DYNAMIC RESPONSE OF A SINGLE TRACK RAILWAY TRUSS BRIDGE

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A lumped mass model of a railway truss bridge is developed. The model considers only the vertical degree of freedom of each truss joint. The vehicle system is idealized as a three degree of freedom model consisting of the carbody and wheel-axle sets. Dynamic interaction equations for the bridge-vehicle system are derived and solved using the numerical integration method. Impact factors for member forces and nodal deflections are generated under the action of a single or a series of three moving vehicles. Finally, a limited parametric study is performed to determine the influence of vehicle speed, vehicle suspension characteristics and sprung mass on impact factors.

The subject of bridge impact* has been of great interest to many investigators and researchers since the middle of nineteenth century. A detailed review on this subject has been given in [1]. During the past two decades, a considerable amount of analytical and experimental research work has been conducted for highway bridges. A general scheme for analysing the vibration problem of highway bridges due to moving vehicles has been given in [2]. This method includes a realistic model of the vehicle to study the interaction between the bridge and the vehicle. To date, only a very limited amount of theoretical work has been done to study such interaction for railway bridges.

The first analytical approach to the railway bridge vibration problem was suggested by Willis [3] in 1849. Subsequently, Robinson [4] in 1884 conducted bridge impact tests and compared them to a simplified analysis. In the early 20th century, significant contributions to the development of a general theory of bridge vibration were made by Timoshenko [5] and Inglis [6]. Schallenkamp [7] in 1937, presented a rigorous closed form solution for the case of a smoothly rolling load in which he included both the mass of the load and the mass of the bridge. Looney [8] in 1944, using a numerical procedure, obtained solutions of the problem with multiple masses.

In the U.S.A. the most complete series of tests on railroad bridges were conducted by the subcommittee of American Railway Engineering Association (AREA) under the direction of Turneaure [9] in 1911. The results of these tests were impact formula that were used in the design of railroad bridges until 1935.

In 1935, Hunley [10] collected data for static and dynamic deflection of 39 different railroad bridges under approximately 300 different locomotives. He also secured additional data on damping coefficients for bridges. The AREA (American Railway Engineering Association) design specifications for railroad bridges were revised in accordance with Hunley's recommendation for impact effects and were used until 1948. Subsequently, extensive field and laboratory tests were conducted by the Association of American Railroads for steel, concrete and timber railroad bridges. These tests are described by Ruble [11]. More recent tests are published in AREA proceedings. Based on these tests the present AREA design specifications for impact on railroad bridges were developed.

The objective of this study is to examine the vertical dynamic response of a truss railway bridge to the passage of one or a series of railway vehicles. Only vehicle in its normal operating condition has been considered. The effects of vehicle hunting or vehicle breaking resulting in lateral or longitudinal motion of the bridge are neglected. The bridge is analyzed as a multi-degree of freedom system; and the vehicle is represented realistically as a four-axle sprung load unit with three degrees of freedom. In the analysis, input due to track irregularities is neglected and only the elastic interaction between the bridge and vehicle is considered. The equations of motion for the vehicle-bridge system, including the vehicle-bridge interaction, are developed and solved to generate dynamic impact factors for member forces and nodal deflections. A limited parametric study has been performed to determine the influence of vehicle speed, vehicle suspension characteristics and sprung mass on impact factors.

Vehicle-Bridge System Model

A lumped mass model of the truss bridge in which the masses are assumed to be concentrated at each truss joint, is considered. The truss joints are assumed to be pin-connected. Half of the mass of each member is contributed to the joints it connects. The floor system of floor-beams, stringers and the track system is lumped to the bottom chord joints. The cross-beams are assumed to be pin-connected to the truss joints and secondary effects are neglected. The wheels of the vehicle are assumed to remain in contact with the track at all times. All displacements are assumed to be small. Only the vertical degree of freedom is assigned to each truss joint. The truss flexibility coefficients are obtained by the deflections caused by a unit load applied at each joint one at a time. The effect of rotary inertia and non-linearity of material or deformation are neglected in the analysis.

