

A CRITICAL EXAMINATION OF THE WAVE EQUATION

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A recent research project sponsored by the Federal Highway Administration produced a new wave equation computer program for the analysis of pile driving (WEAP). While the primary purpose of developing this program was to provide a better model for diesel hammers a number of other improvements were included and an extensive correlation study with dynamic measurements was made. This study together with the authors' extensive field experience pointed out several conditions where wave equation predictions will be inaccurate and unreliable. In this paper the capabilities of the WEAP program will be compared with other commonly used programs. The various factors which can influence the accuracy of a wave equation analysis are considered, evaluated and discussed. The specific topics included are: pile model, soil model, hammer model, and static soil analysis.

The application of the Wave Equation approach to the solution of a variety of pile driving problems has become more and more widespread. For example, the authors have been involved, under a contract with the Federal Highway Administration, in the development of a computer program called WEAP (Wave Equation Analysis of Pile Driving). They also presented a series of eleven seminars around the country as part of FHWA research implementation efforts.

As a result of this work a large number of program copies have been sent out and implementation is proceeding in many State Departments of Transportation and engineering consulting firms. This progress has the advantage of leading to a more realistic preparation of pile driving projects; on the other hand, it may in some instances lead to erroneous conclusions when the basic assumptions of the approach are violated by unusual or unanticipated circumstances.

The Federal Highway Administration sponsored the development of WEAP in response to concerns voiced by wave equation users regarding unsatisfactory results obtained from available programs when applied to diesel hammers. First, it was necessary that the thermodynamic cycle be modeled in complete detail so that combustion chamber forces are determined. Second, since the stroke of a

diesel hammer is dependent on the driving resistance and the dynamics of the pile-hammer system it must be determined during the analysis rather than being treated as an input quantity. The WEAP program satisfies these needs.

It is the purpose of this paper to discuss problems that can arise in wave equation analyses. Since many of these problems occur in cases involving high driving resistance, emphasis will be placed on that aspect of wave equation application.

Basic Approach

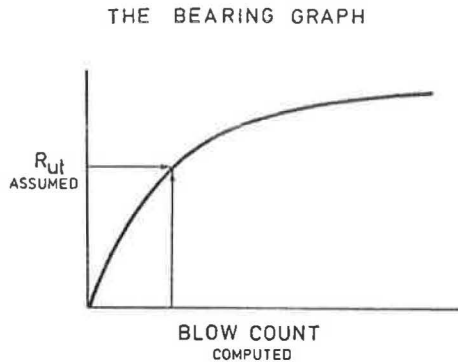
The wave equation method of solving the pile driving problem as devised by E.A.L. Smith in the 1950's is relatively well known to most engineers involved in pile driving. It gives the designer a rational means of designing a pile for driving stresses; it provides a soil resistance versus blow count relation to be used for construction control purposes; and it allows a check on the feasibility of a hammer-pile system, given a certain profile.

Wave equation analysis is usually conducted in the following manner:

1. From a static soil analysis an ultimate capacity versus depth relation is established.
2. For a particular depth the static resistance distribution and the percentage of bottom resistance are determined and the total ultimate resistance, R_{ut} , is found. Wave equation analysis will determine the blow count associated with R_{ut} for the specified driving system.
3. In general, additional analyses are also made for other possible R_{ut} values and a curve, R_{ut} versus blow count (the result of each individual analysis) is constructed. This curve is called a bearing graph and an example is shown in Figure 1.

The bearing graph is usually the desired result as it indicates what capacity has been obtained at a certain blow count or how the pile would drive in the given situation (blow count from soil resistance). The wave equation is not the only source of a bearing graph; for example, the Engineering News Formula or any other dynamic equation can be plotted in the same manner.

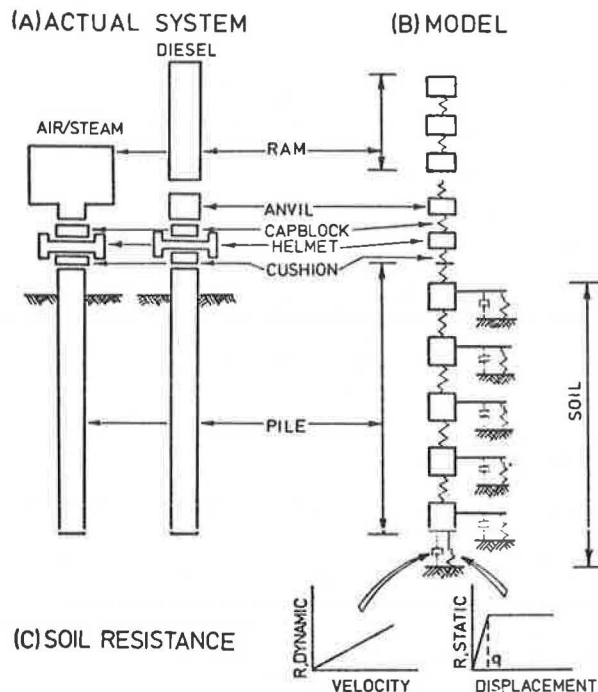
Figure 1. Bearing graph (capacity versus blow count) as constructed from individual wave equation analysis.



