly. Different axle widths may result in entirely different load distribution factors, even if all the other variables are kept the same. Therefore, strictly speaking, the load distribution factors are applicable only to the type of trucks that have the same axle width as those used in the development of the distribution factors. Fortunately, for the majority of trucks, the axle width is a standard 1.83 m (6 feet). Thus this variable has been eliminated from the study.

Longitudinal Axle Spacing

Longitudinal axle spacing has very little effect on the transverse load distributions. Load distribution factors developed for a particular axle spacing may be used for any other types of truck configuration without sacrificing accuracy within practical limits, provided the axle width is standard (1.83 m).

Size of Patch Loads

For the types of bridges considered in this study, this variable is known to have little effect on the lateral load distribution. The sizes of the equivalent patch load for different wheel loads shown in Figure 2 have been used in this study.

Methods of Analysis

Two well-established methods of analysis were used in the current investigation; the orthotropic plate analysis and the grillage analogy.

In the orthotropic plate analysis, the structure is idealized as a plate of constant thickness having different flexural and torsional properties in two mutually perpendicular directions. The series solution presented by Cusens and Pama (7) was used. It covers the torsionally stiff and torsionally soft decks, as well as the isotropic decks. This solution was applied through a well-established computer program, ORTHOP (8). The area of loading for the different wheels is taken into account (Figure 2) and edge beams, if present, are accounted for.

In the grillage analogy method, a structure is idealized as an assembly of beams. This method was applied through a well-established computer program, GRIDS (9). The grillage analogy method was used to provide independent checks on the ORTHOP results as well as to study the effect of diaphragms reported elsewhere (10).

Development of Distribution Schemes

Development of distribution schemes for a code is a complicated task made difficult by the number of variables involved and the need to account for all of them while maintaining simplicity. In this study the task was accomplished in two phases.

1. Preliminary and pilot studies of the different variables and their effect on the load distribution were made in an attempt to determine the importance of each variable and the minimum number of variables to be maintained.
2. Analyses were made for different groups of bridges that cover the practical ranges of variables defined in the first phase. A total of 1,344 hypothetical cases of bridges were analyzed in this phase and the results formed the data base for the suggested schemes.

Figure 3 shows a flow chart of the general methodology used for the development of the distribution schemes.

Observations from the Pilot Studies

To determine effects of different variables on the governing values of moments, pilot studies were conducted on a series of bridges. For each analysis conducted the following three items were extracted:

1. The maximum moments,
2. Intensity factors (defined as the ratio of maximum moment to the average moment in the bridge), and
3. The theoretical equivalent width D (as used in AASHTO to obtain the distribution factor S/D).

Some observations made on the effects of the variables involved in the pilot studies appear below.

Bridge Width

The width of the bridge has an effect on the governing values of the moments in addition to its contribution to the values of θ. Figure 4 shows a typical behavior for the effect of width on the D values for two-lane bridges.

Girder Spacing

The theoretical D values are dependent on the girder spacings. However, it was found that for girder spacing of more than 1.52 m (5 feet), the change in the theoretical D value with girder spacing is small (less than five percent) for all the practical ranges of α and θ.

Bridge Span

A series of analyses for two families of bridges was conducted. The first family had a constant span of 18.29 m (60 feet) while the other family had a constant span of 36.58 m (120 feet). All the possible variations of the properties were considered (for example, α was varied from 0.0 to 2.0 while θ was varied from 0.125 to 2.50). Figure 5 shows a typical member of each family. It was observed that bridges which had the same α and θ resulted in the same theoretical D value independent of the absolute value of the span. Thus, while the critical D value is dependent on α and θ; it is independent of the absolute value of the span. (Note that θ itself is dependent on the span). This approach was repeated for different bridge widths and different numbers of lanes. The conclusion was that two bridges of different lengths but identical α and θ have the same distribution coefficients or D values. As part of these analyses the theoretical D values were calculated for different places within the span. The results showed conclusively that the theoretical D values are constant for most of the span.

