WEB STRESSES IN PRESTRESSED CONCRETE BRIDGE BEAMS HAVING DISCONTINUOUS TENDONS

James R. Libby, President, James R. Libby & Associates, San Diego, California
G. Krishnamoorthy, Professor of Civil Engineering, San Diego State University, San Diego, California
John Revels, Construction Inspector, City of San Diego, San Diego, California

Prestressed concrete bridge girders are sometimes designed with prestressing tendons that terminate within the span rather than at the ends of the girders or at their supports. Tendons of this type are normally significantly inclined at their anchorages due to the clearances required for construction because of the inclination of the tendons at their anchorages. Significant vertical components of prestressing are imposed on the girder. The AASHTO Specifications for Highway Bridges and the Building Code Requirements for Reinforced Concrete (ACI 318-71) do not specify specific methods of analysis for this condition. A finite element analysis of an actual bridge girder was made to determine the conditions of stress in the vicinity of intermediate anchorages. The results of this analysis were compared to those obtained with a principal tensile stress analysis using methods normally employed by bridge designers. It was found both methods of analysis predicted principal tensile stresses of similar magnitude and orientation. The predicted locations of the greatest principal tensile stresses were different for the two methods of analysis. The fact that the greatest principal tensile stresses occur on planes approximately parallel to the paths of some of the post-tensioning ducts is demonstrated.

Description of the Problem

For reasons of economy of prestressing steel, and in some instances in order to confine flexural stresses to acceptable levels, post-tensioning tendons are sometimes terminated within a span rather than at the ends of a girder or at the supports. Bridges containing overhanging girders and a suspended span, as shown in Figure 1, are structures where details of this type of arrangement of tendons are frequently found. The post-tensioning tendons in the overhanging girders may be a combination of tendons that extend from one end of the girder to the other together with tendons that do not. This condition is illustrated in Figure 2, where three groups of tendons are shown. In this case, tendon Groups 1 and 2 can be stressed before the suspended girders

are in place but the stressing of tendon Group 3 must be deferred until the suspended girders are in place. This sequence must be followed in order to avoid flexural overstressing of the concrete in the vicinity of the pier. The area of the girder under consideration in this paper is shown in Figure 2 and in detail in Figure 3, showing the individual tendons of tendon Groups 1, 2, and 3 respectively.

A characteristic of the overhanging girders used in bridges of this type is that the conditions of moment in the girders vary as the construction progresses. At the time tendon Groups 1 and 2 are stressed, the suspended span is not in place. Therefore the maximum positive dead load moment between the supports is larger than it is after the suspended span is in place. In the completed structure, the moments of various sections along the span are very similar to those found in a continuous bridge. In the portion of the girders shown in Figure 3, the moments are relatively low in the completed structure while the prestressing is relatively high.

Tendons which terminate within a span normally have to be inclined rather steeply in order to provide the clearances required to stress the tendon. These tendons may be inclined as much as 30 degrees or more as shown in Figure 3. The relatively great inclinations of the tendons result in vertical components of the prestressing force that are quite significant. The various combinations of the effects of prestressing, dead load and live load that occur during the life of a girder of this type result in interesting and variable conditions of stress states in the vicinity of the intermediate anchorages. The analysis of these stresses is normally referred to as shear design.

The prestressed concrete design provisions of the AASHTO Standard Specifications for Highway Bridges (Reference 1) are based upon the assumption that concrete can withstand a shear force causing a unit stress of 1.242 MPa (180 psi) for concrete strengths of 20.7 MPa (3000 psi) or more without web reinforcement. If the shear force is greater than that which the concrete can carry by itself, web reinforcement must be provided to carry the excess force. These provisions originally appeared in "Tentative Recommendations for Prestressed Concrete" of the ACI-ASCE Joint Committee 323 in 1958

