METHODS OF CALCULATION OF THE WIND-INDUCED RESPONSES OF SUSPENDED-SPAN BRIDGES

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Abstract

An outline is given of the manner in which expressions for the buffeting forces of the wind, together with those for the self-excited aerodynamic forces due to resulting bridge motion, may be used to predict the random response of a long-span bridge to the action of the natural wind. The problem is examined in terms of the individual responses of the several modes of the structure as they are randomly excited, both in space and in time, by wind gusts. The bridge deck modes in question are each considered to have vertical, torsional and lateral sway components. Recent formulations for wind horizontal and vertical gust spectra are employed. More complete literature references are cited for the details of the methodology used.

Introduction

The wind-induced responses of suspended-span bridges include vortex-induced activity, flutter, galloping, and buffeting. Basically, the intrinsic character of each of these phenomena is determined in the first instance by the geometry of the deck cross-section. It has become the practice in the last ten years to abstract the measured unsteady aerodynamics characteristics of the deck— once its geometry is set— from its particular structural characteristics. The two may then be reunited later in whatever way is dictated as appropriate by the structural dynamics of the full bridge.

The aerodynamic forces consist of a) the steady forces, b) the gust, or buffeting forces, and c) the self-excited forces (related to motions of the bridge). Any analysis must provide for all of these.

In modern bridges the vertical, torsional, and sway components of any given natural structural mode must all be considered; modes are not correctly characterized uniquely as being uncoupled in each of these motions. The motion in one component sense will engender structural and aerodynamic forces in one of the others, and these interact. Hence analytic provision must be made for all possible forces as a study progresses.

While section models of bridges remain the investigatory method of choice, they should now be conceived of only as purely geometric sources of aerodynamic data, never as proper analogs of the full prototype. This is true because conditions under which a section model can be conceived of as truly representative of a prototype are indeed very restrictive: the bridge deck must be straight; bending and torsion modes must be strictly uncoupled, and they must possess identical modal form distribution over the entire span. Otherwise, a section model can be a misleading object of study. Further, section models should not be tested exclusively in laminar flow, but the effects of turbulence included (or considered) in order to verify its local effect up deck-section aerodynamic properties.

Structural Modes

It will be assumed here that a single, linear spanwise coordinate x suffices to define any point along the bridge span (even if the latter is curved in plan or elevation). Since the full bridge is three-dimensional, with displacement components $h(x)$, $\alpha(x)$, $p(x)$ in vertical, torsional, and sway directions, respectively, the total deck section displacement may be represented as a superposition of the components in modes i:

$$h(x,t) = \sum_i h_i(x)\, B\, \xi_i(t) \tag{1a}$$

$$\alpha(x,t) = \sum_i \alpha_i(x)\, \xi_i(t) \tag{1b}$$

$$p(x,t) = \sum_i p_i(x)\, B\, \xi_i(t) \tag{1c}$$

where B, the deck width, is a reference length.

Steady Aerodynamic Forces

These are simply given for unit span by

$$\text{Lift:}\quad L_s = \frac{1}{2}\rho U^2 (B)\, C_L\,(\alpha) \tag{2a}$$

$$\text{Drag:}\quad D_s = \frac{1}{2}\rho U^2\, A\, C_D\,(\alpha) \tag{2b}$$

$$\text{Moment:}\quad M_s = \frac{1}{2}\rho U^2\, B^2\, C_M\,(\alpha) \tag{2c}$$

where A is projected area (normal to the horizontal wind) per unit span, ρ is air density, U is mean wind velocity (assumed normal to the span).

Self-Excited Forces

These are given [1] in linearized form (per unit span) by

$$\text{Lift:}\quad L_{s.e.} = \frac{1}{2}\rho U^2 (2B)\,[K H_1^* \frac{\dot h}{U} + K H_2^* \frac{B\dot\alpha}{U} + K^2 H_3^* \alpha] \tag{3a}$$

$$\text{Drag:}\quad D_{s.e.} = \frac{1}{2}\rho U^2 (2B)\,[K P_1^* \frac{\dot p}{U}] \tag{3b}$$

$$\text{Moment:}\quad M_{s.e.} = \frac{1}{2}\rho U^2 (2B^2)\,[K A_1^* \frac{\dot h}{U} + K A_2^* \frac{B\dot\alpha}{U} + K^2 A_3^* \alpha] \tag{3c}$$

where $K = B\omega/U$ is reduced frequency parameter, and H_i^*, A_i^*, $(i = 1,2,3)$ functions of K, are self-excited aerodynamic coefficients.