Number of Loaded Lanes

The number of loaded lanes affect the values of D substantially, in particular for bridges with low values of θ. Figure 6 shows a typical behavior for a two lane bridge. The behaviour shown in this figure suggests that the governing loading case in a design may not necessarily be the fully loaded case since reduction factors for multi-lane bridges have to be introduced first to arrive at the governing design moments.

Edge Distance

Edge distance, defined as the distance between the edge of the bridge and the outermost line of wheel loads, has a major effect on the governing values of the moments, especially when the governing moments are in the vicinity of the free edge. For example, Figure 7 shows a typical behavior of the effect of the edge distance on the theoretically derived D values for a two-lane, 8.84 m (29 feet) bridge with α equal to 0.16.

Other Effects on the Load Distribution

Additional factors that affect the load distribution have been studied and the results have been reported recently by Aziz and Alizadeh (10). Some of the observations that have a bearing on this study are summarized below.

Continuity

While continuity of the bridge may affect the absolute values of the moments, it does not change the moment distribution pattern of the bridge. The zones of negative moments in a continuous bridge behave in a similar manner as the zones of positive moments. The proposed method, although developed from analyses for single span bridges, was found applicable with good accuracy to both positive and negative moments. The only adjustment required for continuous bridges is to utilize the effective span (the distance between contraflexure points) in calculating θ.

Skew

Skew bridges with skew angle less than 15 degrees can be analysed by the proposed method treating them as right bridges. From actual analyses, it was confirmed the error arising from this simplification was small and can be neglected.

Diaphragms

Although the proposed method was developed for bridges without diaphragms, studies on the effects of diaphragms on the transverse distribution of live loads using grillage analogy methods (10,11) concluded that the method is also adequate for bridges with diaphragms. In this case, the proposed method would generally result in a slightly conservative estimate of the distribution factors.

Distribution Charts for AASHTO Loading

Following the pilot studies, a total of 1,344 hypothetical cases of bridges were analyzed by ORTHOP to develop the distribution charts given in Figures 8, 9, 10 and 11 for the AASHTO HS truck. All bridges were assumed to have a minimum curb width of 45.72 cm (1 foot 6 inches) on each side. Thus, the edge distance in these analyses was maintained at a practical limit of 1.07 m (3 feet 6 inches). The values of Θ and α were varied in the range of 0.125 to 2.50 and 0.0 to 2.00 respectively. This should be sufficient to cover all practical bridges.

All the possible loading combinations were considered, from the worst concentric case of loading to the worst eccentric case of loading. In accordance with the AASHTO specifications, when there were three or four lanes loaded, reduction factors of 0.9 or 0.75 respectively were used. All the partial loading possibilities were considered as well. The bridges were loaded to produce the maximum bending moment. The governing theoretical D values were calculated from the moments integrated over a 1.52 m (5 feet) width. The differences observed between an outside girder and an inside girder were not significant enough to justify a special treatment or presentation for each. Finally, contours were drawn to represent the D values for each category of bridge (two lane, three lane and four lane). Because the width of a bridge has an effect on the load distribution in addition to its contribution in Θ, a correction factor chart was devised to deal with bridges having lane widths larger or smaller than 3.35 m (11 feet) (for which the basic contours were drawn). While this correction, C_f, was found to be less than 15 per cent in most cases, it was decided that by developing such a chart the proposed method would be made even more accurate. Finally, a total of 100 bridges were analyzed by the developed charts as well as by ORTHOP and GRIDS. The maximum error observed was very small.

Steps in Applying the Method

After discussing the foregoing distribution charts and the derivation procedures, it is appropriate to summarize here the steps to apply the proposed method, in a format similar to that used in the AASHTO code.

For shallow superstructures (shown in Figure 1) having either right spans or skew angles smaller than 15 degrees, the longitudinal moment due to HS loading is computed as follows.

