DYNAMIC RESPONSE OF BRIDGES

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Bridges can be excited by the action of wind, vehicles, or pedestrians. Whereas there are well developed methods to calculate dynamic behaviour, there have been comparatively few correlations with full scale measurements. The Transport and Road Research Laboratory has therefore undertaken a programme of research to measure the response of selected bridges to such loading. The work has also included tests in which the bridges were excited by an energy input device and damping values were measured from free decays.

Response to wind has been measured for two steel box girder bridges having orthotropic decks. Automatic self-switching equipment was used to record bridge acceleration, wind speed and direction, when wind speeds exceed a threshold level. Response to traffic has been measured for a multi-span steel box girder viaduct having a concrete deck. Response to pedestrians has been measured for a variety of types of footbridge.

Measured values of bridge response are compared with calculated values using procedures recommended in the new British Design Standard.

The dynamic response of bridges is an important aspect of design which has attracted a very considerable volume of research. Most examples of vibration leading to collapse have been due to excitation by wind, for example the Brighton Chain Pier in 1833 and other cases up to the more recent and well publicised Tacoma Narrows Bridge. There have also been cases of serious damage due to pedestrian induced vibration dating as far back as the Broughton case iron chain bridge which collapsed in 1831 and several footbridges have been 'bounced' off their bearings in recent years. There are no known cases of damage to main components due to traffic induced vibrations but there have been examples of fatigue failure of components, such as cross-bracing, that can resonate.

Despite the depth of the literature on dynamic behaviour, it has been found that there are inadequate data in several important areas. For example there have been surprisingly few well documented studies of the damping behaviour of modern steel structures and prior to the work described in this paper no damping data were available for welded steel box girder bridges. Similarly, there have been few comprehensive measurements of response to wind and it has not been possible to check the accuracy of design analyses beyond confirming that behaviour is stable. The situation with regard to traffic induced vibration is more satisfactory and there have been a number of measurements of movements and stresses. Nevertheless, these have been mostly concerned with relatively short span bridges and less attention has been given to the longer spans which are often very lively. Vibration of footbridges is not generally considered to be a major problem because unduly lively behaviour can usually be rectified relatively cheaply. There is however the question of human tolerance. Although active people can tolerate high levels of vibration, elderly and infirm people have a much lower tolerance. In addition it is unwise to build bridges that can deliberately be excited to high amplitudes of vibrations by vandals. Recent work by TRRL on footbridges has been to determine what constitutes lively behaviour and to compare measured response with calculations using formulae given in the new British Standard.

1. Damping

Damping is the term used to describe the dissipation of energy in a vibrating structure. There are a variety of mathematical expressions used to represent damping, the most common ones being logarithmic decrement and fraction of critical. Logarithmic decrement, δ, is most conveniently measured from a free decay and is given by

$$\delta = \frac{1}{n} \mathrm{Log}_e \frac{a_m}{a_{m+n}}$$

where a is amplitude of vibration, m is the number of cycles along the decay at which measurement is started, and n is the number over which the measurement is made. Fraction of critical damping, ζ, often expressed as a percentage, is given by $\frac{C}{C_c}$ where C is damping and C_c is critical damping. The relationship between ζ and δ is $\zeta = \frac{\delta}{2\pi}$ for low values of damping typical of those in the range of superstructure behaviour. In this paper logarithmic decrement is used exclusively.

Values of damping are required for calculation of structural response to transient or cyclic forces. In a recent review of the literature (1) attention was drawn to the distinction between different types of damping. Material damping is related to the steel or concrete and is a function of the energy dissipation by internal mechanisms. Numerous tests have been conducted on a laboratory scale and relatively wide ranges of values have been reported. After elimination of some extreme values and one or two questionable results, consensus ranges of damping are as given in Table 1. (Component damping refers to individual beams).