In the case of vortex-induced oscillations, particularly, the above model may require upgrading by inclusion of nonlinear terms. Typical third-degree additional terms appropriate to lift and moment are

$$\text{Lift:}\quad (L_{s.e.})_{n.\ell.} = \frac{1}{2}\rho U^2 (2B)\,[K H_o^* (\frac{\dot h}{U})^3] \tag{4a}$$

$$\text{Moment:}\quad (M_{s.e.})_{n.\ell.} = \frac{1}{2}\rho U^2 (2B^2)\,[K A_o^* (\frac{B\dot\alpha}{U})^3] \tag{4b}$$

These forces must be determined by recourse to model experiment, as described in Refs. [1], [2].

Buffeting Forces

These are given [3] (per unit span) by the forms

$$\text{Lift:}\quad L_b(x,t) = -\rho U^2 B [C_L(\alpha_o)\frac{u(x,t)}{U} + \frac{1}{2}\left.\frac{dC_L}{d\alpha}\right|_{\alpha=\alpha_o} + \frac{A}{B} C_D(\alpha_o)\, \frac{v(x,t)}{U}] \tag{5a}$$

$$\text{Drag:}\quad D_b(x,t) = \rho U^2 B\, [\frac{A}{B} C_D \frac{u(x,t)}{U}] \tag{5b}$$

$$\text{Moment:}\quad M_b(x,t) = \rho U^2 B^2\, [C_M(\alpha_o)\frac{u(x,t)}{U} + \frac{1}{2}\left.\frac{dC_M}{d\alpha}\right|_{\alpha=\alpha_o} \frac{v(x,t)}{U}] \tag{5c}$$

where α_o is the steady net twist under the mean wind velocity U; C_L, C_D, C_M (functions of bridge deck angle of attack α) are steady lift-, drag-, and moment-coefficients, respectively, and $u(x,t)$, $v(x,t)$ are respectively the horizontal and vertical gust components of the wind.

Net Equations of Motion

These take the form

$$[(M_V+M_L)\, B^2+I]_i\, [\ddot\xi_i+2\zeta_i\, \omega_i\, \dot\xi_i+\omega_i^2\xi] = Q_i(t) \tag{6}$$

where M_V, M_L, I are the generalized inertias of mass $m(x)$ and mass moment of inertia $I_{c.g.}(x)$ about the deck c.g. calculated for the modal components $h(x)$, $p(x)$, $\alpha(x)$:

$$(M_V)_i = \int_{span} m(x)\, h_i^2(x)\, dx \tag{7a}$$

$$(M_L)_i = \int_{span} m(x)\, p_i^2(x)\, dx \tag{7b}$$

$$(I)_i = \int_{span} I_{c.g.}(x)\, \alpha_i^2(x)\, dx \tag{7c}$$

and ζ_i is the mechanical (structural) damping in mode i, which has the natural circular frequency ω_i.

The generalized force Q_i is calculated by noting that

$$\delta W = Q_i \, \delta\xi_i = (L_{s.e.} + L_b)\, \delta h + (D_{s.e.} + D_b)\, \delta p + (M_{s.e.} + M_D)\, \delta\alpha \quad (8)$$

where

$$\delta h = hB\, \delta\xi_i \quad (9a)$$

$$\delta p = pB\, \delta\xi_i \quad (9b)$$

$$\delta\alpha = \alpha\delta\xi_i \quad (9c)$$

The details of Q_i involve intermodal coupling which may be ascribable to the aerodynamics. It is not fruitful here to reproduce full details on the form of Q_i when several modes enter the response. Suffice it to give the flavor of Q_i for the case when one mode alone is assumed to respond tentatively assuming negligible aerodynamic influences from other modes:

