Analysis of Tandem Axle Loads by Elastic Theory

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AASHO Road Test tandem axle loads were analyzed to determine the magnitude of the tandem axle load that causes the same damage as an 80-kN (18 000-lbf/in^2) single axle load. The procedure is outlined and is the same as that used to analyze single axle loads. The essential findings were as follows: One repetition of a 151-kN (34 000-lbf/in^2) tandem axle load appears to cause the same damage as one repetition of an 80-kN single axle load. The relationship of the log of the number of repetitions of load versus the axle load, which is used in Kentucky, appears to be equally valid for single and tandem axle loads. The use of superposition principles and equivalency of repetitions, in combination with terminal serviceability indexes, permits analyses of the tandem-axle-load data and comparison with equivalent Kentucky thickness designs converted to the AASHO structural number. Single-axle-load analyses were combined with the tandem-axle-load analyses and superimposed by Kentucky equivalent designs. The Kentucky designs, based on elastic theory, envelop 91 percent of the AASHO Road Test data. For pavement design, the estimated fatigue for a 20-year design that uses the AASHO damage factors will be reached in 16.2 years if the Kentucky damage factors are used. The Kentucky damage factors more closely approximate observed behavior than do the AASHO factors.

Thicknesses of flexible pavement structures have been determined by several systems that include the Kentucky 1948 and 1959 curves (1, 2), the AASHO interim guide (3), and the Kentucky proposed 1973 design guide (4). The objectives of this paper are (a) to determine the magnitude of the tandem axle load that produces the same equivalent damage as one 80-kN [18 000-lbf (18-kip)] single axle load, (b) to develop appropriate damage factors for tandems by using the above tandem axle load as the base value, and (c) to compare the results obtained by using the above design methods.

BACKGROUND

The 1959 Kentucky design curves were based on empirical tests and observations of pavements that were made in 1948 and 1957. They were generally drawn to separate points representing pavements that were performing satisfactorily from those that were not. The 1959 curves were an extension of the 1948 curves to include the effects of increased traffic volumes.

The 1973 Kentucky design guide is based on elastic theory; the curves (5, 6) are drawn to provide structural thicknesses that have the same strain values. The limiting values of strain were obtained by matching theoretical strain values with field performance data. The matching was based primarily on the performance of a 580-mm (23-in) thick pavement composed of 195 mm (7.75 in) of asphalt concrete and 395 mm (15.25 in) of dense-graded aggregate placed on a soil having a California bearing ratio (CBR) of 7 to withstand a field loading of 8×10^6 applications of 80-kN axle loads.

A stated objective of the AASHO Road Test (7) was to determine significant relationships between the numbers of repetitions of specified axle loads of different magnitudes and arrangements and the performance of different asphalt-concrete pavement structures. The emphasis was on the analysis of pavement structural fatigue and the comparison of load-damage effects.

To make a direct comparison of design thicknesses obtained by the AASHO system (3) and the Kentucky system (4) requires expressing the two systems in equivalent terms. Fatigue data for each pavement section of the test road formed the basis for the AASHO thickness design nomographs. The following equation was used to express the relationship between the number of repetitions (N), the structural number (SN), and the level of serviceability (P_t):

$$\log N = 5.93 + 9.36\log(SN + 1) - 4.79\log(L_x + L_2) + 4.33\log L_2 + (G_t/B_x) \quad (1)$$

where

$G_t = \log[(4.2 - P_t)/(4.2 - 1.5)]$,
$B_x = 0.40 + 0.08(L_x + L_2)^{3.23}/(SN + 1)^{5.19} L_2^{3.23}$
L_x = axle load (kips), and
L_2 = 1 for single axle loads and 2 for tandem axle loads.

Comparison of seasonally weighted data (7) with Equation 1 was deemed appropriate and necessary. The following methodology was used in Kentucky to analyze single-axle-load data from the AASHO Road Test (7, 8):

1. Layer thicknesses for each test section were converted to the corresponding SN by using Equation 2:

$$SN = a_1 d_1 + a_2 d_2 + a_3 d_3 \quad (2)$$

where

d_1, d_2, and d_3 = thicknesses of surface, base, and subbase layers in the pavement structure respectively and
a_1, a_2, and a_3 = 0.44, 0.14, and 0.11 respectively.

(The operations of these equations require the use of U.S. customary units; therefore, SI units are not given in Figures 2 and 7.)