Table 1. Typical values of damping

Material	Component	Bridge
Steel		
0.002 to 0.008	0.004 to 0.03	0.02 to 0.06
Concrete		
0.01 to 0.06	0.02 to 0.06	0.02 to 0.1

Values of component damping are higher than for the constituent materials because there are additional losses of energy associated with the fabricated product. This is less apparent for concrete whose main mechanism of damping is associated with relative movements at cracks and internal flaws which are present in both the basic material and in cast beams. The belief that prestressed concrete exhibits lower damping than reinforced concrete is not supported by such data as are available; laboratory tests on beams with and without prestress have not shown any significant differences in damping (2)(3). Steel has damping values which are approximately one tenth of those for concrete. Damping is influenced by the type of steel but values are unlikely to differ significantly for different constructional steels. Steel beams have differing damping according to the method of fabrication, see Table 2. (derived from references 4 - 8)

In making comparisons between the different forms of fabrication it should be noted that values are influenced not only by testing techniques but are also dependent on amplitude of vibration. Hence comparisons may not be like with like because some investigations involve lower amplitudes than others. Nevertheless the general trends given in Tables 1 and 2 are intuitively correct and are confirmed where different investigations have overlapped. The dependence of damping on amplitude of vibration occurs for both concrete and steel. In all cases the trend is for logarithmic damping to increase with amplitude by up to 500 per cent.

Damping values of bridges are higher than for either the constituent materials or the components, as shown in Table 1. The increases are due to extra forms of energy dissipation such as relative movement at joints and interaction with substructure. Although concrete bridges tend to have slightly higher values of damping than steel bridges, structural form is the dominant feature in determining performance. Few comparisons can be made between steel and concrete because design philosphies differ and it is difficult to find comparable bridges having main features that are similar.

Testing steel bridges presents problems because the most important structures have long spans which require special techniques. Furthermore long-span bridges are usually situated on major roads which cannot be closed to traffic even for brief times during off-peak periods. A variety of methods of excitation have been adopted by different investigators such as the use of a test vehicle driven across a plank, sudden release of deflection, single-pulse loading (rockets) and resonant loading. The latter method has been used almost exclusively in the TRRL work. The excitation equipment, called an energy input device, has a set of weights which are reciprocated in a vertical plane by hydraulic actuators (10), Figure 1. In one exceptional case the

Figure 1. Energy input device and towing vehicle

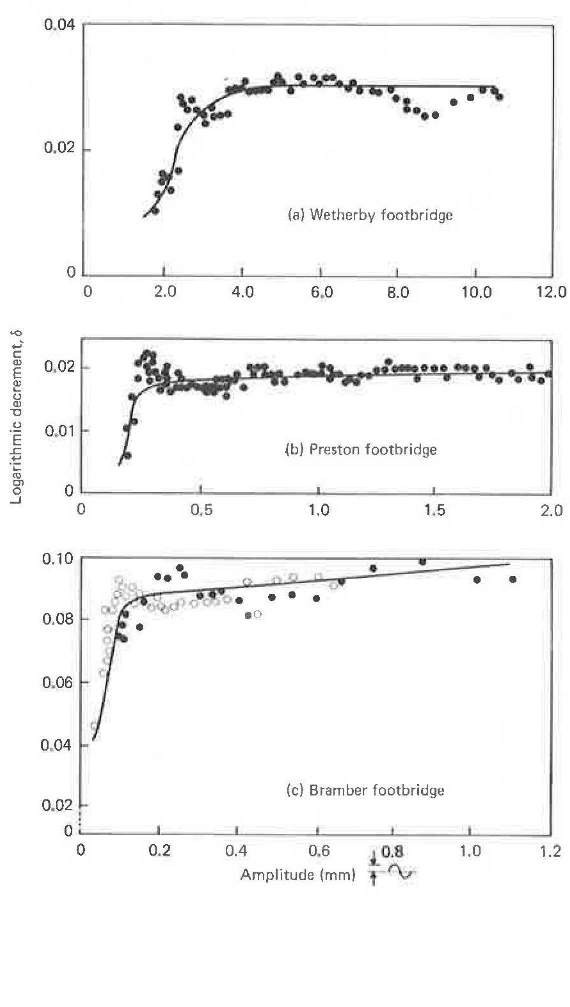
Table 2. Damping values for different types of steel beam

