# Estimation of Left-Turn Saturation Flows 

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This paper addresses the problem of estimating left-turn saturation flows at both signalized and unsignalized intersections. The best known methods for estimating this traffic measure were tested for reliability against field data. A new approach to the problem is also presented. The results are applicable to left-turning traffic flowing through gaps of suitable size in the opposing traffic without the protection of a special signal phase. Both one and two lanes of opposing traffic were considered, as were unsignalized intersections. The gap-acceptance functions that best represent the behavioral patterns of left turners at these intersections are also presented. Time-lapse photography was used to collect data at five different intersections in upstate New York. Approximately 4000 completed left-turning movements were observed. Using these field observations, the ability of several existing methods to estimate left-turn saturation flow was tested by standard statistical analysis techniques. Adjustments were made for any divergence from actual conditions. Most of the original models do not reflect real-world conditions, so a new model is proposed for each type of intersection. The results indicate that the gap-acceptance characteristics of left-turners can be accurately described by a uniform cumulative density function.

Although several methods for estimating left-turn capacity at intersections have been proposed, none has been widely accepted by practicing traffic engineers as being truly representative of real-world conditions. May (1) reports that research on left-turning movements rated second in priority over twenty other items of interest related to intersection capacity, in a 1974 survey.

This paper will be concerned primarily with the leftturn capacity of an intersection, which is easily computed from saturation flow. The saturated condition under consideration is illustrated at unsignalized intersections by a stream of left-turning vehicles moving continuously and restricted only by the presence of the opposing through movement. Pedestrian traffic on the cross street that might interfere with vehicles attempting to turn is assumed to be negligible.

Since the flow at signalized intersections is controlled by the amount of green time allotted, the left-turn saturation flow under these conditions is defined as the flow rate of left-turning vehicles that would be obtained if there were a continuous queue of vehicles given 100 percent green time (2). Left-turn capacity is then given by the actual possible number of left turns in one hour, considering the effects of the signal.

Reliable estimates of left-turn saturation flows have several applications in traffic management and design. Such applications include

$$
\begin{aligned}
& \text { y: } \% \\
& \text { is }
\end{aligned}
$$

,
Table 1. Left-turn saturation flow formulations.

1. Decisions concerning the installation of a traffic signal at unsignalized intersections,
2. Determination of optimum signal timing,
3. Determination of optimum signal-phasing arrangements,
4. Estimation of the average queue length used in the design of left-turn bays, and
5. Estimation of the average and maximum delays for left-turning vehicles.

In an effort to obtain estimates that represent actual conditions, several approaches have been taken, resulting in theoretical or semi-empirical solutions to the problem. The most widely known were considered in this study and are summarized in Table 1,
where
$S_{1}=$ left-turn saturation flow in vehicles per hour,
$\mathbf{Q}_{0}=$ opposing flow in vehicles per hour,
$\mathrm{q}_{0}=$ opposing flow in vehicles per second,
$\tau=$ critical gap in seconds,
$h_{0}=$ mean minimum opposing headway in seconds, and
$\mathrm{h}_{1}=$ mean minimum left-turn headway in seconds.
Tanner's model (3) was initially derived for a single lane of opposing vehicles, but it was proposed that the condition of multilane opposing could be approximated by regarding the opposing vehicles as a single stream with an arrival rate twice that of a single lane and onehalf the minimum headway. Webster and Cobbe (2) used these theoretical equations in developing curves for estimating left-turn saturation flows. Estimations of the model parameters such as critical gaps and minimum headways were based on data collected in the field and on the test track. However, validation of these curves with field data is still lacking (4).

Drew (5) derived an equation based on the analysis of gap-acceptance behavior for drivers merging at freeway ramps. Since this model resembles the condition under question, it was later proposed for estimating the leftturn capacity (6). Using observations to obtain appropriate values for two of these three independent variables in Drew's equation, Fambro, Messer, and Andersen (6) suggested a simplified form of the model, which is also presented in Table 1.