The vehicle system is idealized as a three degree of freedom model of a railway vehicle consisting of a rigid car body and four axle sets, Fig.1. The two, 2-axle trucks are assumed to form a part of the car body as in the actual vehicle. The three degrees of freedom assigned to the vehicle correspond to bounce, pitch and roll motion. In the model, the primary and secondary suspension system are considered as linear springs in series and replaced by an equivalent spring. Damping in the suspension systems is small and consequently neglected. This simplified model has been chosen, after it was found that its bounce, pitch and roll frequencies are quite close to the test values and to a more complex ten degree of freedom model of the vehicle.

Equations of Motion

D'Alembert's principle of dynamic equilibrium is used to write the equations of motion. By equating the external and internal forces we get:

$$[M]\,[\ddot{D}] + [C]\,[\dot{D}] + [K]\,[D] = [F(x,t)] \qquad (1)$$

where

$[M]$ = Mass matrix of the truss

$[K]$ = Stiffness matrix of the truss

$[C]$ = Damping matrix of the truss

or $[C] = a\omega_1[M]+b[K]$, where a and b are scalar constant and ω_1 is the fundamental circular frequency. In the analysis

$$[C] = a\omega_1[M] \qquad (2)$$

is used.

$[F(x,t)]$ = vector of applied nodal loads, due to interaction between the moving vehicle and the bridge.

$[D] = [v_1, v_2, ----v_n]^T$ = joint displacement vector.

Here v is the vertical displacement and subscript denotes the joint number.

$[\dot{D}]$, $[\ddot{D}]$ = joint velocities and accelerations.

Equations of motion for vehicle: v_b^i is the vertical displacement of the bridge associated with the ith wheel of the vehicle at any time t, Fig.2b. The equation of motion of vehicle are expressed as:

$$M^b\ddot{y}^b + \sum_{i=1}^{8} k_y(y^b \pm l_i\phi^b \pm b\theta^b - v_b^i) = 0 \qquad (3)$$

$$I^b\ddot{\phi}^b + \sum_{i=1}^{8} k_y(y^b \pm l_i\phi^b \pm b\theta^b - v_b^i)(\pm l_i) = 0 \qquad (4)$$

$$J^b\ddot{\theta}^b + \sum_{i=1}^{8} k_y(y^b \pm l_i\phi^b \pm b\theta^b - v_b^i)\,(\pm b) = 0 \qquad (5)$$

Where M^b, I^b and J^b are the mass, pitch moment of inertia and roll moment of inertia of the vehicle; y^b, ϕ^b, and θ^b refer to vertical, pitch and roll displacement of the vehicle; k_y is the equivalent spring stiffness of each vertical suspension element; l_i is the distance of the centroid of the vehicle to the ith wheel, and b is the half distance of wheel contact points. A dot superscript denotes differentiation with respect to time.

The interacting force p^i between the rail and the ith wheel of the vehicle is expressed as

$$p^i = M_u(g-\ddot{v}_b^i)+k_y(y^b \pm l_i\phi^b \pm b\theta^b - v_b^i) + M_s g \qquad (6)$$

where

M_u = Unsprung mass

$M_s = \frac{M^b}{8}$ = sprung mass

If the ith wheel is on track segment lying between kth and (k+1) joint, Fig.2a, then v_b^i is expressed in terms of v_k^i and v_{k+1}^i by linear interpolation

$$v_b^i = \frac{x^i}{l_p}\, v_{k+1}^i + (1-\frac{x^i}{l_p})v_k^i = \alpha^i v_{k+1}^i + \beta^i v_k^i \qquad (7)$$