Rolled I-Beams	Bolted	Riveted	Welded	Attached to Concrete Slab
	Loose			
0.003 to 0.007	0.01 to 0.07	0.006 to 0.02	0.004 to 0.008	0.04 to 0.11
	Tight			
	0.005 to 0.03			

main 213m span of the Cleddau Bridge at Milford Haven was excited by the sudden release of deflection produced by suspending a 32.7Mg weight (11), Figure 2. Because of the difficulties in testing

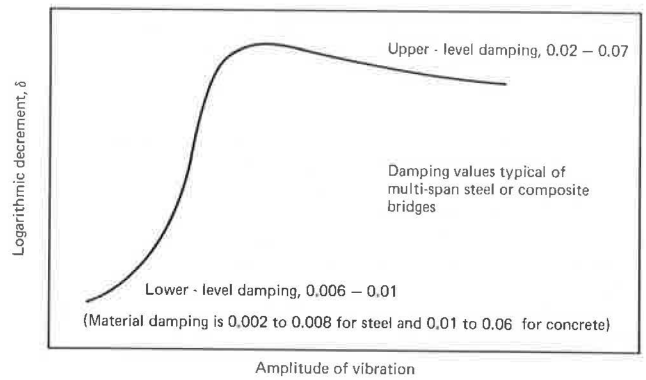
Figure 2. Excitation of the Cleddau Bridge

Figure 3. Influence of amplitude on damping



long-span bridges a programme of tests was conducted on steel footbridges having relevant configurations. The resulting values of damping for six footbridges, had extremes of 0.015 to 0.10 and typical values of 0.02 to 0.06. The measurements of the long-span Cleddau bridge gave damping of 0.043 to 0.059 which is within this range and gives some support to the belief that behaviour of footbridges is relevant to the general study. Typical curves of damping against amplitude of vibration are given in Figure 3. The relationship, shown schematically in Figure 4, involves values of damping which increase from a low value close to the constituent material damping to an upper-level at higher amplitudes. The former

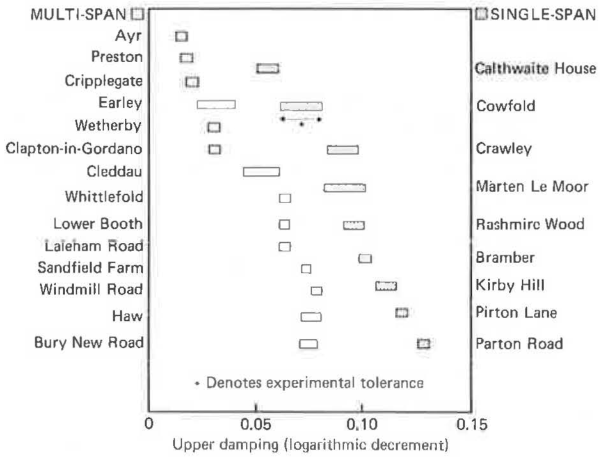
Figure 4. Schematic representation of damping behaviour.



occurs at amplitudes which are too low to have structural significance and it is the upper-level damping that is usually relevant to calculation of bridge response. Values quoted in this paper as being typical are representative of upper-level damping.

When composite bridges were introduced, having concrete decks structurally attached to steel beams, there was concern that the response to traffic induced vibration could be livelier than for earlier types of bridges. As a result several research projects were set up and measurements were made of dynamic characteristics of different types of composite bridge. The damping values from these investigations are in the range 0.05 to 0.10 which is higher than values for wholly steel bridges but similar to slender concrete bridges (1). In the programme of tests by TRRL on sixteen composite bridges (9), damping ranged between extremes of 0.02 to 0.13 but typical values were between 0.05 and 0.07. One of the main points to emerge from the tests on steel and composite bridges was that single-span structures exhibit higher damping values, 0.05 to 0.13, than multi-spans which have values of 0.01 to 0.08, Figure 5. In cases where measurements were made on different parts of the substructure, it was found that significant vibrations occurred so that a substantial contribution to damping may be made at interfaces such as between the ground and an abutment. The higher damping exhibited by single-span steel bridges is a general trend that emerged from the testing of typical examples of modern bridges but it does not follow that single-span bridges need necessarily have high damping. Clearly it is possible to have weak links to the superstructure so that significant movements are not transmitted to the substructure. In cases where damping is associated with the superstructure alone the situation is akin to vibration of a beam and damping values