2. Graphs were made of P_t versus N for each section of each loop.

3. Graphs at various P_ts were made for SN versus N (Figure 1) (7, appendix A).

4. The 1973 Kentucky damage factors (DFs) were calculated by using Equation 3:

$$DF = (1.2504)^{(P-18)} \quad (3)$$

where P = single axle load (in kips) and the constant 1.2504 is the slope of the straight-line, single-axle load versus log repetitions relationship; the base or reference axle load was taken to be 80 kN.

5. For a given P_t, the products of N and DF were plotted on the graph for 80-kN data from lane 1 of loop 4. This procedure converts (by superposition) the N of a given axle load to an equivalent N of an 80-kN axle load. The corresponding SN is not changed.

6. For single axle loads, Equation 1 reduces to

$$\log W_t = 5.93 + 9.36\log(SN + 1) - 4.79\log(L_x + 1) + (G_t/B_x) \quad (4)$$

Solutions of Equation 4 were superimposed on the graphs of step 3.

7. Studies had determined that the soil used at the AASHO Road Test had a CBR of 5.2 as determined by the Kentucky CBR test method. Thicknesses (4) corresponding to a CBR of 5.2 and an asphalt concrete modulus of 3.31 GPa (480 000 lbf/in²) were converted to SNs by Equation 2 and superimposed on graphs from step 6. For P_ts of 2.0 and 2.5, Equation 4 approximated a best fit of the data, and the Kentucky design method required thicknesses (in terms of SN) greater than that required by approximately 93 percent of the Road Test single-axle-load fatigue data. For other P_t graphs, the Equation 4 trend line and the Kentucky design curve reasonably fitted some portions, but not all, of any range of data—except for the 80-kN data at a P_t of 2.5.

8. The relationship for single axle loads shown in Figure 2 was used to choose the appropriate P_t versus axle load combination to be superimposed on the 80-kN base graph for a P_t of 2.5 [i.e., to determine the number of repetitions of 80-kN equivalent axle loads (EALs)] (Figure 3).

9. The following observations were made: (a) The 3.31-GPa asphalt-concrete-modulus Kentucky curve

Figure 1. Relationship between number of repetitions of 80-kN (18 000-lbf/in²) single axle loads and structural number at terminal serviceability of 2.5.

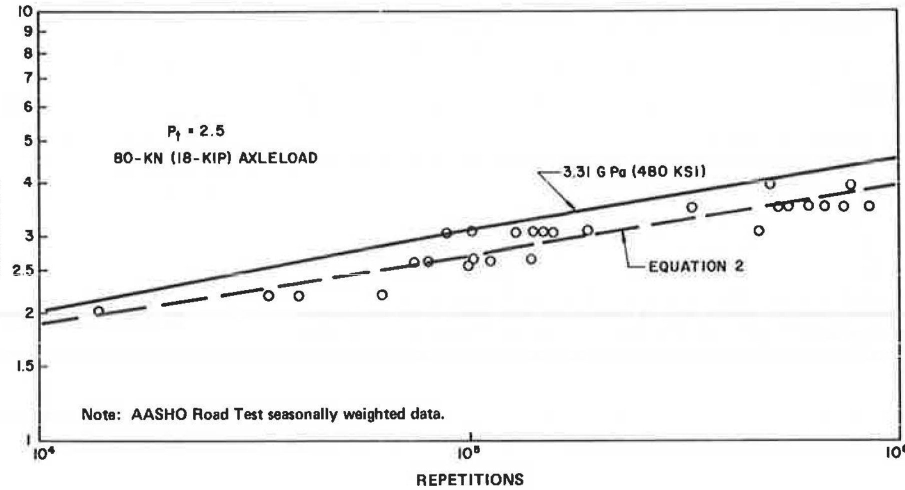


Figure 2. Relationship between level of serviceability and axle load.