where

x^i = distance of ith wheel from joint k

l_p = panel length of truss

$\alpha^i = x^i/l_p$

$\beta^i = (1-\alpha^i)$

Similarly,

$$\ddot{v}_b^i = \alpha^i\ddot{v}_{k+1}^i + \beta^i\ddot{v}_k^i \qquad (8)$$

The load transmitted from the ith wheel to the kth and (k+1)th joints are

$$p_k^i = \beta^i(\frac{c-d}{c})\; p^i$$

$$p_{k+1}^i = \alpha^i(\frac{c-d}{c})\; p^i \qquad (9)$$

where

c = center to center distance of trusses

d = distance between truss and rail

Substituting v_b^i, $\ddot{v}_b^i$ and p^i from equations (6), (7) and (8) into equation (9) gives

$$p^i = \beta^i(\frac{c-d}{c})\;[(M_s+M_u)g+k_y(y^b \pm l_i\phi^b \pm b\theta^b - \alpha^i v_{k+1}^i - \beta^i v_k^i) - M_u(\alpha^i\ddot{v}_{k+1}^i + \beta^i\ddot{v}_k^i)] \qquad (10a)$$

$$p_{k+1}^i = \alpha^i(\frac{c-d}{c})\;[(M_s+M_u)g+k_y(y^b \pm l_i\phi^b \pm b\theta^b - \alpha^i v_{k+1}^i - \beta^i v_k^i) - M_u(\alpha^i\ddot{v}_{k+1}^i + \beta^i\ddot{v}_k^i)] \qquad (10b)$$

Substituting equations (3), (4), (5) and (10) into equation (1) the following dynamic equation is obtained.

$$[M]_R \begin{bmatrix} \ddot{D} \\ \hline [\ddot{y}^b, \ddot{\phi}^b, \ddot{\theta}^b]^T \end{bmatrix} + [C]_R \begin{bmatrix} \dot{D} \\ \hline [\dot{y}^b, \dot{\phi}^b, \dot{\theta}^b]^T \end{bmatrix} + [K]_R \begin{bmatrix} D \\ \hline [y^b, \phi^b, \theta^b]^T \end{bmatrix} = [F]_R \qquad (11)$$

where

$[D]$ = the vector consisting of vertical displacements of truss joints

$[M]_R$ = Mass matrix of bridge-vehicle system including interaction effect

$[C]_R$ = Damping matrix of bridge system

$[K]_R$ = Stiffness matrix of bridge-vehicle system including interaction effect. The stiffness matrix of the bridge is obtained by inverting the flexibility matrix.

$[F]_R$ = Time dependent force vector resulting from bridge-vehicle interaction.

The resulting equations of motion for the bridge vehicle system, (11), are solved using the following step-by-step procedure:
1) Establish a distance vector based on the configuration of axles and the distance between the vehicles. This vector stores the distances between wheel loads.
2) Calculate the position of the wheels and determine the wheel loads on the span.
3) For every load on the span determine its position with regard to the truss panel it occupies.
4) Compute α^i and β^i
5) Form $[M]_R$, $[K]_R$ and $[F]_R$
6) Compute static displacements at truss joints and stresses in truss members.
7) Calculate dynamic deflections using step-by-step numerical integration technique.
8) Compute dynamic load vector by [K] [D] where [K] is the stiffness matrix of the bridge.

Dynamic Response of a Single Track Truss Bridge

Using the formulation discussed previously, a computer program was developed to determine the dynamic response of a single track truss bridge subjected to a single or three moving vehicles system. The bridge analyzed is taken from reference [12] and the main geometry of the bridge is shown in Fig.3. Other relevant data, such as member properties of the bridge etc., and vehicle data is given in Appendix A.

The dynamic response for the single moving vehicle at 60 mph is obtained for two conditions, undamped and damped, using two percent structural damping. The dynamic response for the three moving vehicle system corresponds to the undamped condition at 60 mph. The vehicles are assumed to have no initial vertical motion at the time of entry into the bridge.