1. The values of α, Θ and μ are calculated from:

$$\alpha = \frac{D_{xy} + D_{yx} + D_1 + D_2}{2\sqrt{D_x D_y}} \tag{1}$$

$$\Theta = \frac{W}{2L}\left(\frac{D_x}{D_y}\right)^{0.25} \tag{2}$$

$$\mu = \frac{\text{Lane width} - 3.35\text{ m}}{0.61} \quad \text{(for SI units)}$$

(Maximum of 1.0) (3)

$$\mu = \frac{\text{Lane width} - 11\text{ feet}}{2} \quad \text{(for Imperial units)}$$

where:

W = Width of the bridge.
L = Span of the bridge or equivalent simple span for a continuous bridge (span between contraflexure points for positive or negative moment regions).

2. Corresponding to the values of α and Θ determined previously, D_{design} is found by reading a D value from the applicable chart (Figures 8, 9, and 10), and a correction factor "C_f" from a variation chart (Figure 11), and substituting in the following expression.

$$D_{design} = D\left[1 + \frac{\mu C_f}{100}\right] \tag{4}$$

3. The governing design bending moment is the fraction S/D_{design} of the moment resulting from one line of wheel loads, applied to a girder, or a web of a voided slab, or a unit width of a solid slab,

where:

S = the actual spacing of longitudinal girders; or,
S = spacing of webs, in the case of voided slabs; or,
S = a unit width, in the case of solid slabs or laminated timber bridges.

Examples of the use of these analysis charts appears below.

Examples

1. The proposed method is applied to a three-lane beam and slab bridge with a span of 18.29 m (60 feet) and a width of 13.72 m (45 feet). The cross section of the bridge is shown in Figure 12. The steps in arriving at D_{design}, are:

$$D_x = EI/\text{ (beam spacing)}$$

$$= E\,(5629033.8\text{ cm}^4)/236.22\text{ cm}$$

$$= 23829.62\text{ E}$$

$$D_y = E\ (\text{slab thickness})^3\ /12$$
$$= E\ (19.05\ \text{cm})^3\ /12$$
$$= 576.11\ E$$

neglecting the contribution of the steel I beam to the torsional inertia; since the torsional inertia of an I beam is very small.

$$D_{xy} = D_{yx} = G(\text{slab thickness})^3\ /6$$
$$= \frac{E}{2\ (1 + 0.15)} \quad \frac{(19.05\ \text{cm})^3}{6}$$
$$= 500.96\ E$$

$$D_1 = D_2 = \nu\ (\text{lesser of } D_x \text{ and } D_y)$$
$$= 0.15 \times 576.11\ E$$
$$= 86.42\ E$$

$$\alpha = 0.5\ (500.96 + 500.96 + 86.42 + 86.42)\ /\ (23829.62 \times 576.11)^{0.5}$$
$$= 0.16$$

$$\Theta = 0.5 \times 1371.53\ \ (23829.62/576.11)^{0.25}\ /\ (1828.71)$$
$$= 0.95$$

The lane width is 4.27 m (14 feet), therefore

$$\mu = 0.5\ \frac{(4.27-3.35)}{0.61}$$
$$= 1.5 > 1.0$$

Use μ = 1.0

from Figure 9, the value of D for α = 0.16 and Θ = 0.95 is 1.74 m (5.70 feet) and from Figure 11 the value of C_f is 5.8. Therefore,

$$D_{design} = 1.70\ \text{m}\ (1 + 1.0 \times 5.8/100)$$
$$= 1.84\ \text{m}\ (6.10\ \text{ft}).$$

(The corresponding value of D_{design} according to the current AASHTO method is 1.67 m (5.5 feet.))

2. In this example, the proposed method is used to calculate the load distribution factors for the Conestogo River Bridge, a 2 lane, 3 span continuous plate girder bridge (Figure 13) with a central span of 44.20 m (145 feet) and side spans of 34.75 m (114 feet). The distribution factor for the central span of the bridge was calculated and the results are compared with AASHTO load distribution factors and field test values in Table 1.

While the above examples indicate that use of the AASHTO load distribution formulae generally result in over conservative load distributions, it has also been found (13) that for certain cases, these AASHTO load distribution factors can be unconservative. On the other hand, consistently safe but economical load distribution factors were obtained from the proposed load distribution charts for all cases.