$$Q_i(t) = Q(t) = \rho U^2 B^2 \{ [K\, H_1^*(K)\, G_{hh} + K\, H_2^*(K)\, G_{h\alpha} + K\, A_1^*(K)\, G_{h\alpha} + K\, A_2^*(K)\, G_{\alpha\alpha} + K\, P_1^*(K)\, G_{pp}] \frac{B\dot{\xi}}{U} + [K^2\, H_3^*(K)\, G_{h\alpha} + K^2\, A_3^*(K)\, G_{\alpha\alpha}]\xi + \int_{span} \{ [-C_u h(x) + C_{M1}\alpha(x) + C_p p(x)] \frac{u(x,t)}{U} + [-C_v h(x) + C_{M2}\alpha(x)] \frac{v(x,t)}{U} \} dx \quad (10)$$

with the definitions

$$G_{hh} = \int_{span} h^2(x)\, dx \qquad G_{h\alpha} = \int_{span} \alpha(x)\, h(x)\, dx$$

$$G_{\alpha\alpha} = \int_{span} \alpha^2(x)\, dx \qquad G_{pp} = \int_{span} p^2(x)\, dx$$

$$C_u = C_L(\alpha_o) \qquad C_v = \frac{1}{2}\left(\frac{dC_L}{d\alpha}\bigg|_{\alpha=\alpha_o} + \frac{A}{B} C_D(\alpha_o)\right)$$

$$C_{M1} = C_M(\alpha_o) \qquad C_{M2} = \frac{1}{2} \frac{dC_M}{d\alpha}\bigg|_{\alpha=\alpha_o}$$

$$C_p = \frac{A}{B} C_D$$

Comments on Aerodynamic Response

In principle, with proper section model background experiments, the entire gamut of responses to aerodynamic input can now be calculated, based on the above theory. This is being routinely done on several modern bridges. However, it is often possible, through initial model experiments, to establish bridge deck geometric shapes that are such as to minimize vortex shedding, incipient galloping and flutter [4]. When this type of modern attention to aerodynamic contour treatment is properly paid in the design stage, it typically results in deck sections having $H_1^* < 0$, $A_2^* < 0$, $H_2^* \simeq 0$, $H_3^* \simeq 0$, $A_1^* \simeq 0$, $A_3^* \simeq 0$ for wide ranges of the parameter K. This can considerably simplify the treatment.

For example, the criterion for flutter in the single mode ξ becomes

$$\zeta = \frac{\rho B^4}{2[(M_V+M_L)\, B^2+I]} \{ G_{hh}\, H_1^* + G_{h\alpha} H_2^* + G_{h\alpha} A_1^* + G_{\alpha\alpha} A_2^* + G_{pp}\, P_1^* \} \quad (11)$$

when, by varying K, values of H_i^*, A_i^* (i=1,2) and P_1^* are determined such that the above is satisfied, flutter is present; however, note that if the criteria listed above are already achieved, flutter becomes an impossibility since the righthand side of (11) is intrinsically negative.

There remains only the buffeting problem to examine. This problem remains present even for stable geometric configurations. Many modern bridges that are not flutter-prone have nonetheless not been examined analytically for buffeting. Theory given in [2], [3] and the present paper point out how this calculation can be carried out.

Buffeting Responses

Only the basic outline of this calculation will be reproduced here, the details lying beyond the scope of the present paper. We consider here only the case of buffeting wherein intermodal coupling is negligible.

"Overall" damping $\tilde{\xi}$, including aerodynamic and structural, can be expressed in the form:

$$\tilde{\zeta} = \zeta_i - \frac{\rho B^4}{2[(M_V+M_L)\, B^2+I]_i} \{ G_{hh} H_1^* + G_{h\alpha}(H_2^*+A_1^*) + G_{\alpha\alpha} A_2^* + P_1^* G_{pp} \} \quad (12)$$

and the (slightly) modified natural circular frequency can be given by

$$\tilde{\omega}^2 = \omega^2 \{ 1 \frac{\rho B^4}{[(M_V+M_L)\, B^2+I]_i} H_3^*\, G_{h\alpha} + A_3^*\, G_{\alpha\alpha}] \} \quad (13)$$

Under these conventions, the power spectral density of response ξ_i is given by

$$S_{\xi_i}(\omega) = \frac{1}{(\tilde{\omega}^2-\omega^2)^2 + (2\tilde{\xi}\tilde{\omega}\omega)^2} \left[\frac{\rho U^2 B^2}{[(M_V+M_L)\ B^2+I]_i}\right] S_F(\omega) \qquad (14)$$

where

$$U^2 S_F = \int_{span}\int_{span} [D_u(x_1)\ D_u(x_2)\ S_u(x_1,x_2,\omega) + D_v(x_1)\ D_v(x_2)\ S_v(x_1,x_2,\omega)]\ dx_1 dx_2 \qquad (15)$$