Time histories of the static and dynamic amplification factors for mid-span deflection and "worst case" member stresses are shown in Figs.4 through 8. The amplification factor Λ_d and Λ_s for dynamic (Γ_d) and static (Γ_s) response are defined as

$$\Lambda_d = \frac{\Gamma_d}{\Gamma_{sm}} \quad ; \quad \Lambda_s = \frac{\Gamma_s}{\Gamma_{sm}}$$

where Γ_{sm} is the maximum static response. The impact factor I is defined as $I=\max|\Lambda_d|-1$. Table 1 lists the impact factors for the bridge under damped and undamped condition for both single and three vehicle loading. It may be seen that the maximum impact factor for mid-span displacement is about 5% for a single vehicle without damping and 4% with damping. For the three vehicle system the impact factor is about 23%. This indicates that impact under multiple vehicle loading is significantly larger than the sum of the impact factors resulted from the individual vehicles. Similar observation can be made for member force impact factor, where a single vehicle under undamped condition generates a maximum impact factor of about 6.3% whereas the three vehicles case yiélds a maximum impact factor of about 26%. It should be noted that the member force impact factors are reduced by including structural damping into the system.

A limited parametric study has been performed for the dynamic response of the bridge under a single moving vehicle without structural damping. The study has been performed to establish the influence of vehicle suspension parameters, vehicle speed and sprung mass on impact factors. Fig.9, 10 and 11 show the results of this study. The impact factor for mid-span deflection is increased from about 3.5% to about 8% when the vehicle speed is increased from 40 mph to 80 mph. It may be noted from Fig.9 that as speed is increased from 60 to 80 mph the member force impact factor shows an increase of about 3%.

Fig.10 shows that a stiffer suspension system will result in larger impact factors as expected. Fig.11 indicates that impact factors are somewhat linearly proportional to the sprung mass of the vehicle.

Conclusions

Equations of motion for the bridge-vehicle system have been developed to analyse dynamic response of a single-lane truss bridge under a single vehicle or a three moving vehicle system. It has been found that the impact factors under the multiple vehicles are significantly larger than the sum of the impact factors due to individual vehicles. Incorporation of structural damping into the bridge system resulted in a reduction of the deflection and force impact factors. A limited parametric study shows that largest impact factors result at high speed with a stiff suspension system. Impact factor appears to increase linearly with respect to sprung mass of the vehicle.

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APPENDIX A

Member Properties and Pertinent Data
For Single-Track Truss Bridge

Member	Section	Area (Gross)
Top Chord	cov.24" x $\frac{9"}{16}$	
U_0U_1, U_1U_2	2-L3½" x 3½" x 5/8"	
	2-L3½" x 3½" x 5/8"	58.5 in^2
U_2U_3, U_3U_4	2-web 20" x 11/16"	
	Double Lat.2½" x ½"	
Bottom Chord		
L_0L_1, L_1L_2	4-L3½" x 3½" x ½"	36.63 in^2
L_4L_5, L_5L_6	2-web 21" x $\frac{9"}{16}$	
L_2L_3, L_3L_4	4-L3½" x 3½" x 5/8"	63.51 in^2
	4-web 21" x $\frac{9"}{16}$	
Intermediate Posts		
L_1U_0, L_2U_1	4-L6" x 4" x 3/8"	19.34 in^2
L_3U_2, L_4U_3	Web 13" x 3/8"	
L_5U_4		
Diagonals		
U_0L_2, L_4U_4	4-L6" x 4" x 11/16"	
	Web 13" x 5/8"	34.25 in^2
L_2U_2, U_2L_4	2[15x50	
	Double Lat.2½" x ½"	30.53 in^2
End Posts		
L_0U_0, U_4L_6	cov.24" x 5/8"	
	2-L2½" x 3½" x 5/8"	62.5 in^2
	2-L3½" x 3½" x 5/8"	
	2-Web 20" x 3/4"	
	Double Lat.2½" x ½"	

Wt. of each stringer = 6611 lbs.
Wt. of each cross beam = 4160 lbs.

Wt.of track system coming at each bottom chord joints - 65000 lbs.