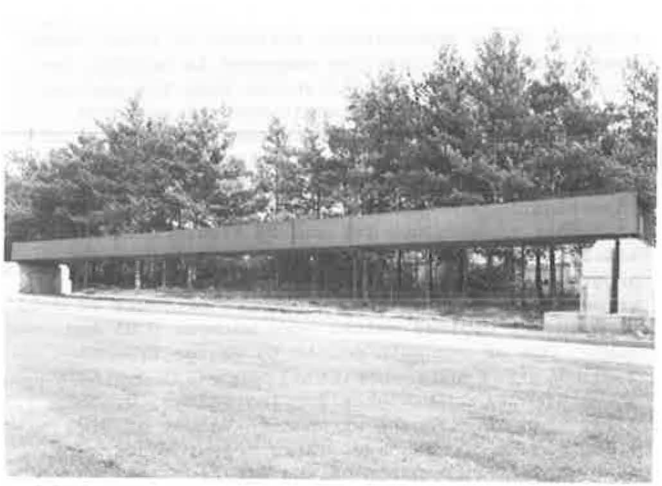
Figure 5. Damping values for single and multi-span bridges



can be very low. Welded box girders, whether having steel or concrete decks, can have very low damping. Of eighteen steel box girder bridges tested, four had upper-level damping values of less than 0.03. These low values are partly due to the low damping of welded plates as distinct from older types of construction such as trusses and partly due to simple supports which permit little interaction with the superstructure.

In order to be able to study damping of steel box girders under more closely controlled conditions, a 30m rectangular steel beam has been erected at TRRL. The beam is 2.44m wide and 0.92m deep, and is supported on concrete piers, Figure 6. The

Figure 6. 30m experimental steel box girder



testing programme involves measurements of damping for several types of bearing and with the addition of different types of surfacing, eg concrete or mastic asphalt. It is intended to assess the contributions to overall damping of these different features, using full scale components in a manner not previously possible. In the work to date, it has been found that with the beam supported on simple rubber pads at each end, the damping is 0,008. This is in the range for intrinsic material damping of steel and it is clear that there is little if any contribution from other mechanisms. With a system composed of rocker bearings at one end and sliding bearings at the other, considerable movement was transmitted to the piers and the damping was 0.012. The extra component was due to dissipation of energy at the sliding bearings and between the piers and the ground.

Most reported work on concrete bridges has been on relatively short spans and there have been very few tests on modern longer span concrete structures. Measured values of damping, excluding some extreme values, are in the range 0.02 to 0.10. There is no clear evidence to support the generally held view that damping of prestressed bridges is any lower than that of reinforced bridges. Two similar concrete footbridges were tested by TRRL, one reinforced the other post-tensioned. The respective span lengths were 10.7m - 36.8m - 10.7m and 15.3m - 33.6m - 15.2m, both had suspended centre spans. The reinforced footbridge had a damping value of 0.044 whereas the post-tensioned footbridge had a value of 0.065. There are several reported cases of prestressed structures having very low damping but this may be due to slim superstructures which can vibrate without interaction with abutments or piers, rather than the fact that the concrete is prestressed.