| Model | No. of Opposing Lanes | Equation |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Tanner | One | $S_{1}=\left[Q_{0}\left(1-h_{0} q_{0}\right)\right] / \exp \left[q_{0}\left(\tau-h_{0}\right)\right]\left[1-\exp \left(-h_{1} q_{0}\right)\right]$ |  |  |  |  |  |
| Tanner | Two or more | $S^{S_{1}}=\left[2 Q_{0}\left(1-h_{h} q_{0}\right)\right] / \exp \left[2 q_{0}\left(T-1 / h_{0}\right)\right]\left[1-\exp \left(-2 h_{1} q_{0}\right)\right]$ |  |  |  |  |  |
| Webster | One |  |  |  |  |  |  |
| Webster | Two or more | $\begin{aligned} & \mathrm{S}_{1}=\left\{\mathrm{Q}_{0}\left[1-(3) \mathrm{q}_{0}\right]\right) / \exp \left[q_{0}(5)-(3)\right]\left[1-\exp \left[-(2.5) q_{0}\right]\right) \\ & \mathrm{S}_{1}=\left\{2 Q_{0}\left[1-(1) q_{0}\right]\right] / \exp \left[2 q_{0}(6)-1 / 2(1)\right]\left[1-\exp \left[-2(2.5) q_{0}\right]\right\} \end{aligned}$ |  |  |  |  |  |
| Drew | Any | $\left.\mathrm{S}_{1}=\mathrm{Q}_{0}\left(\left[\exp \left(-\mathrm{q}_{0} \tau\right)\right] /\left[1-\exp \left(-\mathrm{q}_{d_{1}}\right)\right]\right\}\right]$ |  |  |  |  |  |
| Fambro, Messer, Andersen | Any | $\mathrm{S}_{1}=\mathrm{Q}_{0}\left\{\left[\exp \left[-\mathrm{q}_{0}(4.5)\right]\right] /\left[1-\exp \left[-\mathrm{q}_{0}(2.5)\right]\right]\right\}$ |  |  |  |  |  |
| HCM | Any | $\mathrm{S}_{1}=1200-\mathrm{Q}_{0}$ |  |  |  |  |  |
| Australian Road Capacity Guide | Any | $S_{1}=1200 f$, where $f$ is given by: | $\frac{\text { Q. } 0}{\text { f } 1.0}$ | 200 | 400 | 600 | $\frac{800}{0.45}$ |

The formulations proposed by the Highway Capacity Manual (HCM) (7) and the Australian Road Capacity Guide (8) are brief and easy to follow. The maximum lefttürn flow rate in vehicles per hour (vph) of 1200 , which corresponds to a minimum headway of $3 \mathrm{~s} /$ vehicle and no opposing traffic, is diminished in proportion to the opposing flow. In the case of the HCM, the left-turn saturation flow is equal to the difference between 1200 vph and the opposing flow, with theoretically zero left turns at opposing flows of 1200 vph . However, in capacity calculations, it is stipulated that the number of turns will not be less than two vehicles per signal cycle, regardless of the upposing flow. The Australian method decreases the left-turn flow rate in a nonlinear manner as the opposing volume increases. However, this method is not applicable to opposing flows greater than 800 vph .

The most obvious advantages and disadvantages of the various methods are summarized in the following table.

| Mode! | Advantages | Disadvantages |
| :---: | :---: | :---: |
| Tanner | Distinguishes between one and two opposing lanes <br> Closed-form solution, easy to apply | Lacks sufficient validation Does not adequately agree with field data in any of the cases studied |
| Webster | Distinguishes between one and two opposing lanes <br> Solution easily obtained from graphs | Lacks sufficient validation Overestimates saturation flow at the lower range of opposing flows Does not adequately agree with field data in any oi the cases studied |
| Drew | Closed-form solution, easy to apply Relative agreement with field data in cases with two opposing lanes | Was primarily developed for estimating merging capacity at entrance ramps <br> Oversimplifies assumptions in deriving the solution <br> Does not distinguish between one or two opposing lanes |
| Fambro, Messer, Anderson | Validated with field data <br> Simple, and easy to apply | An extension of Drew's solution and has similar disadvantages <br> Does not adequately agree with field data in any of the cases studied |
| HCM | Relatively accurate for opposing flows <600 vph Easy to apply | Assumes left-turn saturation flow is zero for opposing flow > 1200 vph Underestimates left-turn saturation flow for opposing flows between 600 and 1200 vph <br> Does not distinguish between one or two opposing lanes |
| Australian Road Capacity Guide | Based on a semiempirical gapeacceptance behavioral model <br> Close agreement in cases with two opposing lanes | Valid only for opposing flows $\leqslant 800 \mathrm{vph}$ Does not distinguish between one or two opposing lanes |

Observations made after the evaluation of the models are also indicated in the table. Since none of the methods presented has been unanimously accepted by practicing traffic engineers, field data were collected and a comparative analysis was performed to determine the models best representing actual conditions (9).

Traffic was observed at both signalized and unsignalized intersections having exclusive left-turn lanes including the effects of opposing traffic in one or two lanes. Although existing methods were found to be fairly realistic in some instances, their application is limited to only
a small number of cases or to a certain range of volumes. The general disagreement of the field data with the theoretical results led to the development of the alternate method presented in this paper. The new method allows a choice among the most suitable of the existing models for a particular case or statistical models developed from the collected data.

Data were also examined microscopically to determine the distribution of gap sizes accepted by left-turners confronted with opposing traffic. Thus, gap-acceptance functions were derived by allowing the estimation of the percentage of drivers accepting a gap of a particular size.

## DATA COLLECTION AND ANALYSIS

By using a time-lapse camera, approximately 4000 completed left-turn movements were observed at five different upstate New York intersections. The test sites were located in a typical suburban environment near central shopping areas. A film exposure of one frame per second was selected as desirable for recording intersection data of this type. A total of 11 h of selected data were collected. This was considered a sufficient sample size for making statistical inferences concerning the general behavior of left-turning vehicles.