Basic Data For a 4-Axle Railway Vehicle

Dimensional Data:

$\bar{b}$	- Half length of wheel base	=	54 in (137 cms)
b	- Half distance between wheel contact points	=	29.5 in (75 cms)
L	- Half distance between truck centers	=	204 in (518 cms)

Mass & Inertia Data:

M^b	Carbody Mass	=	546 lbs-sec^2/in (247 kg)
	Truck frame mass	=	19.4 lbs-sec^2/in (8.7 kg)
M^u	- Wheel set mass	=	22.1 lbs-sec^2/in (10 kg)
I^b	- Carbody roll moment of inertia	=	1.2x10^6 lbs-in-sec^2 (1.36x10^{10} kg-cm^2)
J^b	- Carbody pitch moment of inertia	=	1.8x10^7 lbs-in-sec^2 (2.04x10^{11} kg-cm^2)

Spring Rates Data:

k_y	- Vertical spring stiffness/wheel (Includes Primary & Secondary stiffness)	=	7,000 lbs/in (12,250 N/cm)
Distance from last wheel of front vehicle to the first wheel of rear vehicle		=	81.5 in (217 cm)

Table 1 - Maximum Axial Forces in Members of a Truss Bridge and Impact Factor in Percentages

Note: All Forces are in KIPS

Member Number	Member Forces Under dead load	SINGLE VEHICLE CASE Max.Live Load Static Force	Undamped Bridge Max.L.L. Dynamic Force	 Impact Factor	2% Bridge Damping Max.L.L. Dynamic Force	 Impact Factor	3 VEHICLE CASE Max.L.L. Static Force	Undamped Bridge Max.L.L. Dynamic Force	 Impact Factor
2 or 3	76.35	82.06	84.38	2.83	83.19	1.4	158.2	186.7	18.02
4 or 5	139.6	139.6	145.9	4.51	141.3	1.22	274.5	342.3	24.7
6 or 7	76.35	81.62	86.37	5.82	84.63	3.69	159.1	190.1	19.48
8	19.79	54.3	54.88	1.07	54.82	0.96	90.46	108.0	19.39
9	69.66	92.42	94.44	2.19	94.36	2.1	154.5 -11.95	181.4 -16.4	17.41 -
11	-23.74	34.15 -63.62	35.65 -66.18	4.39 4.02	35.04 -66.03	2.6 3.8	34.98 -77.86	35.82 -87.48	2.0 12.36
12	22.68	54.99	58.48	6.3	56.61	2.95	90.46	102.0	12.76
13	-23.74	34.7 -65.96	29.79 -69.24	 4.9	32.87 -67.79	 2.77	34.93 -79.01	36.77 -85.7	5.0 8.48
15	69.66	91.77 -11.73	94.94 -7.814	3.45	95.2	3.73	154.4 -8.99	188.5 -16.4	22.09 -
16	19.79	55.26	56.77	2.73	55.89	1.14	90.46	114.0	26.0
18 or 19	-123.5	-128.9	-131.9	2.33	-130.1	0.93	-252.0	-308.9	22.58
20 or 21	-123.5	-128.7	-130.4	1.32	-129.8	0.77	-252.0	-312.8	24.13

Figure 1. Idealized vehicle model.