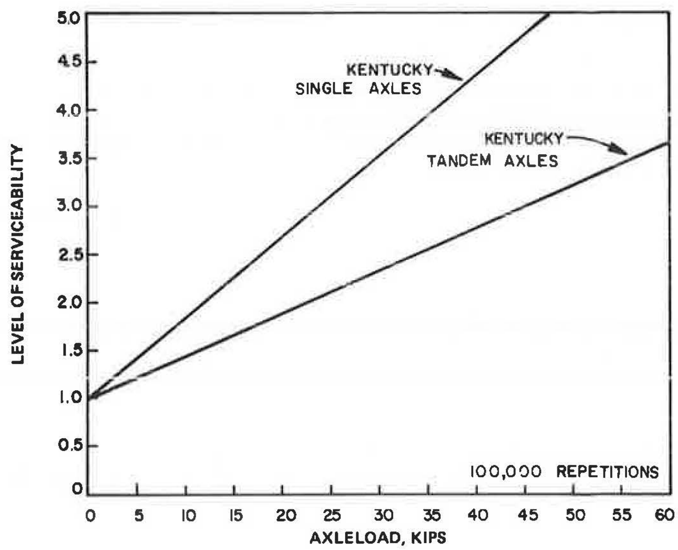
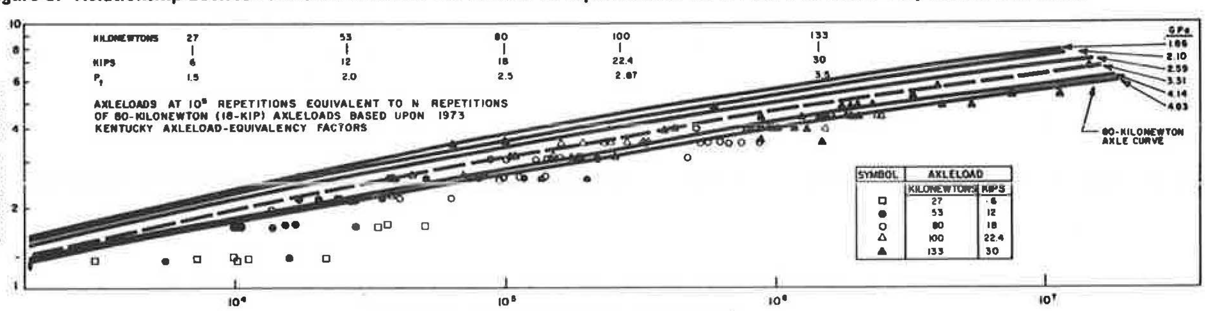


Figure 3. Relationship between structural number and number of repetitions of 80-kN (18 000-lbf/in²) equivalent axle loads.



shown in Figure 3 requires an SN greater than that required by approximately 93 percent of all AASHO single-axle-load data points [Figure 3 also includes Kentucky moduli lines for 1.86, 2.10, 2.59, and 4.14 GPa (270 000, 300 000, 375 000, and 600 000 lbf/in^2)]; (b) the 80-kN single-axle-load curve for P_t = 2.5 from Equation 4 was superimposed onto Figure 3, showing that the SN for 59 percent of the data points was less than that from Equation 4, has the same general shape suggested by the data points [except for the 27-kN (6000-lbf) set], and is almost a direct fit of the family of asphalt-concrete elastic-moduli lines; (c) the combination of a and b confirms that the 1973 Kentucky design guide (4) does provide for variable terminal serviceability levels, as a function of the EAL level, with a high probability of success in the design life; (d) elastic theory provides a sound basis for accurately describing the observed performance data of both Kentucky highways and the AASHO Road Test; (e) one level of terminal serviceability should not be used for a wide range of EALs (as suggested by AASHO thickness design nomographs); (f) AASHO nomographs provide for an approximate 60-percent probability of successful pavement performance throughout the design life; and (g) the semi-logarithmic relationship between single axle load and repetitions that is used in the 1973 Kentucky design guide is not only a reasonable assumption, but has been confirmed by the analysis of the AASHO Road Test data (7).

Kentucky had not developed its own damage factors for tandem axle loads, nor had it determined the magnitude that produces the same damage as one repetition of an 80-kN single axle load. The purpose of this investigation is to provide these answers.

ANALYSIS

Single Versus Tandem Axle-Load Equivalency

Analysis of the single-axle-load data (8) from the AASHO Road Test (7) proved to be rewarding enough to warrant an analysis of the tandem-axle-load data (7, appendix A). AASHO (3) had determined that an 80-kN single axle load and a 147-kN (33 100-lbf) mean tandem axle load both produced the same damage. However, it was noted that the equivalent tandem axle load varied and was a function of SN and P_t. Again, it was deemed appropriate and desirable to investigate the tandem-axle-load data by steps 1 through 3 used to analyze the single axle loads. Thus, the 142-kN (32 000 lbf) tandem-axle-load data were plotted for P_ts of 1.5, 2.0, 2.5, 3.0, and 3.5; Figure 4 (7, appendix A) for P_t = 2.5 is an example. The dashed line is the solution of Equation 1, and the 3.31-GPa line is the same Kentucky solution shown in Figure 1. The similarity between Figures 1 and 4 prompted plotting of the 107-, 178-, and 214-kN (24 000, 40 000, and 48 000 lbf) tandem-axle-load data for the same five values of P_t. The same trends were evident; this is represented by the latter portion of step 8 (except that the reference axle load was 142 kN).