Although we initially intended to investigate all possible left-turn conditions, personnel and time constraints precluded the inclusion of left-turn movements from optional through and left-turn lanes. Thus, intersections with the following characteristics were selected:

1. With or without signalization,
2. With opposing traffic moving in one or two lanes,
3. With an exclusive left-turn lane, and
4. Without a separate signal phase for left turns.

In addition to these traffic control conditions, certain other factors were considered in the selection of test sites:

1. Intersection isolation to ensure random arrivals,
2. Sufficiently high left-turn demands to allow continuous left-turn queues during most of the observation periods,
3. Full range of opposing traffic,
4. Grades (a zero grade was sought on all approaches),
5. Clear visibility,
6. Parking (no parking allowed on any approach),
7. Approach speed [an average free-flow speed of $48-56 \mathrm{~km} / \mathrm{h}(30-35 \mathrm{mph})$ was considered in all the cases], and
8. Good pavement conditions and satisfactory pavement markings.

After the selection of test intersections representative of the conditions stated, the most influential traffic variables affecting left-turn saturation flows were identified according to observed findings and an exhaustive literature review. Thus, it was concluded that the dominant independent traffic variables are flow rate of the opposing through movements, critical gap, minimum left-turn headway, and minimum opposing headway.

Among other traffic variables considered were average approach speed and percentage of trucks, buses, and motorcycles. It was found, however, that these variables either have secondary importance or they are indirectly included in the above four.

Time-lapse photography was selected as the most efficient method of data collection because of its clearly superior advantages over other alternatives. The advantages of this system include: time savings, easy op-
eration, low operating costs, reliability of data (precise and complete records), permanent record of data, technical dependability, and minimum personnel requirements. The benefits of the time-lapse photography ensured an exact quantitative account of all traffic variables in question for subsequent reconstruction of the test conditions.

Observations were made at intersections where a queue of left-turning vehicles had formed because of opposing through traffic. The maximum flow rate of the turning vehicles was then recorded for the time period when a queue existed and converted to vehicles per hour of green. Since left-turning drivers at signalized intersections use both the initial green period and the yellow at the end of the phase, there is little lost time. For this reason, opposing flow was measured for the entire time available for making turns while left-turn demands were present. This time included at most the entire green and the yellow clearance intervals.

With the necessary information available and in usable form, a quantitative comparison of existing methods for estimating left-turn saturation flows was carried out. Classical statistical analysis techniques were employed in testing the reliability of the models. The measures used to compare the models were the coefficient of determination ( $\mathrm{R}^{2}$ ) and the standard error of the estimate $\left(\mathrm{S}_{\mathrm{\varepsilon}}\right)$. The F -test, t-test, and chi-square test were also employed for qualitative testing in the analysis.

Using the available data, the existing models were subsequently modified to obtain a higher degree of correlation with the observations. Regression analyses were performed on each of the equations of Table 1 to adjust each model to the data. (Because the Australian model is given by tabulated values, rather than an equation, it was not adjusted.) The general form of the adjusted models is
$\mathrm{S}_{\mathrm{L}}=\mathrm{b}_{0}+\mathrm{b}_{1} \mathrm{X}$
where

$$
\begin{aligned}
S_{L} & =\text { left-turn saturation flow, } \\
X & =\text { original form of the model, and } \\
b_{0} \text { and } b_{1} & =\text { regression coefficients. }
\end{aligned}
$$

In the statistical analysis, the BMDP2R computer program package developed by the University of California (10) was used. This package has been extensively tested in the past and has been widely accepted for statistical analyses of a similar nature.

Because many of the models were too inaccurate, even after the adjustment, a new model was developed for each condition under consideration, using multiple regression to achieve the closest fit to the observed data. By assigning qualitative variables to represent the number of opposing lanes of traffic and the presence of a signal, a composite model was also devised, using the data collected from all the test sites. This general model allows the traffic engineer to easily arrive at a reasonably accurate estimate of the left-turn saturation flow for any combination of roadway characteristics previously deseribed.

In the gap-acceptance study, the probability density functions most widely employed in studies of similar nature were tested with standard statistical tests, and the most appropriate for each case were singled out.

## RESULTS

Based on the results of the statistical analyses performed for the original and adjusted models, the models were ranked according to their ability to match the observed data. A summary of the results is given in Table 2 with the standard error of the estimate ( $\mathrm{S}_{\mathrm{\varepsilon}}$ ) and the coefficient of determination ( $\mathbf{R}^{2}$ ) given for a quantitative comparison. A case-by-case study was performed to detect the significance of signalization and the number of opposing

Table 2. Summary of results.