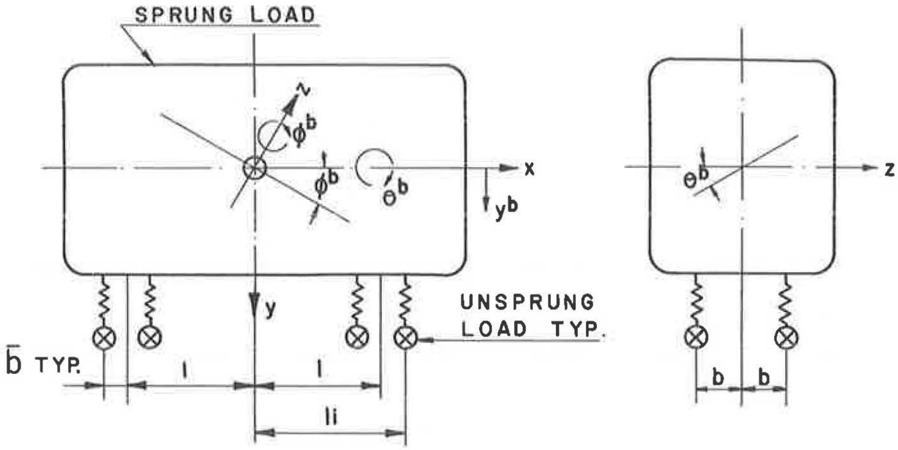

Figure 2a. Plan view of configuration of wheel load in a panel of bridge span.

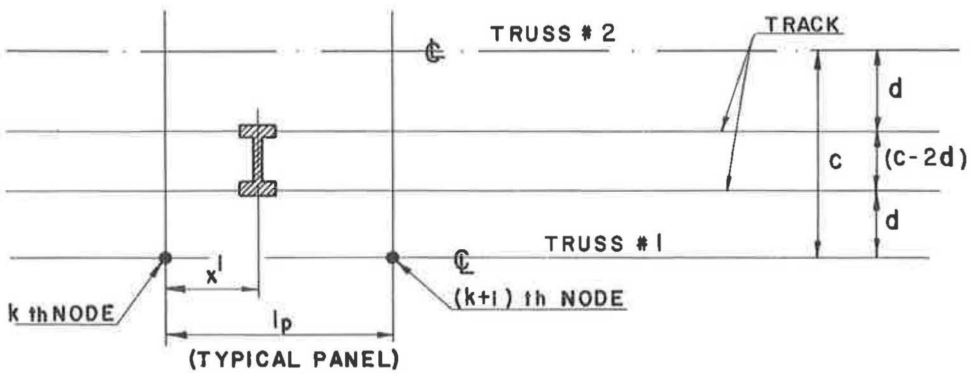

Figure 2b. Conversion of wheel loads into joint loads.

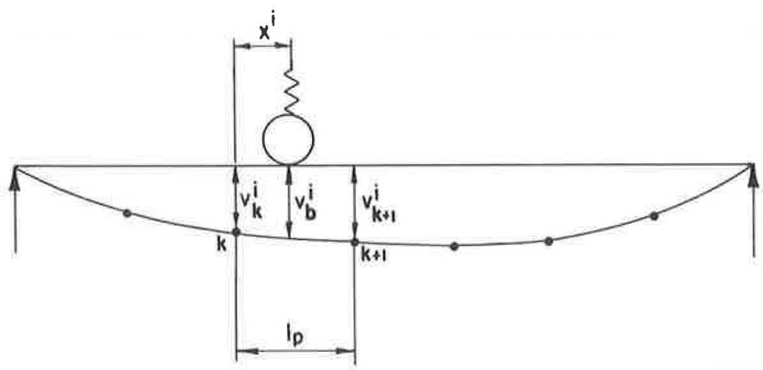

Figure 3. Geometry of the bridge and masses concentrated at each joint.