Pile Model

The analysis itself is carried out using a discrete spring mass model of the pile as shown in Figure 2. The pile is divided into segments, each two to at most, ten feet long. The pile accelerations are computed from Newton's Second Law for a given set of forces at an element at a certain time.

Figure 2. The Wave Equation representation of pile driving. A. The system to be analyzed. B. The Wave Equation model. C. The components of the soil resistance model.



The pile velocities and displacements are obtained by integration of the acceleration over a small time increment. The pile forces are in turn computed from the segment displacements and the spring stiffnesses. The process is then repeated for a new time increment. More elaborate and accurate computational procedures are frequently used but they will not be discussed here in any further detail.

Soil Model

On each embedded pile segment a soil resistance force is acting during driving. This force is dependent on the soil characteristics and is a complex function of both pile and soil motion. A static soil analysis is performed and an ultimate static resistance, R_u , is obtained for each pile element. The sum of all R_u -values is, of course, the total pile bearing capacity, R_{ut} .

Smith's soil model distinguishes an elastic and a plastic portion of soil behavior. It introduces the "quake" as that displacement at which the elastic, static behavior becomes plastic, i.e. that pile displacement at which the ultimate resistance, R_u , is reached. In addition to the static resistance a dynamic resistance force is also modelled. It is treated as proportional to the pile element velocity. The soil resistance forces acting during driving are functions of element displacements and velocities. The characteristics of these forces are shown in Figure 2. The parameters defining these forces must be provided by wave equation program users.

Hammer Model

The wave equation hammer model can be more or less sophisticated depending on the particular computer program used and on the type of hammer employed. If the program does not permit a realistic treatment of hammer operation it cannot be expected that consistent reliable results will be obtained. For the simplest hammer type, such as the drop or the single acting air/steam hammer, the ram is usually modelled as a single mass, striking a capblock that is modelled by a bilinear spring which in turn exerts forces on a helmet that strikes the pile, sometimes through a cushion.

For diesel hammers, the WEAP program uses several elements to represent the more slender rams typical of these machines. An initial stroke is assumed, the ram is allowed to fall past the exhaust ports and impact on the anvil after going through the precompression phase. The combustion chamber pressures are calculated from the Gas Law and the effect of these pressures on impact velocity is included in the computation. The dynamic analysis is continued after impact until ram separation occurs on the upstroke. The ram motion is then computed including the effect of the combustion chamber pressure and the rebound stroke is determined. If the rebound stroke is different than the starting stroke the computation is repeated until convergence. Thus, ram stroke is available as a function of blow count for use in construction control. The WEAP program is the only available program having this capability.

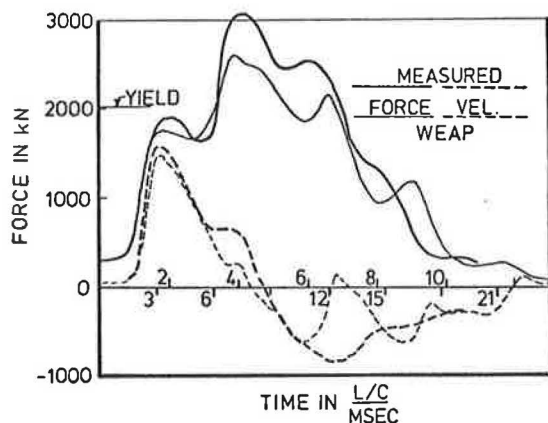
Examples

A few examples of problems that may be encountered when using the Wave Equation approach will be discussed here. It should be understood that these examples are unusual and should not be considered to be proof of a failure of the wave equation but rather an illustration of problems that can occur.

1. The Pile Model

The lumped mass model, representing the pile, is probably the most reliable portion of the wave equation idealization. In the context of high capacity piling the assumption of elastic behavior should be examined. The measured and computed curve of pile top forces shown in Figure 3 were obtained for an HP 10x42 pile driven by a Kobe K25

Figure 3. Pile top forces and velocities both measured and computed by WEAP for a case where stresses exceed yield.



Note: 1 kN = 0.225 kip.