In (15) the definitions are used:

$$S_{\substack{u\\v}}(x_1,x_2,\omega) = S_{\substack{u\\v}}(\tilde{\omega})\ e^{-C(x_1-x_2)/L} \qquad (16)$$

$$C = \frac{16\ \tilde{n}\ L}{U} \qquad (17)$$

where L = span, $\tilde{\omega} = 2\pi\ \tilde{n}$, and S_u, S_v define "standard" wind spectra.

From (14) formulas can be derived (as in [3]) which enable the calculation of the variance of the displacement components h, α, or p. As an example the formula for the single-mode variance $\sigma_h^2(x)$ of the vertical displacement of a bridge curved in plan but with negligible sway is given as

$$\sigma_h^2(x) = \frac{\pi h^2(x)}{4(2\pi\tilde{n})^3\tilde{\zeta}} \left[\frac{\rho B^3 L}{M_V\ B^2 + I}\right]^2 \left[\frac{2(C-1)}{C^2}\right] \times [G_{Du}\ S_u(n) + G_{Dv}\ S_v(n)]\ U^2 \qquad (18)$$

where

$$G_{Du} = \frac{1}{L}[C_u^{\ 2}\ G_{hh} - 2\ C_u C_{M1}\ G_{h\alpha} + C_{M1}^2\ G_{\alpha\alpha}] \qquad (19a)$$

$$G_{Dv} = \frac{1}{L}[C_v^{\ 2}\ G_{hh} - 2\ C_v C_{M2}\ G_{h\alpha} + C_{M2}^2\ G_{\alpha\alpha}] \qquad (19b)$$

Typical results for high excursions $3\sigma_h$ at the center of the hypothetical bridge are listed below for various mean wind velocities at bridge height:

U(mph)	40	50	60	70	80	90	100	110
$3\sigma_h$(ft)	0.43	0.64	0.89	1.27	1.56	1.86	2.26	2.68

Full details on the example bridge being omitted, the above results are merely illustrative as to trends. A very complete expose' of the methods described here, with additional commentary on stability under buffeting, is given in Ref. 5.

Summary

The present paper has sketched, though extremely briefly, the methods and approaches available to the calculation of aerodynamic responses of suspended-span bridges. It has been pointed out how the necessary bases for analytical insights into the problem can be obtained and used. Intrinsic to the method is the postulated use of aerodynamic data developed from bridge deck section models.

The point of the study is that if basic section model data of the proper sort are first made available, extensive prototype response calculations can be made reliably on a theoretical basis.

The methods alluded to briefly herein permit of considerable extensions and generalizations beyond those mentioned. A notable one of these is to demonstrate the fact that flutter of a full bridge under turbulent flow may be delayed to a higher velocity when many modes participate, due to buffeting; and the general buffeting response will exhibit an evergrowing mean square amplitude with increasing mean velocity U.

References

1. Scanlan, R. H. and Tomko, J. J.: "Airfoil and Bridge Deck Flutter Derivatives" Jnl. Eng. Mech. Div. ASCE, Vol. 97, No. EM6, Dec. 1971, pp. 1717-1737.

2. Simiu, E. and Scanlan, R. H.: Wind Effects on Structures New York, John Wiley and Sons. (in press, 1978).

3. Scanlan, R. H. and Gade, R. H.: "Motion of Suspended Bridge Spans Under Gusty Wind" Jnl. Struct. Div. ASCE, No. ST.9. Sept. 1977, pp. 1867-1883.

4. Scanlan, R. H. and Wardlaw, R. L.: "Aerodynamic Stability of Bridge Decks and Structural Members" Proc. Symposium on Cable-Stayed Bridges, Federal Highway Administration Pasco, Wash., Dec. 6-7, 1977.

5. Scanlan, R. H.: "The Action of Flexible Bridges under Wind: I. Flutter Theory; II. Buffeting Theory" Jnl. Sound and Vibration Southampton, U.K. (in press, 1978).