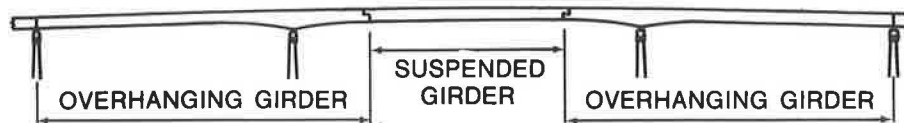Figure 1. Elevation of girders.
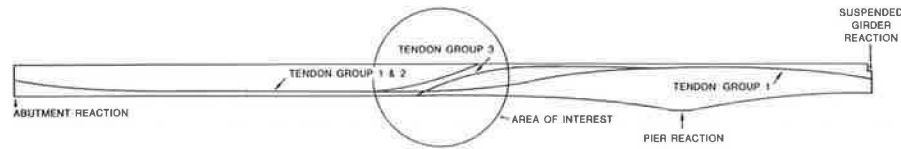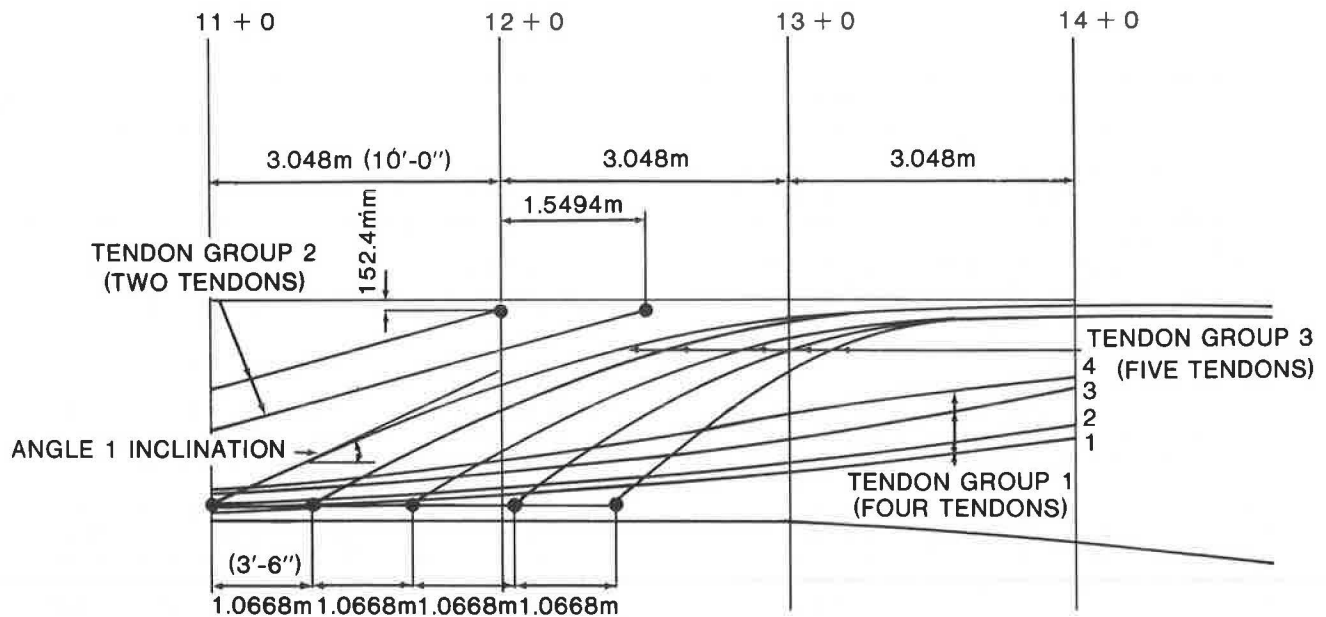
Figure 2. Girder and center line of cables-profile.

Figure 3. Non-composite cross section.

(Reference 2) which were based upon a limited number of tests on beams conducted at the University of Illinois. These requirements were intended to be conservative as the test data available at that time were limited.

Subsequently additional tests were performed at the University of Illinois on beams with moving loads to simulate bridge beams. The results of the later test result in the shear provisions which appeared in the 1963 edition of the Building Code Requirements for Reinforced Concrete (ACI 318-63) (Reference 3). They are also included in ACI 318-77 (Reference 4) in a slightly simplified version. Comparison of the results obtained with the current ACI & AASHTO relationships will reveal the AASHTO Specifications are conservative in areas where flexural cracking is precluded and unconservative in areas where flexural cracking can occur.