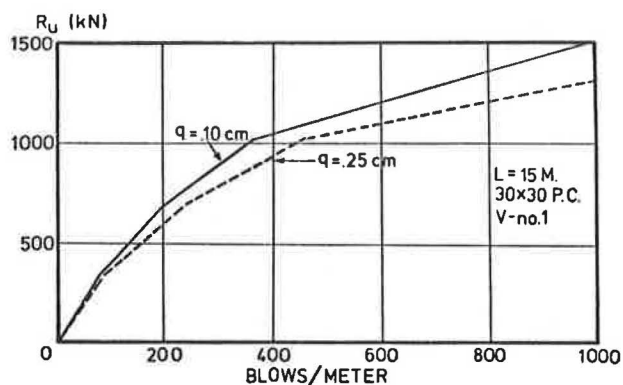
hammer to a hard limestone rock. The measurements were made using strain transducers attached to the pile web. Thus, when yield was reached a force was measured that was higher than actually occurred due to the assumption of a constant elastic modulus. The wave equation showed that the forces at the pile top were smaller since yielding and hence a reduction of the pile strength was not modelled.

2. The Soil Model

Among the various error sources inherent in the simplified soil model used in wave equation analysis those introduced by the assumption of linear elastic - ideal plastic behavior will be discussed. These error sources are of particular interest in hard driving.

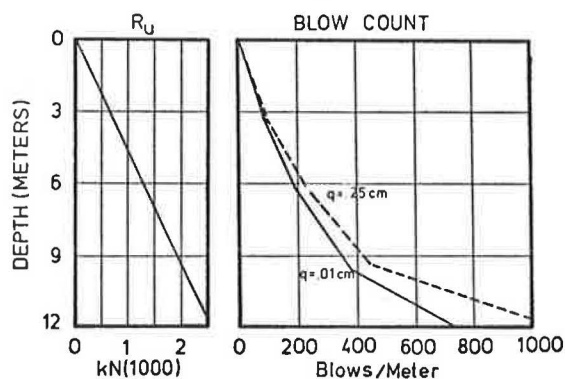
First consider differences in bearing graphs when the quake is changed from the usual 0.25 cm. (0.1 in.) to a low value of 0.10 cm. (0.04 in.). The two bearing graphs are shown in Figure 4. At a capacity of 2670 kN (300 tons) the larger quake gives a 35% higher blow count. The errors introduced are smaller when determining a capacity from a given blow count. If the goal is to estimate blow count at a particular depth the error is

Figure 4. Bearing graphs from WEAP for two different quakes.



larger. This characteristic is illustrated in Figure 5 where the blow counts are plotted as a function of depth for a given R_u versus depth

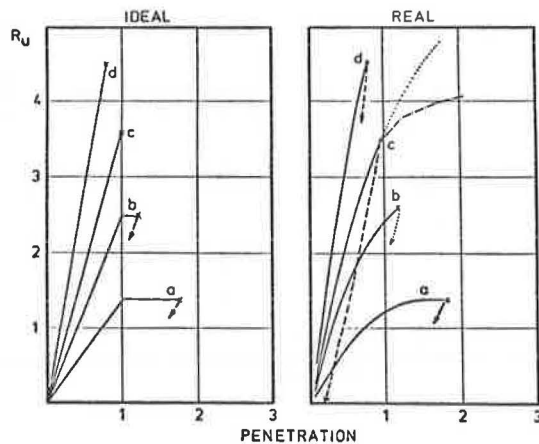
Figure 5. Blow counts versus depth from the two bearing graphs of Figure 4 via an assumed R_u versus depth relation.



relationship. Since the value of the quake is usually not accurately known and since the usual assumption of 0.25 cm. (0.1 in.) has only proven to give good predictions of capacity for "normal" cases, this factor should be considered when the analysis gives high blow counts or refusal.

Another effect may lead to erroneous results in hard driving cases. This effect, resulting from the real non-linear behavior of the soil is of importance when the final sets during driving become very small; say less than 0.13 cm. (0.05 in.) or more than 790 blows per meter (240 blows per foot). In Figure 6 real force deformation curves are shown together with the wave equation idealization. For these cases it is seen that for Cases a and b the idealization approximates the real curve quite well. For the other two cases the results will be poor. The idealized soil law would predict complete refusal (no permanent set) where actually penetrations still occur. This error source limits the applicability of the wave equation approach since R_u becomes unknown for a high blow count. In fact, at high blow counts higher capacities than those predicted by the wave equation are likely.

Figure 6. Nonlinear static soil resistance law - real and idealized.



3. The Hammer Model

Among the various quantities that describe the hammer model the efficiency is probably most important for air/steam hammers. It is very well possible that errors are introduced into results due to an erroneous assumption of hammer energy and that these errors are initially thought to result from improper soil data.