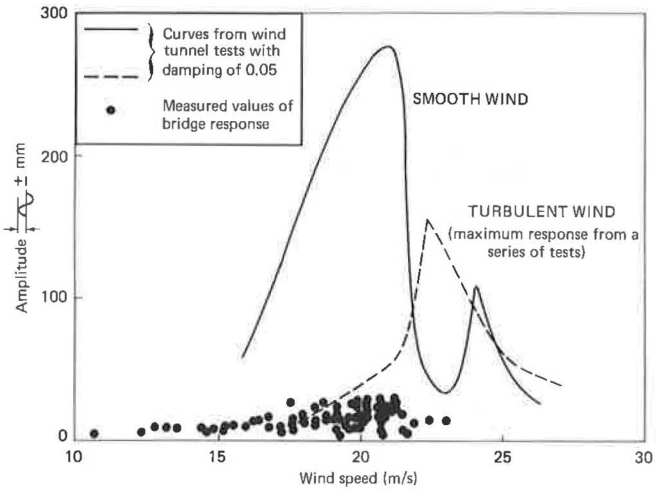
2. Measured Bridge Response

2.1 Wind Induced Vibration

Measurements of wind induced vibrations by TRRL have been made on two long-span steel box girder bridges (12). The first was the Cleddau Bridge at Milford Haven mentioned in Section 2. Earlier wind tunnel tests had predicted that large wind induced vibrations could develop and consequently a damping device was fitted inside the box (13). Measurements were made of wind induced vibrations during a 5½ month period to check behaviour with the damper inoperative. Accelerometers were positioned at the centre of the span and at quarter points. Wind speed and direction were measured using an anemometer and wind vane at the top of a 6m high mast positioned at the leading edge of the structure for winds blowing onshore. The indicated wind speed was corrected to free stream wind speed using a factor of 20 per cent which was obtained from wind tunnel tests. During the time the measurements were taken, the predicted critical wind speeds of 21 and 23m/s were experienced on several occasions. The maximum amplitude of movement never exceeded ±28mm in any of the occurrences and was substantially lower than predicted by the wind tunnel tests for either laminar or turbulent flow, Figure 7. However the response could be under-estimated from these measurements because there are insufficient data for the higher wind speeds and it is possible that the maximum respones could develop at speeds above the predicted values. Furthermore, the wind speeds may not have been sustained for long enough to cause the bridge to develop maximum response.

The second bridge whose wind response has been monitored is the Wye Bridge which has a 235m cable-stayed main span. During the first two years in service, between 1966 and 1968, there were several reported instances of relatively severe vibration but since that time the response has apparently been considerably less. Wind tunnel tests have predicted that a response of ±64mm can develop for damping of 0.03 and a wind blowing at 7.8 to 9.2m/s normal to the superstructure. The measurements on Wye Bridge were after the work on Cleddau Bridge and used

Figure 7. Comparison of measured and predicted deflection for Cleddau Bridge

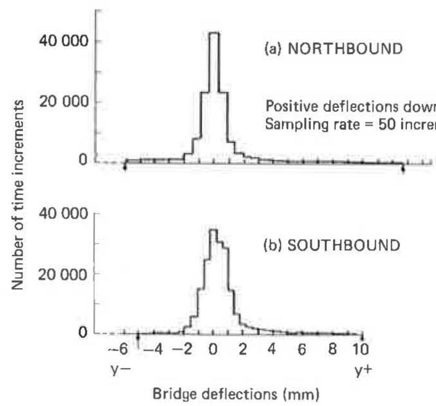


specially constructed equipment for automatic data acquisition, ADA, not previously available (12). This equipment constantly monitors three anemometers and an accelerometer. The anemometers are mounted orthogonally at the end of a 7m long horizontal boom positioned outwards from the leading edge at the centre of the span. The accelerometer is fixed inside the box also at the centre of the span. By suitable combinations of the signals from the anemometers the ADA is constantly informed of the direction and magnitude of the wind vector. When it is in the horizontal plane, within $\pm 40^{o}$ of the normal to the bridge, and its magnitude exceeds a predetermined level, outputs of the four sensors are recorded on analogue tape for a period of 8 minutes. The equipment was first installed on the bridge in March 1976 and will remain in use for several years. It has not been continually operational because it was necessary to move it when maintenance operations were carried out on the box. Up to 1978 there have been a number of occasions when sustained oscillations developed in the structure. The maximum measured response is ±18mm at 0.46Hz.

2.2 Traffic Induced Vibration

Measurements of traffic induced vibrations have been made on the Tinsley Viaduct. This structure had been found to require extra strengthening when reassessed using updated design criteria. It was decided to measure traffic induced vibrations before the strengthening in order to assess the dynamic performance. The bridge is two-level with twenty spans and a total length of 1032m. The superstructure is composed of steel box girders with concrete decks (12). Measurements were made of the deflections of the upper motorway level of the eighteenth span under normal traffic, using a cantilever deflection gauge. The data were recorded on analogue tape during four one-hour periods. The superstructure deflected as a continuous beam so that uplift as well as downward deflections were recorded. Vehicles were assessed by observation and those which appeared to be less than 15kN were excluded. During the four hours recording there were 914 north-bound vehicles and 744 south-bound vehicles exceeding 15kN. It was found that the maximum uplift was 5mm and the maximum downward deflection was 14.4mm. Most occurrences were in the range ±2mm, see histograms in Figure 8. These move-

Figure 8. Histograms of traffic induced deflections measured on Tinsley Viaduct



ments are very small in relation to the size of the structure; the maximum vibration component of ±2.6mm is less than half the recommended tolerance limit to pedestrian comfort.