Two type of shear-related cracking are now recognized. The first of these is termed flexural-shear cracking and the second is referred to as web cracking. The analysis of the former involves an investigation of the flexural cracking that can occur under the design loads. Web cracking is predicted by the determination of the conditions of loading that result in a principal tensile stress equal to the tensile strength of the concrete.

In the area of the beam under consideration, the conditions of prestressing and moments due to the dead and live loads preclude flexural cracking and hence, the analysis of the safety of the member can be made by determining the principal tensile stresses that exist in the member under various conditions of loading and comparing these to the tensile strength of the concrete. The tensile strength of the concrete is normally taken as $0.33\sqrt{f'_c}$ MPA ($4\sqrt{f'_c}$ psi).

Stress analysis in the vicinity of curved tendons, which terminate at various locations such as shown in Figure 3, is very complex due to the effects of the anchorage zone stresses, the vertical component of the prestressing resulting from the curvature of the tendon and the reduced width of the web resulting from the presence of the post-tensioning duct. These factors have to be taken into account

for a realistic analysis.

Principal Tensile Stress Analysis

For beams prestressed longitudinally, not pro-vided with prestressed stirrups and subject to trans-verse loads, stress analysis is generally confined to the investigation of the principal tensile stresses at the centroidal axis of the member. Vertical pre-stressing of the web resulting from the vertical curvature of the tendons is normally neglected because it cannot be easily evaluated, and the ef-fect of neglecting this is generally small. This is a conservative procedure. Because the moment due to transverse loads do not cause flexural stresses at the centroidal axis, the principal tensile stresses are computed from the shear stresses resulting from the transverse loads and the axial stresses due to prestressing. It should be recognized the stress at

the centroidal axis due to the longitudinal component of the prestressing is the quotient of this force and the area of the beam cross-section. The shear stress can be computed using the classical flexural-shear stress relationship:

$$v = \frac{VQ}{Ib} \tag{1}$$

in which:

b = horizontal width of the beam at the centroi-dal axis.

I = moment of inertia of the beam cross section with respect to the horizontal plane passing through the centroidal axis.

Q = the first moment of the beam area lying above or below the horizontal plane passing through the centroidal axis, with respect to the plane.

V = the shear force at the section under con-sideration

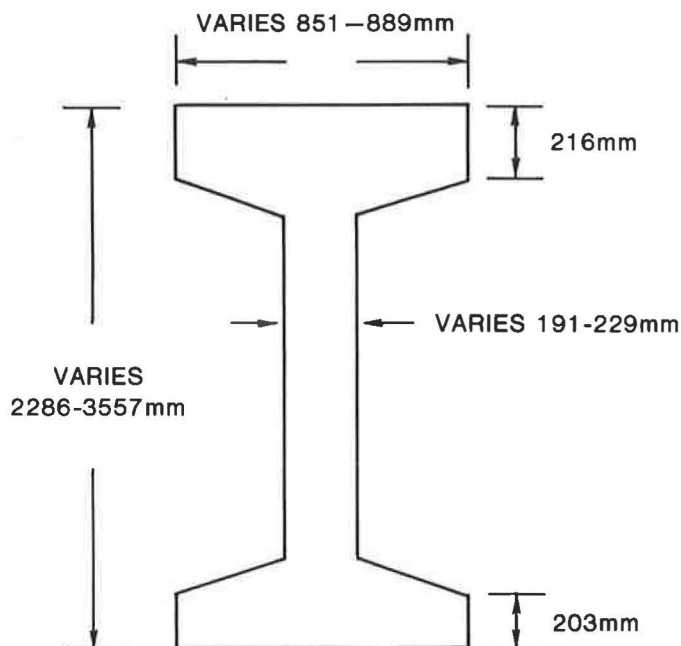v = unit shear stress

Figure 4. Non-composite cross section.

Table 1. Principal tensile stresses at centroidal axis-gross web thickness MPa (psi).