| Case | Ranking | Model | $S_{\varepsilon}$ | $\mathrm{R}^{2}$ | Ranking | Adjusted Model | $S_{\varepsilon}$ | $\mathrm{R}^{2}$ | Regression Models |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  | Form | $S_{5}$ | $\mathbf{R}^{2}$ |
| 1 | 1 | Drew | 142 | 0.73 | 1 | Fambro | 141 | 0.74 | Polynomial ${ }^{\text {b }}$ | 139 | 0.76 |
|  | 2 | Australian* | 169 | 0.54 | 2 | Drew | 142 | 0.73 |  |  |  |
|  | 3 | Fambro | 175 | 0.59 | 3 | Tanner | 150 | 0.70 |  |  |  |
|  | 4 | HCM | 303 | 0.00 | 4 | HCM | 162 | 0.65 |  |  |  |
|  | 5 | Tanner | 324 | 0.00 | 5 | Webster | 168 | 0.62 |  |  |  |
|  | 6 | Webster | 366 | 0.00 |  |  |  |  |  |  |  |
| 2 | 1 | Australian ${ }^{\text {* }}$ | 168 | 0.50 | 1 | Fambro | 157 | 0.57 | Polynomial ${ }^{\text {b }}$ | 148 | 0.62 |
|  | 2 | HCM | 208 | 0.23 | 2 | Drew | 165 | 0.53 |  |  |  |
|  | 3 | Webster | 221 | 0.13 | 3 | Webster | 165 | 0.53 |  |  |  |
|  | 4 | Tanner | 245 | 0.00 | 4 | Tanner | 170 | 0.50 |  |  |  |
|  | 5 | Fambra | 269 | 0.00 | 5 | HCM | 176 | 0.46 |  |  |  |
|  | 6 | Drew | 341 | 0.00 |  |  |  |  |  |  |  |
| 3 | 1 | Australian ${ }^{\text {* }}$ | 107 | 0.83 | 1 | Tanner | 94 | 0.85 | Polynomial ${ }^{\text {b }}$ | 92 | 0.86 |
|  | 2 | Drew | 136 | 0.69 | 2 | Webster | 104 | 0.82 |  |  |  |
|  | 3 | HCM | 195 | 0.34 | 3 | Fambro | 105 | 0.81 |  |  |  |
|  | 4 | Fambro | 248 | 0.00 | 4 | Drew | 106 | 0.80 |  |  |  |
|  | 5 | Webster | 269 | 0.00 | 5 | HCM | 136 | 0.68 |  |  |  |
|  | 6 | Tanner | 301 | 0.00 |  |  |  |  |  |  |  |
| 4 | 1 | Tanner | 194 | 0.25 | 1 | Fambro | 122 | 0.70 | Polynomial ${ }^{\text {b }}$ | 114 | 0.74 |
|  | 2 | HCM | 214 | 0.05 | 2 | Webster | 133 | 0.64 |  |  |  |
|  | 3 | Webster | 247 | 0.00 | 3 | Tanner | 137 | 0.61 |  |  |  |
|  | 4 | Australian ${ }^{\text {a }}$ | 263 | 0.00 | 4 | Drew | 138 | 0.58 |  |  |  |
|  | 5 | Drew | 286 | 0.00 | 5 | HCM | 143 | 0.57 |  |  |  |
|  | 6 | Fambro | 427 | 0.00 |  |  |  |  |  |  |  |
| $1,2,3 \text {, and } 4$combined | 1 | Australian ${ }^{\text {a }}$ | 206 | 0.48 | 1 | Fambro | 169 | 0.55 | Composite ${ }^{\text {b }}$ | 137 | 0.71 |
|  | 2 | Drew | 237 | 0.11 | 2 | Drew | 172 | 0.54 |  |  |  |
|  | 3 | HCM | 247 | 0.03 | 3 | HCM | 197 | 0.40 |  |  |  |
|  | 4 | Fambro | 280 | 0.00 |  |  |  |  |  |  |  |
| 2 and 4 combined | 1 | Tanner | 217 | 0.16 | 1 | Tanner | 164 | 0.54 | - | - | - |
|  | 2 | Webster | 232 | 0.03 | 2 | Webster | 178 | 0.45 |  |  |  |
| 1 and 3 combined | 1 | Tanner | 300 | 0.00 | 1 | Tanner | 136 | 0.73 | - | - | - |
|  | 2 | Webster | 342 | 0.00 | 2 | Webster | 264 | 0.65 |  |  |  |

${ }^{a}$ Tested for opposing flows $<800 \mathrm{vph}$. ${ }^{\text {b }}$ See Table 3 or the regression models.
lanes in the selection of the best model. However, since most of the methods do not make any such distinction, an overall comparison was also made.

The six tested models are listed according to increasing standard error of estimate and the corresponding decrease in the coefficient of determination for each case. $R^{2}$ values close to zero indicate that the model is unrealistic in estimating left-turn saturation flows. According to these results, Drew's original formulation is very close to the best fit of the data for signalized intersections with two Ianes of opposing traffic. However, when all cases are combined, the standard error of the estimate and the coefficient of multiple determination are 237 and 0.11 vph , respectively, suggesting failure of the model.

The Australian method presented in Table 1 can be used only for opposing flows less than 800 vph , and therefore it was tested for only this range of opposing flows. Observation of the results presented in Table 2 leads to the conclusion that this is the best of the existing methods for two of the four cases studied, assuming the restricted volume range indicated earlier.