Table 1. Comparison of distribution factors obtained by AASHTO, proposed method and reference 12

Methods	Distribution Factors (Wheel Load)	D_{design}	
		Meters	Feet
AASHTO Code	1.58	1.67	5.5
Proposed Method	1.44	1.83	6.0[a]
	1.31	2.01	6.6[b]
Testing (12)	1.04	2.43	8.0

[a] If μ is limited to Maximum Value of 1.0
[b] If actual μ is used (μ = 3.0)

Conclusions

A simplified lateral load distribution procedure has been presented. The method is believed to provide a more accurate load distribution than the AASHTO and CSA-S6 codes while maintaining the simplicity of an AASHTO-type approach. Analysis charts have been developed and presented for AASHTO HS vehicles. It was confirmed that the longitudinal axle spacing of the truck has little effect on the lateral load distribution factors. Therefore, the analysis charts developed for AASHTO loading may be used with reasonable accuracy for other truck configurations, provided that the reduction factors for multi-lane loading are the same as those in the AASHTO specifications. This has in fact been done for the Ontario Highway Bridge Design Load proposed for the new Ontario Bridge Code (10, 13).

The methodology for developing a simplified method as presented, can be utilized for developing distribution factors for other types of trucks and reduction factors.

Acknowledgements

The original concept and objective to develop a rational and simplified method of analysis for lateral load distribution for highway bridge decks was initiated by the Transportation group, Public Works Canada. The project was sponsored and funded by Public Works Canada and the Department of Supply and Services.

References

1. American Association of State Highway and Transportation Officials. Standard Specifications for Highway Bridges. Eleventh Edition, Washington, D.C., 1973
2. Canadian Standards Association, S6-1974. Design of Highway Bridges. Ontario, 1974.
3. D. Motarjemi and D.A. Van Horn. Theoretical Analysis of Load Distribution in Prestressed Concrete Box-Beam Bridges. Lehigh University, Fritz Engineering Laboratory Report 315.9, October 1969, Bethlehem, Pennsylvania.
4. W.W. Sanders Jr. and H.A. Elleby. Distribution of Wheel Loads on Highway Bridges. National Cooperative Highway Research Program Report 83, 1970.
5. American Association of State Highway and Transportation Officials. Interim Specifications, Bridges. Washington, D.C., 1974.
6. R.A. Dorton and P.F. Csagoly. The Development of the Ontario Bridge Code. Paper prepared for the 1977 National Lecture Tour of the Canadian Society for Civil Engineering,

Structural Division.
7. A.R. Cusens and R.P. Pama. Bridge Deck Analysis. John Wiley and Sons Ltd., London, 1975.
8. B. Bakht and R.C. Bullen. Analysis of Orthotropic Right Bridge Decks. Highway Engineering Computer Branch/B/15 (ORTHOP), Department of Environment. U.K.
9. R. Sen. Pogram for the Grillage Analysis of Slab or Pseudo-Slab Bridge Decks. Highway Engineering Computer Branch/B/9 - GRIDS, Department of Environment U.K.
10. T.S. Aziz and A. Alizadeh. Transverse Distribution of Vehicle Loads on Highway Bridges. Report for the Technological Research and Development Branch, Public Works Canada, 1978.
11. G.A. Culham and A. Ghali. Distribution of Wheel Loads on Bridge Girders. Canadian Journal of Civil Engineering, Vol. 4, Number 1, 1977.
12. R.A. Dorton, M. Holowka and J.P. King. The Conestogo River Bridge - Design and Testing. Canadian Journal of Civil Engineering, Volume 4, Number 1, 1977.
13. B. Bakht, M.S. Cheung and T.S. Aziz. Application of A Simplified Method of Calculating Longitudinal Moments to the Proposed Ontario Highway Bridge Code. To be published in Canadian Journal of Civil Engineering.