Measurements of traffic induced vibrations on other bridges have given similar data.

2.3 Response to Pedestrians

Modern long-span bridges, although designed to be stable under the action of wind, tend to be lively under traffic. The vibration is often very noticeable to pedestrians and may be uncomfortable to motorists in stationary vehicles because movements can be amplified by the suspension system. This is not usually considered to be a serious problem because the vibrations generate negligible stresses in the superstructure. The consideration of human tolerance in such cases has not been a serious issue either. This is partly because the vibrations are not high enough to cause serious concern and partly because pedestrians who walk along heavily trafficked long-span bridges tend to have a higher tolerance. Footbridges excited by the pedestrians themselves present a different situation because they serve a large proportion of elderly and infirm people. In several cases it has been neccesary to amend designs at a late state in construction or after completion. The question of pedestrian tolerances is very subjective and limits are dependent on features such as frequency and amplitude of vibration, duration of exposure, whether walking or standing, and the psychological attitude of the pedestrian. After consideration of earlier work, the tolerance limit proposed for the British Standard is an acceleration of $\pm\frac{1}{2}\sqrt{f_o}$ where f_o is the fundamental natural bending frequency in Hz. Frequencies above 5Hz are not considered to present any problem. The background to the recommendations is given in

Reference 14. Simplified and generalised formulae are given for calculation of bridge response. For bridges of up to three spans the simplified method gives acceleration by the expression

$$4 \pi^2 f_o^2 Y_s K \psi \quad m/s^2$$

where y_s is static deflection due to a vertical load of 700 Newtons at mid point of the longest span.

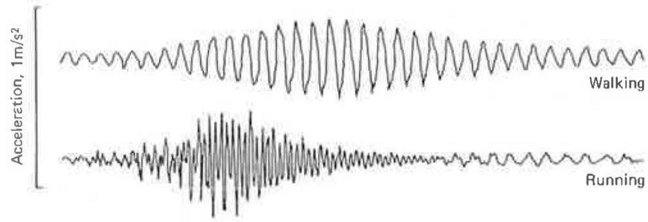
K is a configuration factor

ψ is a dynamic response factor depending on damping. Values recommended for damping are 0.03 for steel, 0.04 for composite (steel beams with concrete deck) and 0.05 for concrete bridges.

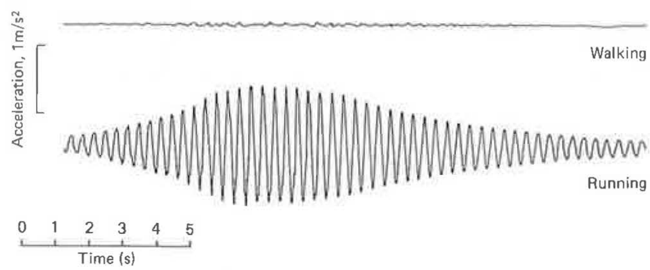
For frequencies greater than 4Hz the calculated acceleration may be reduced by an amount varying linearly from zero reduction at 4Hz to 70 per cent reduction at 5Hz. Alternatively response can be calculated by a general method where the load applied by a pedestrian is given by 180 Sin $2\pi f_o t$ Newtons, moving at a speed of 0.9 f_o m/s, t being time in seconds.