For unsignalized intersections with one opposing lane, none of the existing models adequately estimates the observed values. The low coefficients of determination in most of the unadjusted models clearly indicate that it is important to consider more complex formulations for each particular case. Thus, as a first step the existing models were adjusted according to Equation 1, and the resuits are presenteu in Table 3.

In this table, values of $b_{0}$ and $b_{1}$ close to 0 and 1 respectively indicate that the original model is realistic. Each model has individual characteristics, so the ranking of the adjusted models is not identical to that of the unadjusted models. Inspection of Table 2 reveals that most of the models are substantially improved by the adjustment. The most dramatic improvement is observed from the Fambro, Messer, and Andersen model, which is ranked as the best in three of the four cases studied and also in the combined category.

It can also be seen from Table 2 that the results obtained from the best adjusted models are fairly close to those of the polynomial regression models (where $S_{L}$ is left-turn saturation flow, $Q_{0}$ is flow in the opposing arm, and T is critical gap), which are based on a 99 percent confidence level. The polynomial regression models, however, are closer to the observed data for all cases as expected. The regression equations are presented below.

Polynomial Model Case

| 1 | Signalized, two opposing lanes | $\begin{aligned} \mathrm{S}_{\mathrm{L}}= & -0.875 \mathrm{Q}_{\mathrm{o}}+0.000012 \mathrm{Q}_{0}^{2} \mathrm{~T} \\ & +1145 \end{aligned}$ |
| :---: | :---: | :---: |
| 2 | Signalized, one opposing lane | $\begin{aligned} \mathrm{S}_{\mathrm{L}}= & -1.245 \mathrm{O}_{\circ}+0.000014 \mathrm{O}_{\circ}^{2} \mathrm{~T} \\ & +1165 \end{aligned}$ |
| 3 | Unsignalized, two opposing lanes | $\begin{aligned} \mathrm{S}_{\mathrm{L}}= & -0.277 \mathrm{Q}_{\mathrm{o}} \mathrm{~T}+0.000012 \mathrm{O}_{\mathrm{o}}^{2} \mathrm{~T}^{2} \\ & +1172 \end{aligned}$ |
| 4 | Unsignalized, one opposing lane | $\begin{aligned} \mathrm{S}_{\mathrm{L}}= & -0.324 \mathrm{Q}_{0} \mathrm{~T}+0.000012 \mathrm{Q}_{0}^{2} \mathrm{~T}^{2} \\ & +1142 \end{aligned}$ |

where
$S_{\mathrm{L}}=$ left-turn saturation flow in vehicles per hour, $Q_{0}=$ flow in opposing arm in vehicles per hour, and $\mathrm{T}=$ critical gap in seconds.

For all cases combined, the composite model equation is
$\mathrm{S}_{\mathrm{L}}=-0.233 \mathrm{Q}_{\mathrm{e}} \mathrm{T}+0.000015 \mathrm{Q}_{0}^{2} \mathrm{~T}^{2}+126 \mathrm{~L}+103 \mathrm{~S}+995$
where
$\mathrm{L}=0$ if there is one opposing lane and
$\mathrm{L}=1$ if there are two opposing lanes; and
$\mathrm{S}=0$ if the intersection is unsignalized and
$S=1$ if the intersection is signalized.
As mentioned earlier, in addition to the independent variables and the terms which appear in the final form of the equations, others were also considered but they were found to be statistically insignificant.

It is evident from the form of the equations that signalization affects left-turn saturation flow. Further, it is observed that both signalized cases (1 and 2) are consistent in form, as are unsignalized cases (3 and 4). The relative ease with which the independent variables are obtained allows direct field application for all practical purposes.

In order to further simplify the application of the polynomial models, the results were combined to form the single comprehensive model presented above (Equation 2) as the composite model. The effects of signalization and number of opposing lanes of traffic were considered by adding qualitative (dummy) variables to the general form used in the development of the individual models.

Figure 1 illustrates the results obtained from the composite model representing the four cases studied. The four distinct curves verify that there is a significant difference between the left-turn saturation flows at signalized and unsignalized intersections with one or two opposing lanes. It should be noted that the composite model yields values that are very close to those of the first four equations of the polynomial model.

As depicted in Figure 1, the curves corresponding to signalized intersections (cases 1 and 3 ) are shifted up by approximately 100 vph in comparison with the unsignalized pair of curves (cases 2 and 4). This phenomenon is accounted for by the change in driver behavior caused by psychological stress when turning at a signalized junction. Since the driver who is waiting to turn faces the possibility of being delayed for a full cycle if the turn is not completed before the red begins, every attempt will be made to pass through the intersection before the amber period. Thus, the flow of turning vehicles is hastened and the maximum output is increased. It is possible that the driver's critical gap will be decreased in the process, but this is not entirely necessary.