To proceed to step 4, the first task was to determine the tandem axle load that produced the same equivalent damage as an 80-kN single axle load. The criteria to be satisfied were that

1. The level of serviceability must be equal,
2. The same number of repetitions must be used,
3. One value of an AASHO SN must be chosen to evaluate the equivalent effects of single and tandem axle loads, and
4. The tandem-axle-load damage factors must be compatible with the 1973 Kentucky (4) single-axle-load damage factors.

Kentucky has associated 80-kN single axle loads and a P_t-value of 2.5. After determining (8) that the 1973 Kentucky design guide (4) and the AASHO Road Test results (7) were compatible, the same serviceability value of 2.5 was chosen. As shown in Figures 1 and 4, the AASHO Road Test data (7) at a serviceability of 2.5 had a span of repetitions from 1000 to 1 000 000; 100 000 repetitions was chosen for analysis only because that point is the middle of the log scale. From Figure 1, the 3.31-GPa line intersects the 100 000 repetitions line at an SN value of 3.07. Thus, the criteria values were determined.

The 3.31-GPa line exceeded the SN for 93 percent of the AASHO Road Test single-axle-load data (Figure 3) and, as shown in Figure 1, one data point was above the 3.31-GPa line. The 3.31-GPa line was superimposed on Figure 4, and similar figures for a serviceability value of 2.5 and tandem axle loads of 107, 178, and 214 kN were prepared. A line of equal offset to the 3.31-GPa line was constructed such that one data point was above the constructed line. The SN value at 100 000 repetitions was graphically determined and transferred to Figure 5. These four pairs of axle load versus SN values were connected with a smooth curve. Thus Figure 5 shows that, at SN = 3.07, an 80-kN single axle load

Figure 4. Relationship between number of repetitions of 151-kN (34 000-lbf/in^2) tandem axle loads and structural number at terminal serviceability of 2.5.

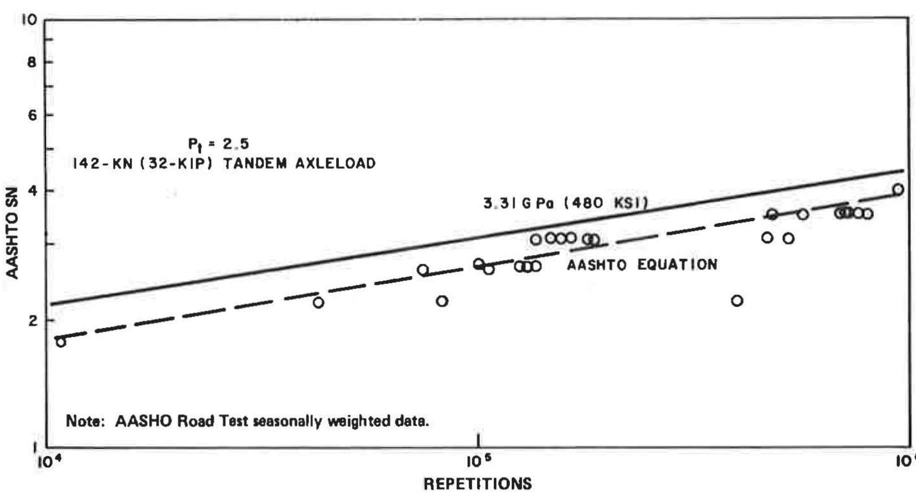
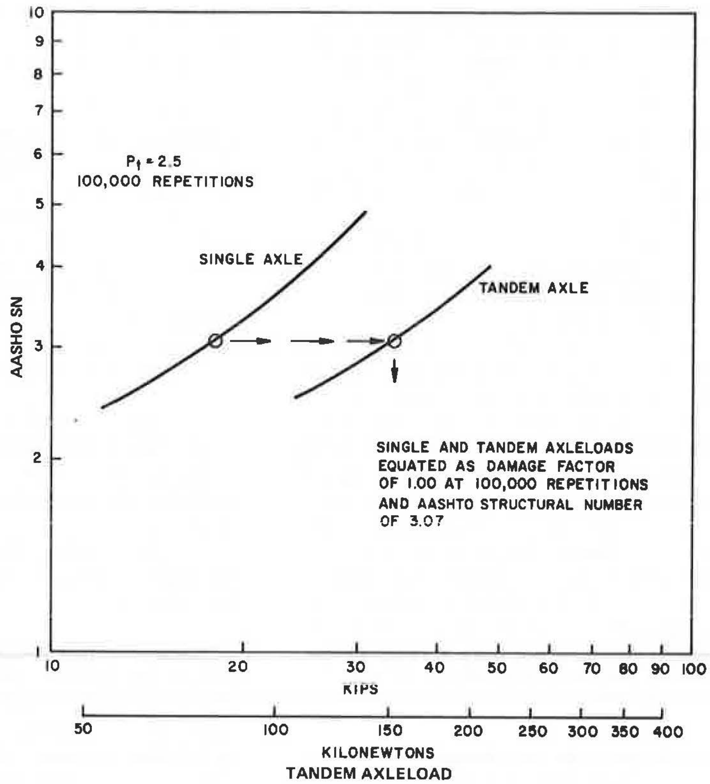