In the tests on footbridges by TRRL characteristics that are measured include static stiffness under concentrated loading, bending frequencies, mode shapes, damping, and response to pedestrians. Measured data of this type can be used to check the accuracy of calculated frequencies and response. Although considerable attention has been given to the development of accurate methods to calculate dynamic behaviour, few comparisons have been made with field measurements. In practice, the accuracy of calculations is influenced by features such as the support conditions which are difficult to express numerically. The methods and assumptions used to calculate dynamic behaviour of superstructures in the TRRL work have been described by Wills (15). Briefly, the calculations of frequency were made for free undamped vibration. Effects of rotary inertia and shear deformation are neglected. Bridge superstructures of variable depths have been considered. In calculating flexural rigidities, the whole cross-section including parapet upstands for continuous superstructures, is assumed to contribute fully to stiffness without reduction for shear lag. Hand rails and surfacings are ignored. Reinforcement is neglected and the dynamic modulus of concrete is used. This varies with concrete strength but gives a modular ratio of 5 to 6. Using these assumptions, the fundamental frequency can usually be calculated to within ±10 per cent of the measured value. Four of the lively footbridges, two steel and two concrete, are examined in relation to calculated response to pedestrians, Table 3. The resulting figures confirm that the two formulae give similar results. Furthermore the calculated accelerations correlate reasonably well with the measured ranges which are themselves very dependent on the pedestrians in question. The calculated response for the Wetherby footbridge exceeds the tolerance limit but measured responses for a single pedestrian on all the bridges were below the tolerance limit. Where responses were measured for two pedestrians walking in step, the values were approximately twice as high as for single pedestrians. Responses for walking and running across the two concrete footbridges are shown in Figure 9.

Figure 9. Responses of concrete footbridges to one person walking and running, measured at mid-span



(a) COTTESMORE



(b) BRACKENDALE ROAD

In cases where bridges are found to be unduly lively remedial action can usually be taken relatively easily. This is illustrated by a slender steel box girder footbridge at Clapton-in-Gordano which could not be adequately assessed before construction due to doubts about what damping value should be assumed. When built, the bridge was found to have damping of 0.005 for amplitudes of vibration of up to ±6mm. This damping is unusually low and permitted unacceptably lively behaviour because the tolerance limit converted into deflection, is ±6mm, which could easily be exceeded by a pedestrian. The lively behaviour was cured by fitting friction dampers against the abutment at the free end of the bridge and in the sleeve joints between the hand rails. The addition of the handrail damper increased the overall damping to 0.012 at ±4mm rising to 0.035 at ±24mm. The damper at the bearing raised the values to 0.055 at ±3mm, Figure 10. The small reduction in damping at higher amplitudes is typical of friction damping. In cases of continuous multi-span bridges it may not be convenient to add friction devices and use of tuned dynamic absorbers is a more attractive solution. Vibration absorbers have been used successfully for bridges found to be susceptible to wind induced vibration (13) (16). Their performance has been demonstrated recently on the 30m steel box girder at TRRL. The dynamic characteristics were investigated by exciting the beam using a small version of the energy input device ie a vibrating mass reciprocated in a vertical plane by a hydraulic actuator. The excitation was carried out at different frequencies so that response/time curves could be plotted for the beam with and without addition of the dynamic absorber, Figure 11. The reduction in response can be seen to be very significant.

Table 3. Calculated and measured values of dynamic response of footbridges

Bridge	Frequency, f_o, Hz: Calculated	Frequency, f_o, Hz: Measured	Calculated response m/s^2: Simplified method	Calculated response m/s^2: General method	Measured response m/s^2	Tolerance limit $\pm\frac{1}{2}\sqrt{f_o}$
Steel						
Craigie Park, Ayr	3.30	3.11	0.58	0.57	0.09-0.54	0.91
River Wharfe, Wetherby	2.62	2.62	0.97	0.88	0.06-0.49	0.81
Concrete						
Brackendale Road, Camberley	3.04	2.82				0.87
1 pedestrian			0.24	0.20	0.16-0.63	
2 pedestrian			0.48	0.40	0.58-1.03	
Cottesmore, Oxford	1.69	1.89				0.65
1 pedestrian			0.28	0.22	0.37-0.55*	
2 pedestrian			0.56	0.44	0.83-1.03*	

*Pedestrians walking in step with a metronome

Figure 10. Influence of extra dampers on first mode behaviour of Clapton-in-Gordano footbridge