Table 3. Calibration coefficients of adjusted models.

| Model | Equation | Case 1 |  | Case 2 |  | Case 3 |  | Case 4 |  | Cases 1, 2, 3 , and 4 Combined |  | Cases 2 and 4 Combined |  | Cases 1 and 3 Combined |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{b}_{0}$ | $\mathrm{b}_{1}$ | $\mathrm{b}_{0}$ | $\mathrm{b}_{1}$ | $\mathrm{b}_{0}$ | $\mathrm{b}_{1}$ | $\mathrm{b}_{0}$ | $\mathrm{b}_{1}$ | $\mathrm{b}_{0}$ | $\mathrm{b}_{1}$ | $\mathrm{b}_{0}$ | $\mathrm{b}_{1}$ | $\mathrm{b}_{0}$ | $\mathrm{b}_{1}$ |
| Tanner | $\mathrm{S}_{6}=\mathrm{b}_{0}+\mathrm{b}_{1}$ | 306 | 0.794 | 246 | 0.459 | 316 | 0.776 | 39 | 0,702 | - | - | 160 | 0.544 | 307 | 0.787 |
| Webster | $\mathrm{S}_{\mathrm{L}}=\mathrm{b}_{\mathrm{o}}+\mathrm{b}_{\text {l }}$ | 365 | 0.658 | 303 | 0.487 | 292 | 0.666 | 65 | 0.852 | - | - | 223 | Q. 475 | 347 | 0.656 |
| Drew | $\mathrm{S}_{\mathrm{L}}=\mathrm{b}_{0}+\mathrm{b}_{1}$ | -41 | 0.926 | -264 | 0.862 | -115 | 1.070 | -256 | 0.958 | 28 | 0.715 | - |  | - | - |
| Fambro | $\mathrm{S}_{\mathrm{t}}=\mathrm{b}_{0}+\mathrm{b}_{1}$ | -44 | 0.777 | -75 | 0.812 | -106 | 0.827 | -370 | 0.954 | , | 0.684 | - | - | - | - |
| HCM | $\mathrm{S}_{6}=\mathrm{b}_{0}+\mathrm{b}_{1}$ | 459 | 0.414 | 310 | 0.502 | 292 | 0.520 | 61 | 0.630 | 367 | 0.345 | - | - | - | - |

Figure 1. Saturation flows given by composite regression model.


The left-turn saturation flow was observed to be higher with two opposing lanes of traffic than with one, for the same type of intersection (signalized or unsignalized) for any given value of opposing flow. This is due to the ability of the opposing traffic to move simultaneously on two lanes rather than one and results in a larger number of acceptable gaps for the same value of opposing flows. It should be noted that with two opposing lanes it is possible to observe gaps close to zero, but for a given opposing flow, there is a higher probability of larger gaps, so that overall there is more opportunity to turn.

As can be seen in Table 2, the composite model has a fairly high coefficient of determination and a standard error of 137 vph , which is substantially lower in comparison with the existing left-turn saturation flow models. These statistics indicate that the model is reliable for the estimation of the desired values, although the particular polynomial models are slightly more accurate on a case-by-case basis.

The effectiveness of the composite and regression models can also be visualized in Figure 2, in which they are plotted along with the observed data, the best unadjusted, and the worst unadjusted models for comparison purposes. The data correspond to the case of a signalized intersection with one opposing lane. However, it should be noted that similar results were obtained for the remaining cases. Tanner's model is one of the least accurate unadjusted models, and the figure indicates that it generally overestimates saturation flows when opposing traffic is less than 900 vph , while above this value leftturn saturation flows are underestimated. The figure also illustrates that the polynomial models are best for the entire opposing flow range, while the Australian method yields results similar to these models for opposing volumes less than 800 vph . Incidentally, it should be pointed out that the Australian method is not adjusted, since it presents results in a tabular form rather than in a closed-form expression.

Knowledge of left-turn saturation flows allows estimation of the left-turn capacity that can be expected at the intersection. In the case of an unsignalized intersection, the left-turn capacity is simply equal to the saturation flow, since a continuous queue of drivers is capable of turning left without the interruption of a signal. However, at a signalized intersection, the actual number of
cars that can turn left in 1 h is affected by the traffic signal. Thus, considering the discharge time required for the queue formed at the beginning of green, the following equation is applicable (8)
$\mathrm{C}_{\mathrm{L}}=\mathrm{S}_{\mathrm{L}}\left[\mathrm{S}_{0} \mathrm{~g}-\mathrm{q}_{\mathrm{o}} \mathrm{C} / \mathrm{C}\left(\mathrm{S}_{0}-\mathrm{q}_{\mathrm{o}}\right)\right]+3600(\mathrm{~K} / \mathrm{C})$
where
$\mathrm{C}_{\mathrm{L}}=$ left-turn capacity in vehicles per hour,
$\mathrm{S}_{\mathrm{L}}=$ left-turn saturation flow in vehicles per hour of
$\quad$ green,
$\mathrm{S}_{0}=$ saturation flow of opposing traffic,
$\mathrm{g}=$ effective green time,
$\mathrm{C}=$ cycle length, and
$\mathrm{K}=$ average maximum number of turns per phase
$\quad$ change.