Figure 5. Relationship between structural number and axle load.



would be equivalent to a 151-kN (34 000 lbf) tandem axle load.

Tandem-Axle-Load Equivalency Factors

The tandem-axle-load equivalency factors must be compatible in methodology with the single-axle-load damage factors. Thus, the Kentucky single-axle-load relationship of axle load versus log number of repetitions (Figure 2) required the use of the same axle load versus log number of repetitions format for tandem axle loads. Also, Kentucky has associated 8 000 000 repetitions of 80-kN single axle loads as a reference pavement. Thus, a 151-kN tandem axle load must be associated with 8 000 000 repetitions on the same pavement.

Equation 5 was used to express the single axle load versus number of repetitions relationship labeled as the 1973 Kentucky line in Figure 6:

$$EAL = N(1.2504)^{(P-18)} \quad (5)$$

When EAL =8 000 000 and P (single axle load) = 0, N = 446 654 133. If we assume this same intercept value for a tandem axle load and use the point for a tandem axle load of 151 kN at 8 000 000 repetitions, we find that the resulting straight line in Figure 6 is given by

$$EAL = N(1.1254)^{(P-34)} \quad (6)$$

Here, the constant 1.1254 is the slope of the tandem axle loads versus log number of repetitions relationship. The tandem-axle-load equivalency factors shown in Figure 7 were derived as step 4 from

$$\text{Traffic equivalency factor} = N_{34}/N_p \quad (7)$$

where

N_{34} = number of repetitions for 151-kN axle load and

N_p = number of repetitions for any other given tandem axle load.

Equation 7 can be expressed in the form of Equation 3 as

$$DF = (1.1254)^{(P-34)} \quad (8)$$

The traffic-equivalency-factor relationships for the 1973 Kentucky tandem axle load and the 1972 AASHO interim guide tandem axle load at SN = 5.0 and P_t = 2.5 are shown in Figure 7.

AASHO Serviceability Ratings

Figure 3 is the result of combining data for five pairs of axle-load and serviceability levels (step 5) and is shown in Figure 2 as the line labeled as single axle load. The Kentucky 3.31-GPa line (step 7) and the line from Equation 4 (step 6) are superimposed.

Tandem-axle-load data were processed in the same manner as single-axle-load data except that the reference axle load was 151 kN. In step 6, the data for lane 2 of loop 4 were used as the base graph. In step 8, the tandem relationship shown in Figure 2 was used.

The tandem axle load versus serviceability level relationship shown in Figure 2 was used to determine the serviceability level for the four levels of tandem axle loads used in the AASHO Road Test (step 8). Road Test data (7) were plotted for each axle load (step 3), and the repetitions at the respective serviceability levels were interpolated and plotted in a manner similar to Figure 4. By using the 1975 Kentucky traffic-equivalency factors for tandem axle loads shown in Figure 7, the interpolated data were equated to equivalent 151-kN repetitions as shown in Figure 8. Approximately 89 percent of the AASHO Road Test tandem data required an SN value less than the Kentucky 3.31-GPa line although only 47 percent was beneath the line obtained for tandem-axle-load solutions of Equation 1.