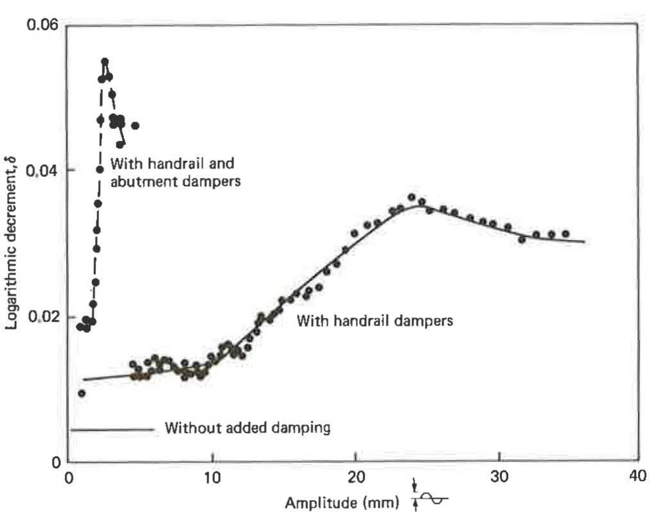
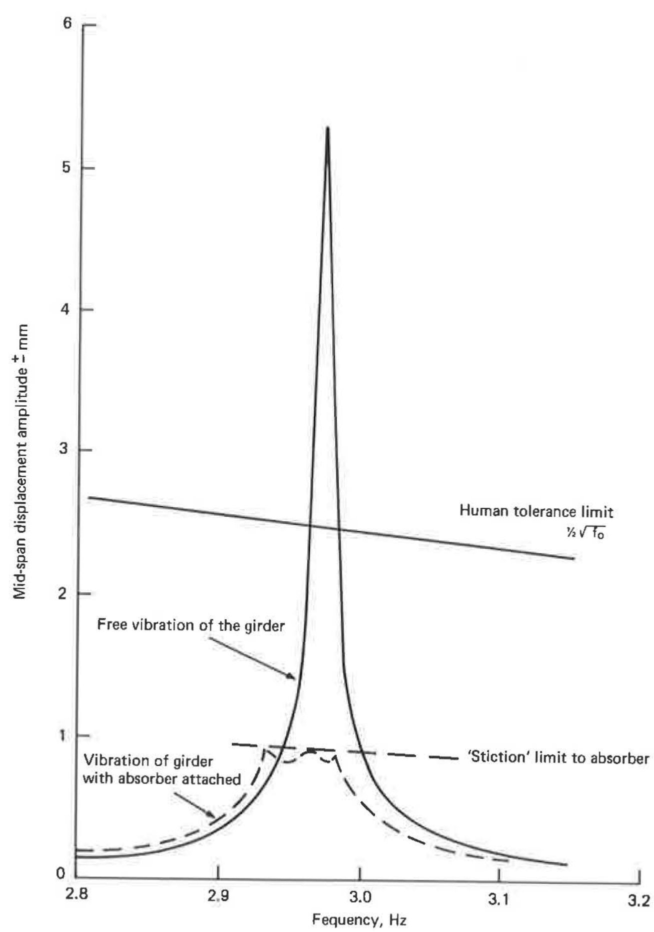


Figure 11. Effect of tuned absorber on vibration of 30m steel box girder



3. Conclusions

1. Damping values of bridges and their component materials increase with amplitude of vibration. Typical upper-level values are 0.02 to 0.06 for steel, 0.05 to 0.10 for composite (steel beams with concrete decks) and 0.02 to 0.10 for concrete bridges.

2. Measurements made to date, of the response of two long span steel bridges to wind, indicate that amplitudes of vibrations are less than values predicted from wind tunnel tests.

3. Traffic induced vibrations are not a serious problem for modern bridges. Amplitudes of movement are relatively low and involve low stresses and accelerations below the levels for human tolerance.

4. Pedestrian induced vibration of footbridges can sometimes present problems. However calculated responses correlate with measured behaviour and show that proposed design procedures give adequate guidance.

5. Lively bridges can be deadened by the addition of extra damping or vibration absorbers. Such methods are considered preferable to structural modifications such as stiffening the superstructure or reducing span lengths.

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