Station	All of tendon group 1 plus tendons 1-4 of group 3	All of tendon group 1 plus tendons 1-4 of group 3	All of tendon groups 1-3
	Without shear stress from 5 of group 3	Including shear stress from tendon 5 of group 3	Including shear stress from tendon 5 of group 3
12 + 4	1.566 (227)		
12 + 5	7.015 (292)		
12 + 6	0.794 (115)	1.69 (245)	1.532 (222)
12 + 7		1.518 (220)	1.373 (199)
12 + 8		1.359 (197)	1.228 (178)

Table 2. Principal tensile stresses at centroidal axis--net net thickness MPa (psi).

Station	All of tendon group 1 plus tendons 1-4 of group 3 Without shear stress from 5 of group 3	All of tendon group 1 plus tendons 1-4 of group 3 Including shear stress from tendon 5 of group 3	All of tendon groups 1-3 Including shear stress from tendon 5 of group 3
12 + 4	2.167 (314)		
12 + 5	2.015 (292)		
12 + 6	1.097 (159)	2.339 (339)	2.125 (308)
12 + 7		2.104 (305)	1.904 (276)
12 + 8		1.884 (273)	1.697 (246)

The cross sectional dimensions of the beam studied in this paper vary as shown in Figure 4. Principal tensile stresses, computed by usual methods, for several stations along the beam shown in Tables 1 and 2. The stresses listed in Table 1 are based upon the gross web thickness of 228.6 mm (9 in.) while those in Table 2 are based upon the net web thickness of 165.1 mm (6.5 in.). Based upon this analysis, which ignores the effect of vertical prestressing resulting from the curvature of the group 3 tendons, the greatest principal tensile stress occurs at (station 12 + 6). This is the case if the shear stress resulting from the vertical component of group 3 tendon 5 is or is not considered to be effective at this station.

Finite Element Analysis

The structural model consisted of a simply supported span of 9.15 m (30 feet) with cantilevers of 3.05 m (10 feet) on each end as shown in Figure 5. This configuration was selected as a means of simulating the moments and shears acting upon the beam between stations 11 + 0 and 14 + 0 where the principal tensile stresses were under study. This model permitted moments to be applied through forces of variable intensity at the ends of the cantilevers while the shears were applied through the reactions at stations 11 + 0 and 14 + 0. The effects of prestressing were modeled with forces as illustrated in Figure 6. The post-tensioning ducts were not included in the finite element model used in this study.

The analysis of the structure was conducted by using SAP IV computer program (Structural Analysis Program for Static and Dynamic Response of Linear Systems) developed at University of California Berkeley, (Reference 5). The beam was modeled using three dimensional finite elements with three translational degrees of freedom. In the flanges, three dimensional variable node isoparametric elements (i.e., more than eight nodes) were used. In the web, eight node "brick" elements which employ incompatible modes were used. Isotropic properties were assumed. The elastic modulus and Poissons ratio employed were 27,600 MPa (4000 ksi) and 0.20, respectively. Additional dead and live loads within the span were simulated by surface pressure loads. The two typical cross sections assumed for the analyses are shown in Figures 4 and 7.

The structure was modeled with 1498 nodes and 639 three dimensional finite elements for the non-composite case. Additional 368 nodes and 275 elements were added for analyzing the composite case.

In the first set of analysis, the finite element model was restrained along the lateral direction of the girder. In order to ascertain the effect of Poisson's ratio, the model with all the degrees of freedom was also analyzed. It was found that for planar loadings of the type considered here, the effect of Poisson's ratio was not significant.

The analysis resulted in six global stresses at the centroid of each finite element. From these stresses, the magnitude and direction of principal tensile stresses at various locations of the girder were computed and plotted.

The following load conditions were analyzed and the maximum principal tensile stresses were plotted in order to obtain the critical load case.

1. Dead Load + Diaphragm and drop in girder + Slab + 90% initial load due to tendon group 1 and tendon group 2 + 100% initial load due to tendon group 3.
2. Dead Load* + 80% initial load due to tendon groups 1, 2, and 3 + Maximum moment (Mmax) + shear (V).
3. Dead load + 80% initial load due to tendon groups 1, 2, and 3, + Mmin and V concomitant.
4. Dead load + 80% initial load due to tendon groups 1, 2, and 3 + Vmin and M concomitant.
5. Dead load + 80% initial load due to tendon groups 1, 2, and 3, + Vmax and M concomitant.