Notation

C_f	=	A correction factor for width effects.
D	=	Load distribution factor.
D_{design}	=	Load distribution factor, used in design.
D_x	=	Longitudinal flexural rigidity per unit width.
D_{xy}	=	Longitudinal torsional rigidity per unit width.
D_y	=	Transverse torsional rigidity per unit span.
D_1	=	Coupling rigidity per unit width.
D_2	=	Coupling rigidity per unit span.
E	=	Young's Modulus.
G	=	Shear Modulus.
I	=	Moment of Inertia.
L	=	Span of bridge.
N_L	=	Number of lanes
N_G	=	Number of girders.
S	=	Girder spacing
W	=	Width of bridge.
α	=	Torsional parameter $\dfrac{D_{xy} + D_{yx} + D_1 + D_2}{2(D_x D_y)^{0.5}}$
θ	=	Flexural paramater $\dfrac{W}{2L}\left(\dfrac{D_x}{D_y}\right)^{0.25}$
ν	=	Poisson's ratio.

Figure 1. Bridges of common use for which the proposed method applies.

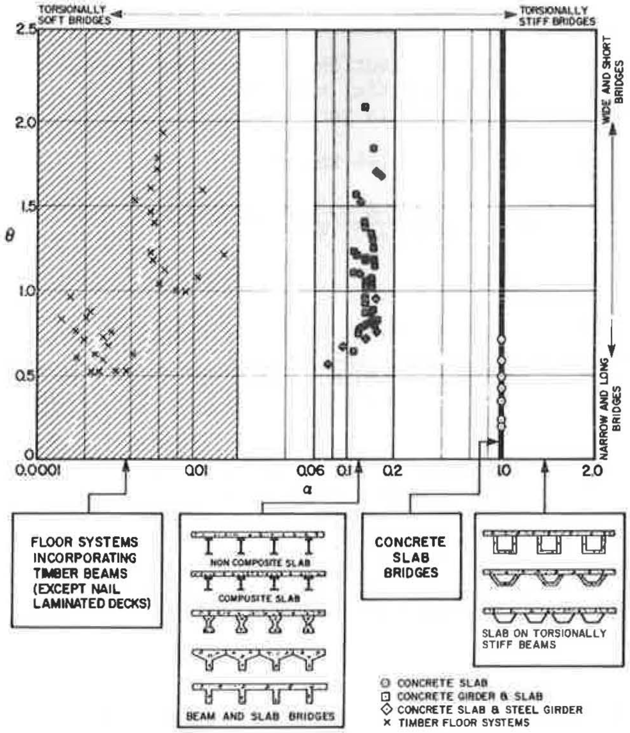
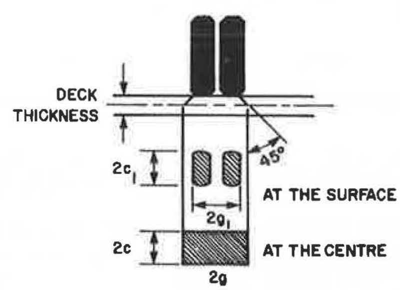


Figure 2. Wheel bearing area at the surface, and load area at the center of the deck.



NOMINAL WHEEL LOAD		DIMENSIONS OF THE LOAD AREA AT THE CENTRE OF THE DECK			
		2g		2c	
kN	Tons	cm	in.	cm	in.
35.59	4	71.12	28	45.72	18
35.38	6	71.12	28	45.72	18
71.17	8	81.28	32	45.72	18

Figure 3. Analysis flow chart.

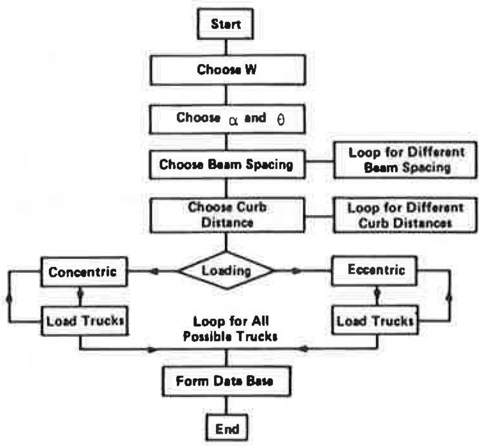


Figure 4. Effect of width on D values for beam and slab bridges.