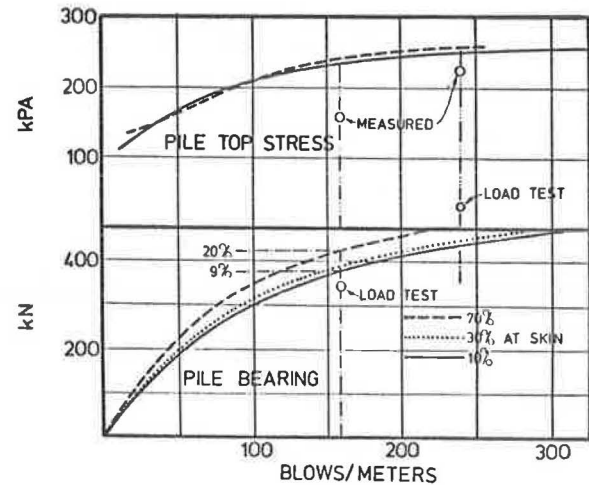
An example is given to illustrate this problem: A 13 meter (43 feet) long pipe pile was driven into coarse grained material that exhibited increasing strength with depth. The hammer was a Vulcan No. 1 single acting air/steam hammer which is commonly thought to supply an energy that is independent of soil resistance. The final blow count was 157 blows per meter (48 blows per foot).

The soil resistance distribution was calculated by a static analysis and the total amount of skin friction was varied using 10, 30 and 70%. Correspondingly three different bearing graphs were obtained that differed from the load test by 9 to 20%. The bearing graphs together with the stress-blow count plots are shown in Figure 7.

The conclusion that an erroneous amount of skin friction was primarily responsible for the difference between dynamic prediction and static capacity is not justified. Note the low magnitude of the measured dynamic stresses suggesting that the hammer was not 80% efficient as assumed in the wave equation input.

A second load test was performed on an almost identical, nearby pile which was driven to 236 blows per meter (72 blows per foot). It indicates a better agreement of stresses and an underprediction of static capacity. Measurements taken during driving of both piles actually did indicate an increase of hammer energy with blow count.

Figure 7. Results from WEAP analyses and from load tests for a pipe pile driven into coarse grained soil.



Note: 1 kpa = 0.145 psi.

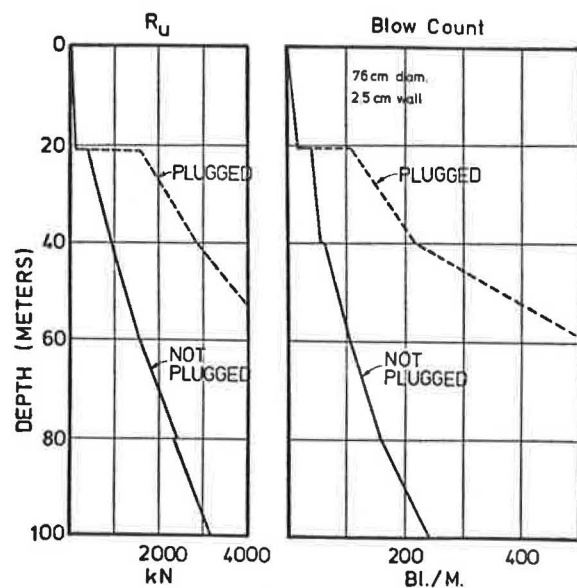
Static Soil Analyses and Driveability

It was shown that an uncertainty in the magnitude of the quake can result in a rather large error in the estimate of the number of blows necessary to drive a pile. In that example of Figures 4 and 5 it had been assumed that the static soil analysis was correct.

Uncertainties in static formulas, however, were the reason why the wave equation approach was developed and accepted. Thus, a driveability study must always be considered with some suspicion since it depends on the ability to make an accurate static analysis.

The next example demonstrates the rather dramatic effect of an unknown resistance force at the pile bottom. The example was taken from a driveability study on an open ended conductor pipe driven into primarily sandy material. The pile had a drive shoe consisting of a portion of pipe having increased wall thickness over the bottom two feet of pile. The pile shoe extended to the inside thus reducing the diameter of the soil entering the pile and the friction on the inside of the pile. Frequently this technique is unsuccessful and a plug may form that activates a soil resistance equal to that of a closed end pile. In Figure 8, blow count versus depth curves are shown for the two cases. A drastic difference results.

Figure 8. R_u versus depth and resulting blow count versus depth from WEAP analysis for an offshore conductor pipe.



Conclusions

Experience with the WEAP program has shown that it can be an effective tool for evaluating pile driving systems. It is particularly useful in dealing with open end diesel hammers since the total thermo-mechanical system is modelled.

Care must be used in applying WEAP or any other wave equation program in that errors in input parameters or cases where the model contained in the program is not a proper representation of the real physical system can produce poor results. Of particular concern are three problems:

1. The program depends on a knowledge of hammer efficiency and since it can vary from hammer to hammer it is unknown in advance.
2. The soil resistance model of elastic-plastic spring and linear dashpot can produce poor results for high blow count driving.
3. The inability to perform an accurate static soils analysis make driveability studies unreliable.