*Girder + Diaphragm + Drop in Girder + Slab + Curb + Railing + Future D.L.

Condition 1 was analyzed with the non-composite section; the remaining with the composite section. The principal tensile stresses for the above beam sections and loading conditions are plotted in Figures 8 - 12.
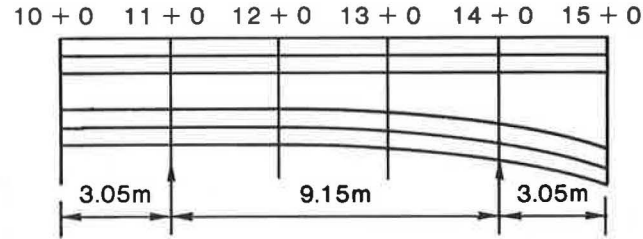
Figure 5. Structural model.

10 + 0   11 + 0   12 + 0   13 + 0   14 + 0   15 + 0

3.05m   9.15m   3.05m
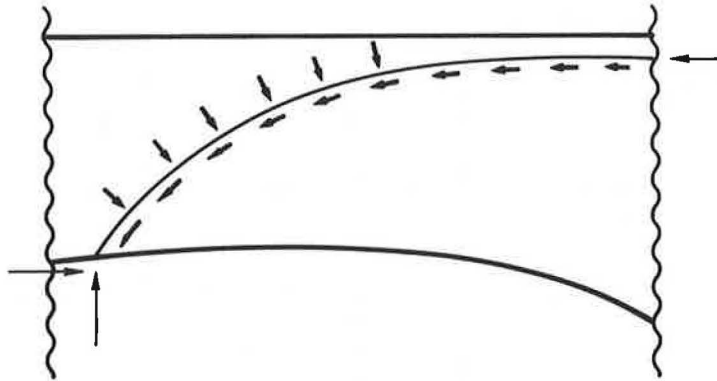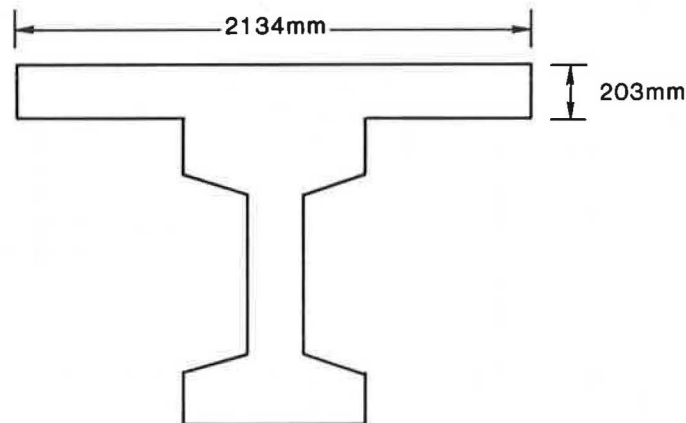
Figure 6. Forces in prestressed cables.

Figure 7. Composite cross section (see Figure 4. for dimensions not shown).

—2134mm—

203mm

Conclusions

Comparing the principal stresses obtained by conventional methods of computation, as shown in Table 1, to those obtained by the use of the finite element method, as illustrated in Figures 8 - 12, leads one to conclude the former method is conservative for the conditions studied. The former method predicts maximum values of maximum principal tensile stress at locations different from those predicted by a finite element analysis. The difference in magnitude and location of principal tensile stresses predicted by the two methods is at least partially explained by the effects of vertical prestressing of the web being neglected in the conventional computations but not in the more sophisticated method.

The orientation of the principal tensile stresses as shown in Figure 9 shows the direction of these stresses approximately follow the paths of the group 3 tendons. Comparison of the values of the principal tensile stresses shown in Table 1 and 2 clearly illustrates the importance of basing principal tensile stress computations on net rather than gross web thicknesses in conditions such as these.

Conventional methods of computing principal tensile stresses are adequate for usual cases encountered in bridge design. The cost of three dimensional finite element analysis can only be justified under unusual circumstances.