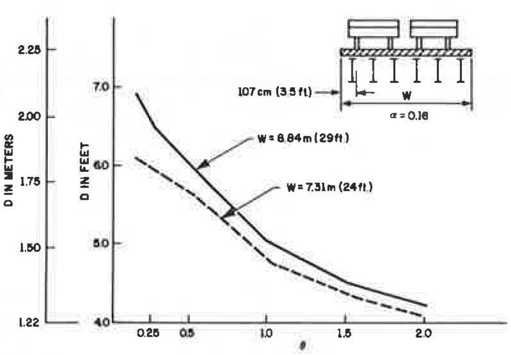


Figure 5. Critical positions for moment.

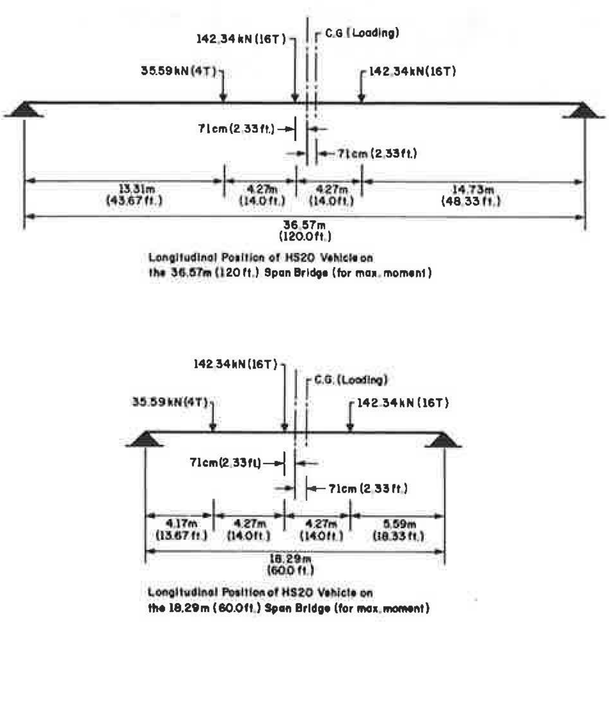


Figure 6. Effect of number of loaded lanes on D values for beam and slab bridges.

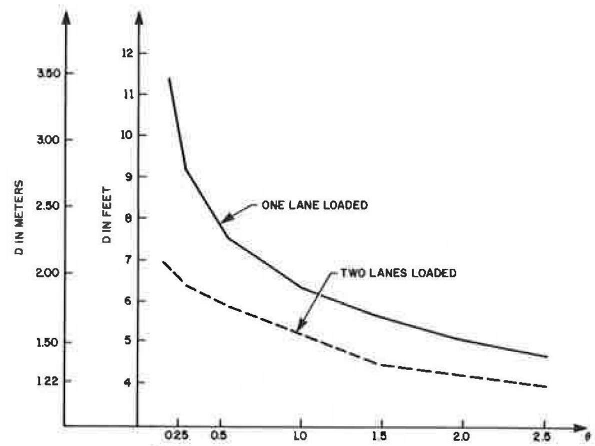


Figure 7. Effect of edge distance on D values for beam and slab bridges.

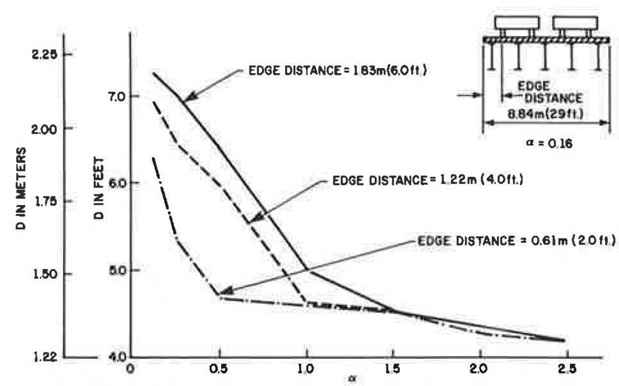


Figure 8. D values for 2 lane bridges - AASHTO loading.