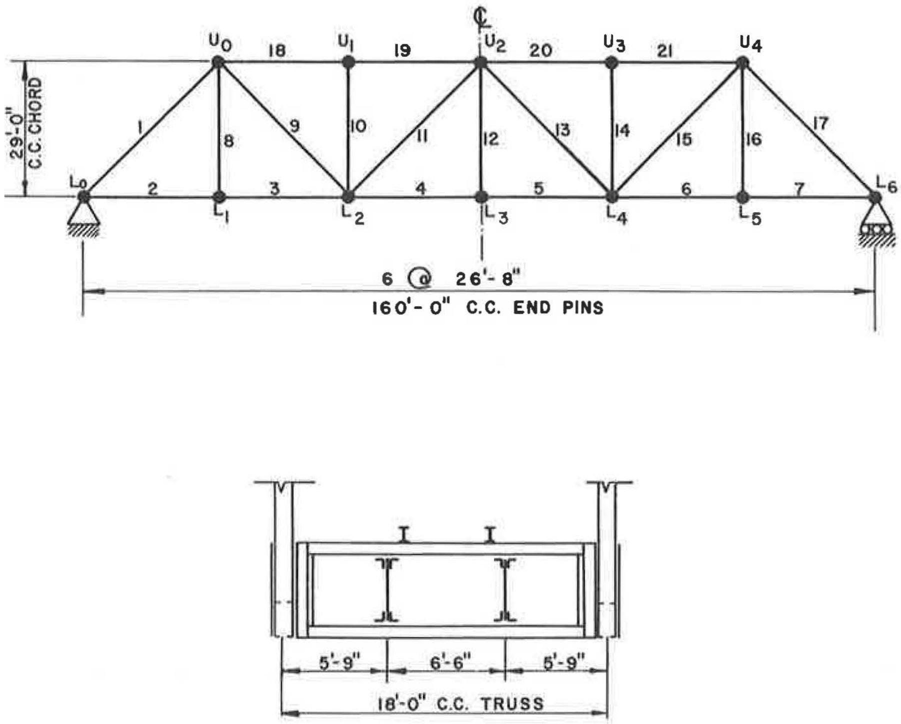

Figure 4. History curves of mid span deflection of truss bridge w/o damp.

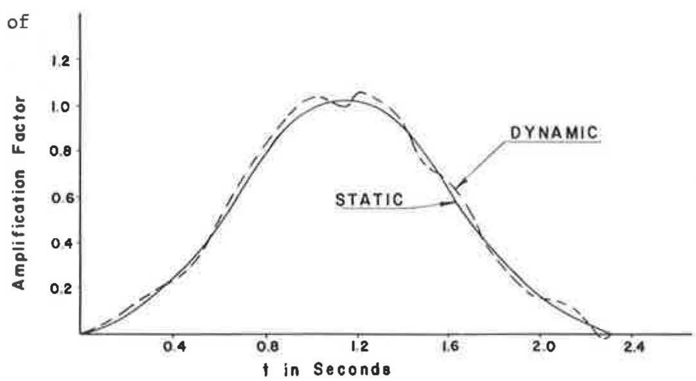

Figure 5. History curves of mid span deflection of truss bridge w/2% damping.

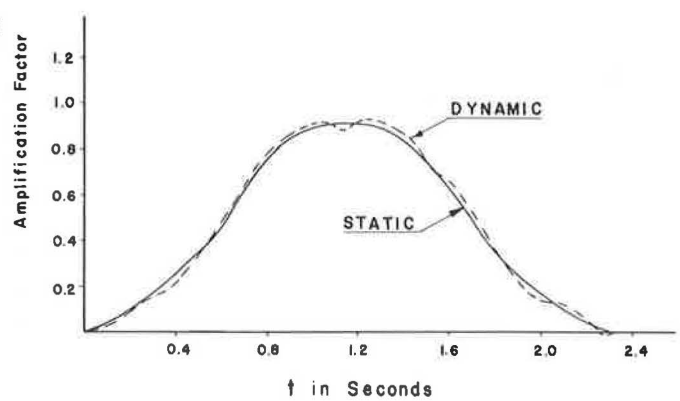

Figure 6. History curves of stress in member U_2L_3 w/one vehicle.

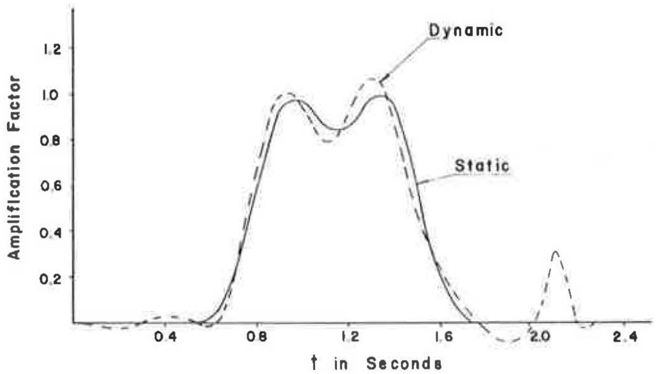

Figure 7. History curves for mid span deflection for single track truss bridge w/3 vehicles w/o damping.