50

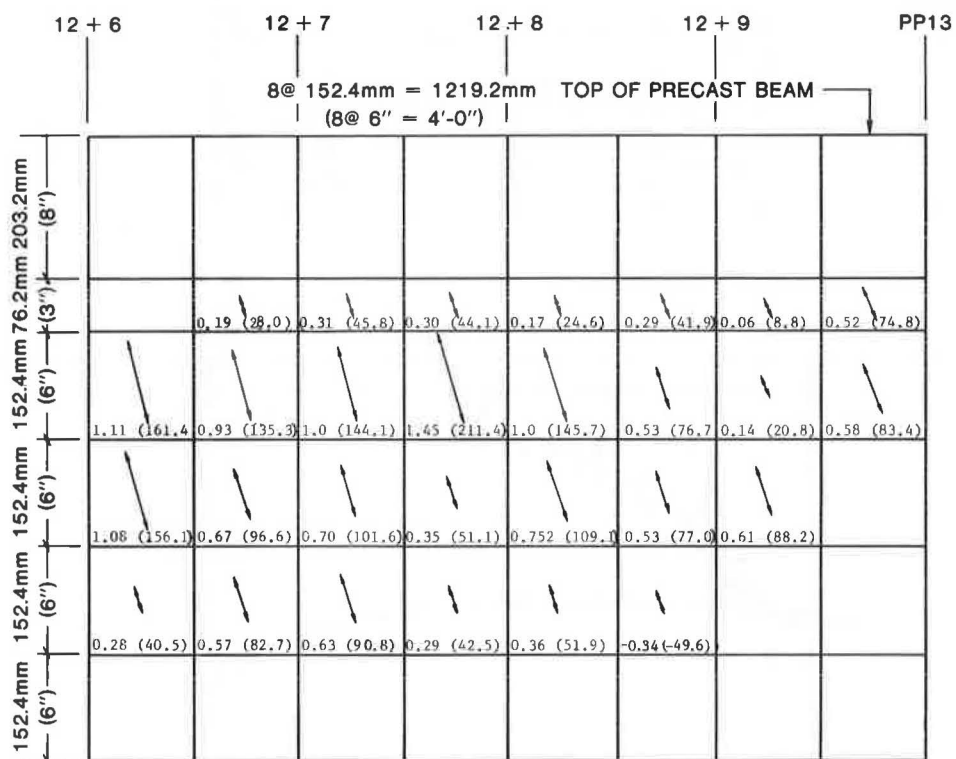Figure 8. Principal tensile stress plot MPa (psi) for load condition 1.

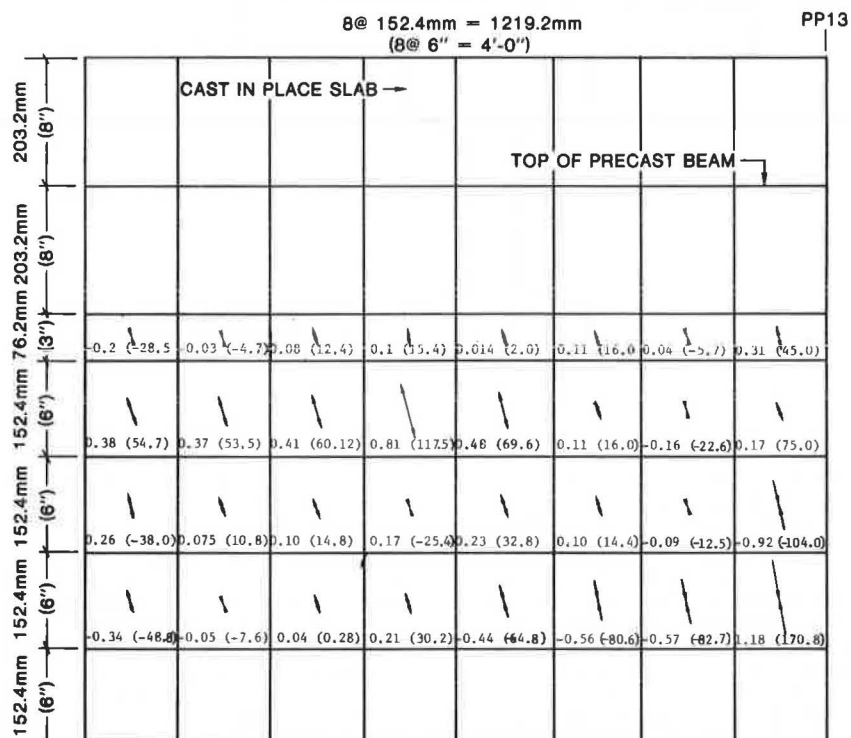Figure 9. Principal tensile stress plot MPa (psi) for load condition 2.