Caution should be exercised when the above equation is used in conjunction with the proposed models, since the effects of cars turning ahead of time at the beginning of green or during the yellow interval should be taken into account. Furthermore, Equation 3 suggests that left-turn movements can discharge at rates to saturation flow only after the queues of the opposing traffic, formed at the beginning of green, are dispersed.

## GAP-ACCEPTANCE CHARACTERISTICS

A secondary objective of this study was to determine the probability distributions that best describe the behavior of left-turning drivers at signalized and unsignalized intersections with one or two opposing lanes. Knowledge of gap-acceptance characteristics is needed in traffic simulation, the design of control systems, the computation of delays resulting from left-turning traffic, and so on.

The distributions most widely employed to describe gap-acceptance functions are uniform (trapezoidal) distribution, shifted negative exponential distribution, Erlang distribution, and log-normal distribution (5).

Using time-lapse films, an analysis was performed in order to determine the gap size ( T ) accepted by each

Figure 2. Left-turn saturation flow versus opposing flow at signalized intersection with one opposing lane.

driver. From these data a plot was obtained showing the cumulative number of gaps accepted (expressed in percentages) for gap intervals ranging from 1 to 10 s in 1-s increments. This cumulative function was then compared with the cumulative density functions of the above distributions.

The uniform distribution was found to be a very good description of actual driver behavior in all four cases. The resulting probability density functions are presented in Table 4, and a representative cumulative function is shown in Figure 3.

Chi-square tests were also performed on all equations to test goodness of fit, which was found acceptable at the 99 percent confidence level. The coefficients of determination ( $\mathrm{R}^{2}$ ), which are very close to 1 , indicate that very little error is encountered when describing these data with a linear relationship. The computed standard errors of the estimate revealed deviations within less than 1 percent of the regression line.

It can be safely concluded, therefore, that for this set of intersections the uniform distribution realistically represents the actual gap-acceptance characteristics. Theoretically, then, an equal number of drivers will accept one gap as will accept any other gap between the upper and lower bounds of the equation.

The data also suggested that the first case, signalized intersection with two opposing lanes, has a longer range of acceptable gaps. The span is approximately 10 s long as compared to 9 and 8 s for the other cases, primarily because the larger gaps are required for completion of the turning movement across two traffic lanes. However, when the road junction is unsignalized, the number of opposing lanes of traffic appears to have little effect on the range of acceptable gaps. For instance, the gap accepted by 100 percent of the drivers ( $\mathrm{c}_{1}$ ) and the minimum gap size (c) are almost identical for cases 3 and 4.

## CONCLUSION

The most widely known methods for estimating leftturn saturation flow were analyzed and found, in general, to be unsatisfactory for realistically predicting the saturated conditions observed in the field. Depending on the model and type of intersection tested, the estimates deviate from the actual values by as much as 400 vph . In a few instances, however, the existing models were found to be essentially as good as the regression models derived from the data.

For example, Drew's equation, when used at signalized intersections with two opposing lanes, provides a

Figure 3. Gap-acceptance distribution at signalized intersection with one opposing lane.

very reliable estimate of left-turn saturation flows. The Australian method is fairly satisfactory for signalized intersections with one opposing lane and for unsignalized intersections with two opposing lanes. For the case of an unsignalized intersection with one opposing lane, none of the models was found satisfactory.

Since most of the existing methods failed to match all data satisfactorily, they were subsequently modified to better reflect real-world conditions. The adjusted forms were found to result in substantial improvements over the original models, but even with this improvement estimation of left-turn saturation flows for some of the cuses studied was still unsatisfactory.

For this reason, a new model was developed for each type of intersection studied. From the extensive statistical analysis of the factors affecting left-turn saturation flows, it was concluded that the dominant independent variables are the opposing flow, the critical gap, signalization, and the number of opposing lanes. Other variables uscd in theoretical derivations performed by earlier researchers, such as the minimum opposing and left-turn headways, did not prove to be statistically significant.

Naturally, since the statistical models fit the observed data more closely, one would be inclined to recommend their use over the others. However, in using these models, caution must be exercised, since it can be argued that the collected data represent only a limited number of intersections. It is primarily for this reason that the existing models were not ignored in Table 2.

Use of this table is recommended as a guideline in selecting the appropriate model for a particular situation. The close agreement of some of the existing models with the statistical ones suggests that there should be a reasonable degree of confidence to the collected data. It should be mentioned, however, that further validation of the proposed models is desirable. Validation of the composite model should be of particular interest due to its ability to represent all the cases combined.

Finally, it must be pointed out that the case of multilane opposing traffic can be taken into account in a manner similar to Tanner's (3), i.e., by considering the opposing vehicles as a single stream with arrival rate three times that of a single lane (for the case of three opposing lanes for example) and increased critical gap.