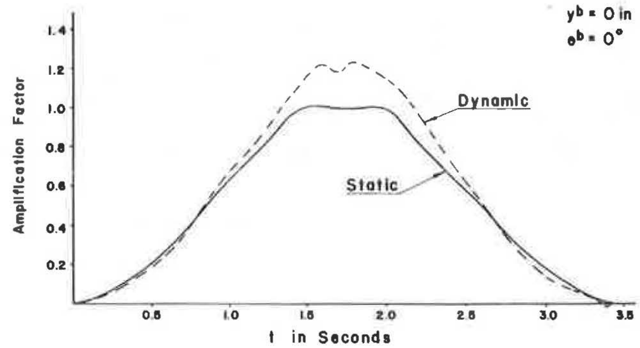

Figure 8. History curves for forces in member L_5U_4 w/3 vehicles w/o damping.

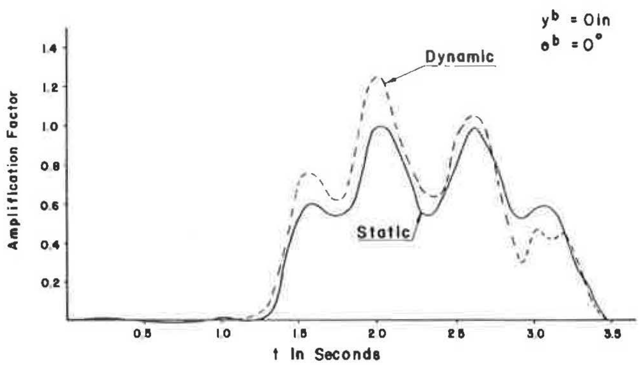

Figure 9. Impact percentage versus speed of vehicle (single vehicle on bridge).

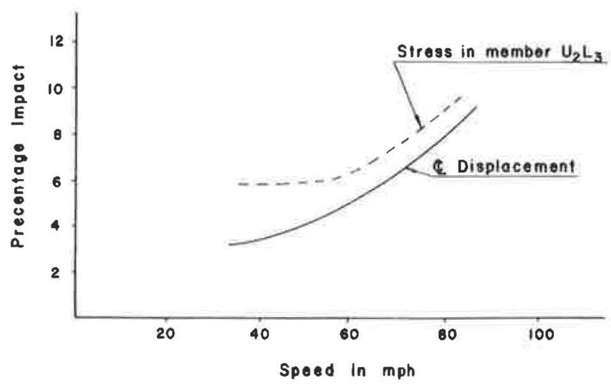

Figure 10. Impact percentage versus spring stiffness (single vehicle on bridge).

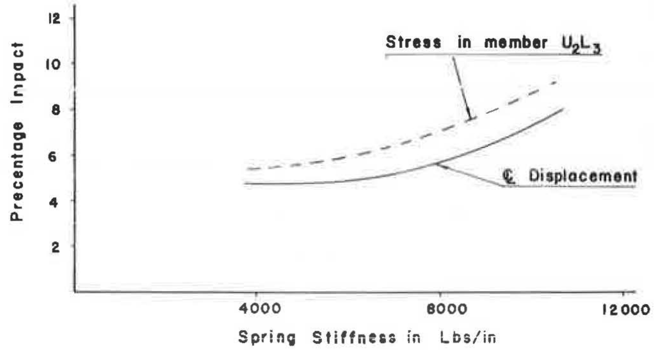

Figure 11. Impact percentage versus sprung load (single vehicle on bridge).