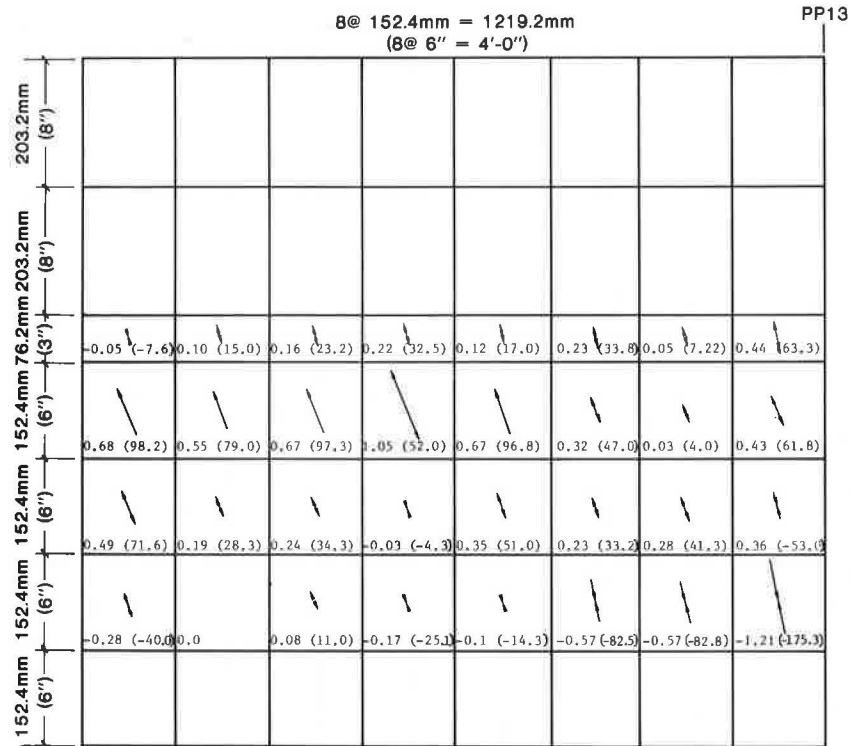Figure 10. Principal tensile stress plot MPa (psi) for load condition 3.

8@ 152.4mm = 1219.2mm   PP13
(8@ 6″ = 4′-0″)

Figure 11. Principal tensile stress plot MPa (psi) for load condition 4.