From the gap-acceptance study, it is concluded that cumulative accepted gaps are uniformly distributed over the range of permissible sizes. Inspection of the gapacceptance functions (Table 4) leads to the conclusion that there appears to be insignificant difference in gapacceptance characteristics when the opposing traffic moves in one or two lanes at unsignalized intersections. Finally, critical gaps were found to be shorter at signalized intersections, as expected.

Table 4. Gapacceptance distributions.

| Case | Equation | Conditions | $\mathrm{R}^{2}$ | $\mathrm{~S}_{\mathrm{E}}$ |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | if $\mathrm{T}<2.33$ | 0.98 | 0.040 |
|  | $\mathrm{P}(\mathrm{T})=(\mathrm{T}-2.33) /(12.37-2.33)$ | if $2.33 \leq \mathrm{T} \leq 12.37$ |  |  |
|  | 1 | if $\mathrm{T}>12.37$ |  |  |
| 2 | 0 | if $\mathrm{T}<1.91$ | 0.94 | 0.080 |
|  | $\mathrm{P}(\mathrm{T})=(\mathrm{T}-1.91) /(10.91-1.91)$ | If $1.91 \leq \mathrm{T} \leq 10.91$ |  |  |
|  | 1 | if $\mathrm{T}>10.91$ |  |  |
| 3 | 0 | if $\mathrm{T}<2.70$ | 0.96 | 0.072 |
|  | $\mathrm{P}(\mathrm{T})=(\mathrm{T}-2.70) /(10.80-2.70)$ | if $2.70 \leq \mathrm{T} \leq 10.80$ |  |  |
|  | 1 | if $\mathrm{T}>10.80$ |  |  |
| 4 | 0 | if $\mathrm{T}<2.73$ | 0.87 | 0.070 |
|  | $\mathrm{P}(\mathrm{T})=(\mathrm{T}-2.73) /(10.80-2.73)$ | if $2.73 \leq \mathrm{T} \leq 10.80$ |  |  |
|  | 1 | if $\mathrm{T}>10.80$ |  |  |

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# Signal Cycle Length and Fuel Consumption and Emissions 

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#### Abstract

A microscopic network simulation model (NETSIM, formerly UTCS-1) was used to evaluate the relationship between fuel consumption and signal cycle length. A single intersection was simulated for three scenarios having different traffic characteristics. It was found that the cycle length that minimizes delay also minimizes fuel consumption and hydrocarbon and carbon monoxide emissions. A regression analysis showed that fuel consumption and these emissions are strongly correlated with vehicle average speed but that the relationship is not linear. Differences between the results in this work and previous results are discussed.


Since the passage of the Clean Air Act of 1970 and the oil embargo crisis of 1973, the issues of automobile fuel consumption and emissions have greatly increased in importance. Thus, it has been proposed that more emphasis be placed on the measures of effectiveness (MOEs) for fuel consumption and emissions and on such traditional measures as speed, stops, and delay. Thus, various types of policies affecting traffic flow would be evaluated as to their effect on the fuel-emission MOEs, speed, and so forth.

In recent years a number of authors (1, 2, 3, 4, 5, 6, 7) have addressed themselves to the issue of fuel consumption in urban traffic. Bauer (1) and Courage and Parapar (2) investigated the relationship between signal cycle length and fuel consumption. Lieberman and Cohen (3) and Honeywell (4) addressed the issue of finding the effects of different traffic control strategies on fuel efficiency (measured in distance traveled versus fuel consumed). Evans, Herman, and Laur (5) addressed the problem of relating fuel consumption to other traffic MOEs such as average speed, while Pattersen (6) and Cohen (7) examined the problem of estimating the concentration profile of traffic-generated carbon monoxide at signalized intersections.

Of particular interest are the findings of Bauer (1) and Courage and Parapar (2). The analysis performed
by these authors showed that at an isolated intersection the cycle length at which fuel consumption is minimized is very much longer than the cycle length at which delay is minimized.

In the present study, we shall describe an analysis of this finding that was conducted using the network flow simulation [NETSIM, (8), formerly the UTCS-1] model. Our result differed from others ( 1,2 ) in that fuel consumption and the hydrocarbon (HC) and carbon monoxide (CO) emissions were found to be minimized at approximately the same cycle length as delay. Another finding of interest was that MOE stops did not always follow Webster's expression ( 9,10 ), which predicts that number of stops decreases as the cycle length increases.

A regression analysis was performed to examine relationships between the average speed and MOE fuel consumption and emissions. It was found that there is a strong correlation between these measures but that the relationships are not linear.

## PROBLEM DESCRIPTION AND TECHNICAL APPROACH

In order to isolate the relationship between signal cycle length and average speed, stops, fuel consumption, and emissions, we confined ourselves to the analysis of single isolated intersections.

The initial configuration involved the analysis of a two-phase pretimed signal. In future work, we plan to analyze more complicated situations, in particular multiphase signals and varying geometric configurations.

Our approach to the problem was to use NETSIM as modified to compute fuel consumption and emissions (4). This approach is particularly appropriate for analyzing fuel versus emissions impacts, because it is difficult to measure the former directly in the field and impossible to measure the latter.