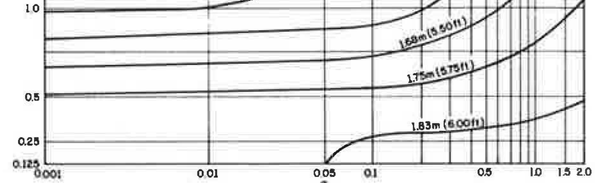


Figure 9. D values for 3 lane bridges - AASHTO loading.

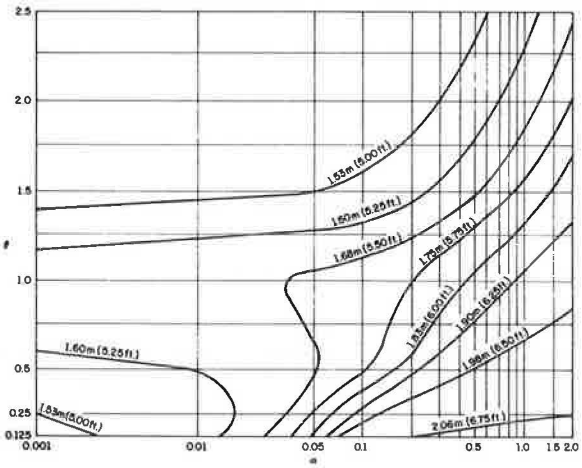


Figure 10. D values for 4 lane bridges - AASHTO loading.

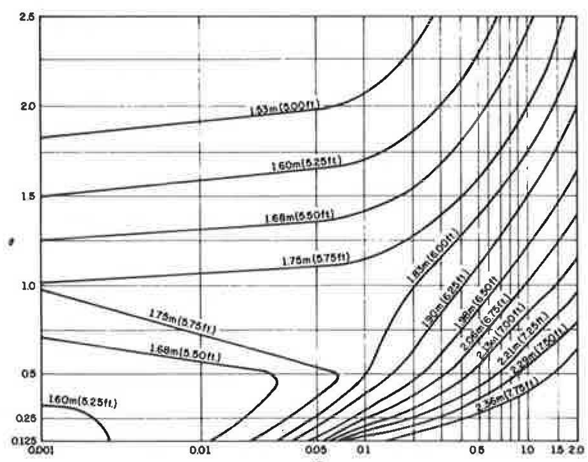


Figure 11. Variations of correction factors for AASHTO loading - 2, 3 and 4 lanes (percent).

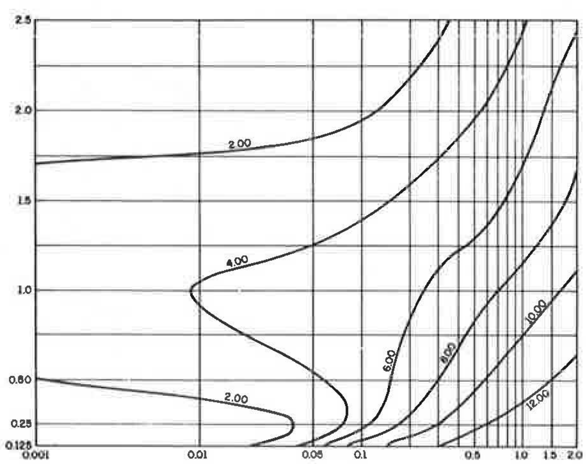


Figure 12. Example bridge number 1.

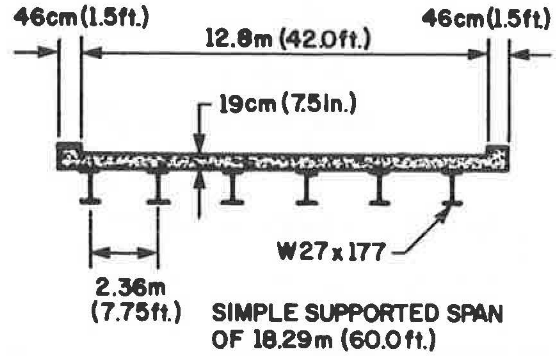


Figure 13. Example bridge number 2 (Conestogo River Bridge).