8@ 152.4mm = 1219.2mm   PP13
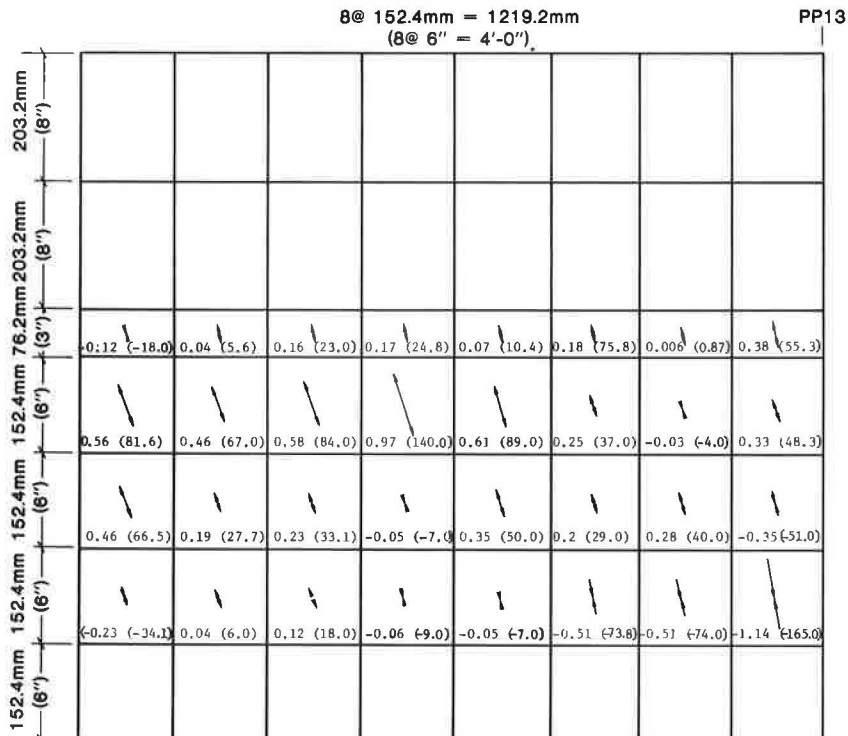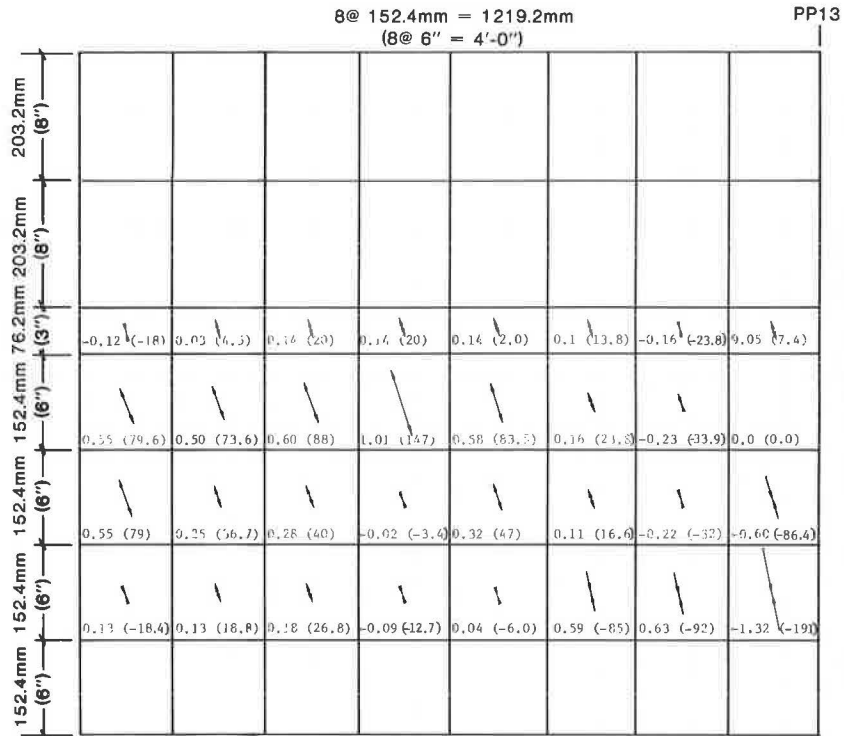**(8@ 6″ = 4′-0″)**

Figure 12. Principal tensile stress plot MPa (psi) for load condition 5.

References

1. American Association of State Highway and Trans-
   portation Officials. Standard Specifications for
   Highway Bridges. 11th ed., Washington D.C.,
   1973.
2. ACI-ASCE Joint Committee 323. Tentative Recom-
   mendations for Prestressed Concrete. Journal of
   the American Concrete Institute. 29, No. 7,
   pp. 545-578 (January 1958).
3. ACI Committee 318. Building Code Requirements of
   Reinforced Concrete. American Concrete Insti-
   tute. Detroit, 1963.
4. ACI Committee 318. Building Code Requirements of
   Reinforced Concrete. American Concrete Insti-
   tute. Detroit, 1971.
5. K. Bathe, E. Wilson, and F. Peterson. SAP IV, A
   Structural Analysis Program for Static and
   Dynamic Response of Linear Systems. Report No.
   EERC 73-11, June 1973, College of Engineering,
   University of California, Berkeley, California.