Figure 6. Relationship between number of repetitions of load and axle-load equivalencies.

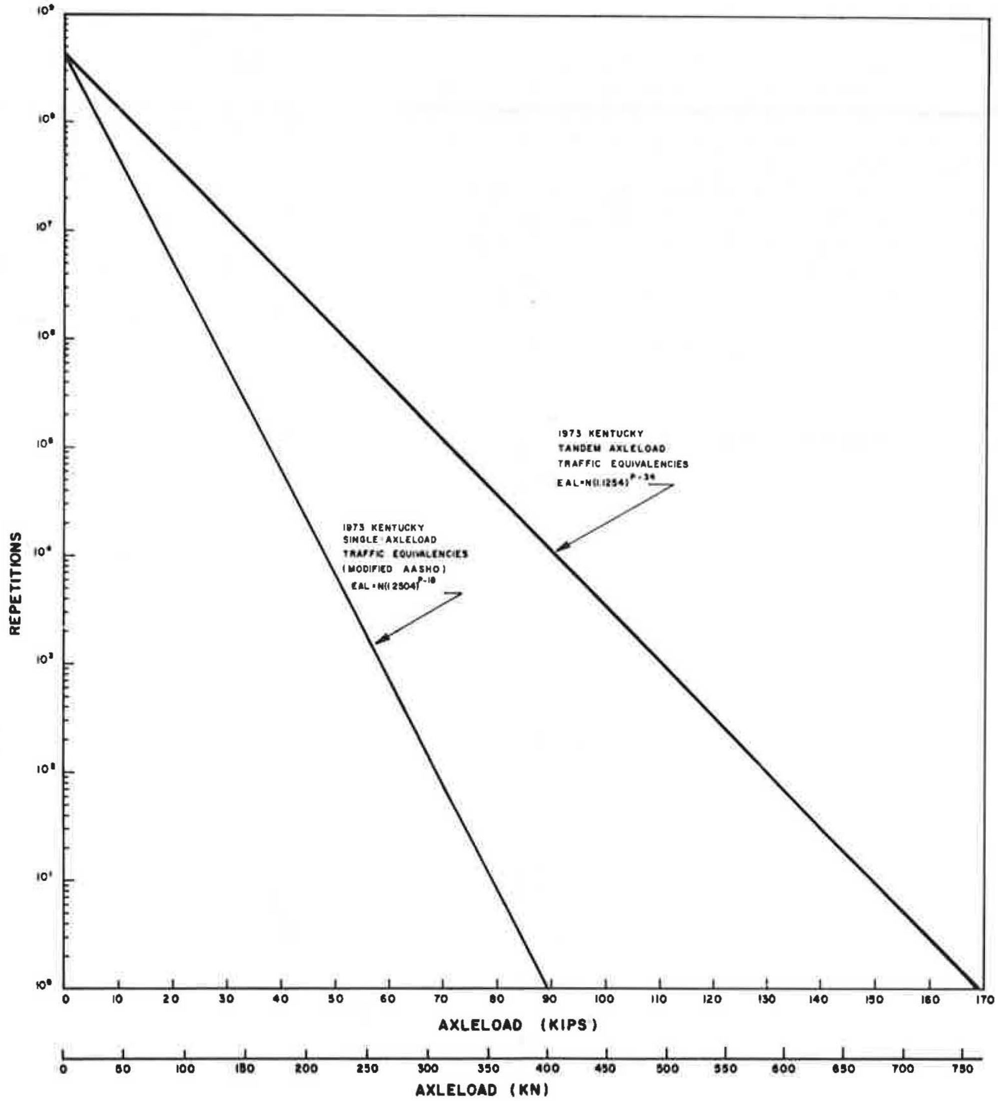


Figure 9 is a combination of single- and tandem-axle-load data from Figures 3 and 8. The Kentucky 3.31-GPa line is an envelope for 91 percent of the combined AASHO Road Test data; the Equation 1 solutions require an SN greater than that required by approximately 50 percent of the data.

Net Effect of Kentucky Traffic-Equivalency Factors

To determine the net effect of the new, Kentucky single- and tandem-axle-load traffic-equivalency factors, data in the W-4 tables for 1959 through 1973 were analyzed. The number of axle loads of various groupings of loaded and empty trucks and truck combinations of each type weighed were summed by axle-load groups within the single- and tandem-axle-load divisions. These sums were multiplied by the corresponding traffic-equivalency factors for the 1959 and 1973 Kentucky design systems and the 1974 AASHO interim guide factors for SN = 5.0 and $P_t = 2.5$. The total number of accumulated EALs are shown in Figure 10 and indicate that the AASHO (3) factors would produce a 20-year design that would be reached within 17.5 years by using the 1959 Kentucky factors and 16.2 years by using the 1973 Kentucky factors.

Until this current analysis, there has not been a set of tandem-axle-load traffic-equivalency factors for application in Kentucky. The practice has been to weigh the individual axles of the tandem arrangement and use the single-axle-load equivalency factor for each axle load. Thus, the calculated total accumulated EALs have been higher than the 1974 AASHO values but less than the 1973 Kentucky values.

SUMMARY

The AASHO Road Test tandem-axle-load data were analyzed by using the procedure reported previously (8) for a single-axle-load analysis. The following are the major points in this report:

1. All other factors being equal, one 151-kN tandem axle load produces the same fatigue damage as one 80-kN single, rear axle load.
2. The logarithm of the number of repetitions versus axle load relationship used by Kentucky (5, 6) appears to be equally as valid for tandem axle loads as for single axle loads.
3. A correlation between serviceability index and

axle load was developed for single and tandem axle loads.

4. The combination of a terminal serviceability that varies with axle load and the equation for single and tandem axle loads permits combining the AASHO Road Test single- and tandem-axle load versus number of repetitions data (Figure 9).

5. The superpositioned Kentucky design curve requires SN values greater than the SN for 91 percent of the combined single- and tandem-axle-load data from the AASHO Road Test.

6. Pavements designed by using the 1974 AASHO thickness design nomographs have been shown to reach the design fatigue level in approximately 80 percent of the expected life when adjusted for increased volume. This discrepancy is explainable by the 1973 Kentucky axle-load damage factors.

RECOMMENDATIONS

The AASHO damage factors used in the Kentucky W-4 tables should be replaced with the following factors:

1. For single axle loads—damage factor = $(1.2504)^{(P-18)}$.

2. For tandem axle loads—damage factor = $(1.1254)^{(P-34)}$.

Figure 7. Traffic-equivalency-factor relationships.

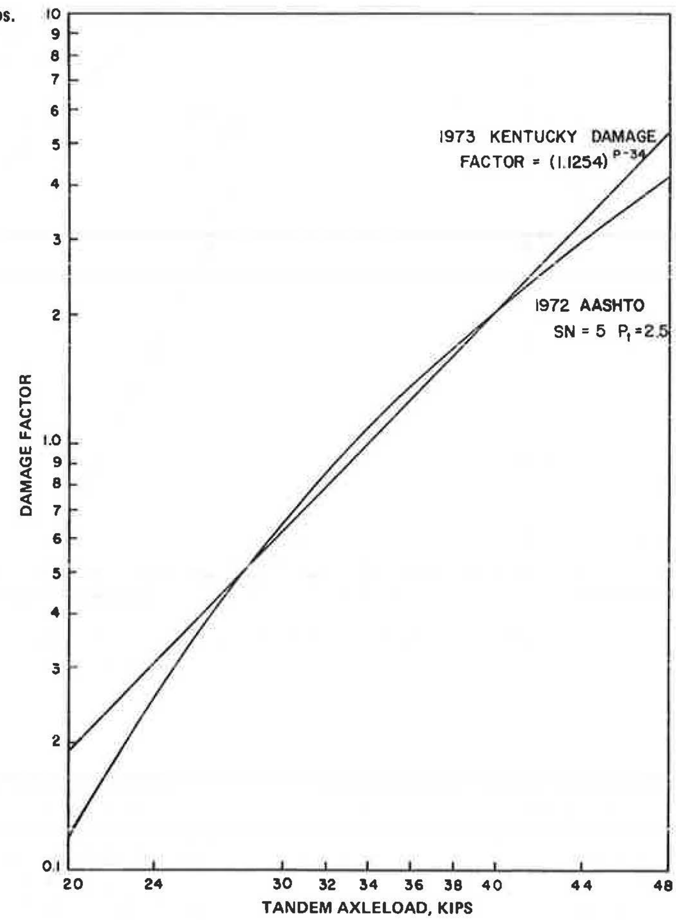


Figure 8. Relationship between structural number and number of repetitions of 151-kN (34 000-lbf/in²) tandem axle loads.

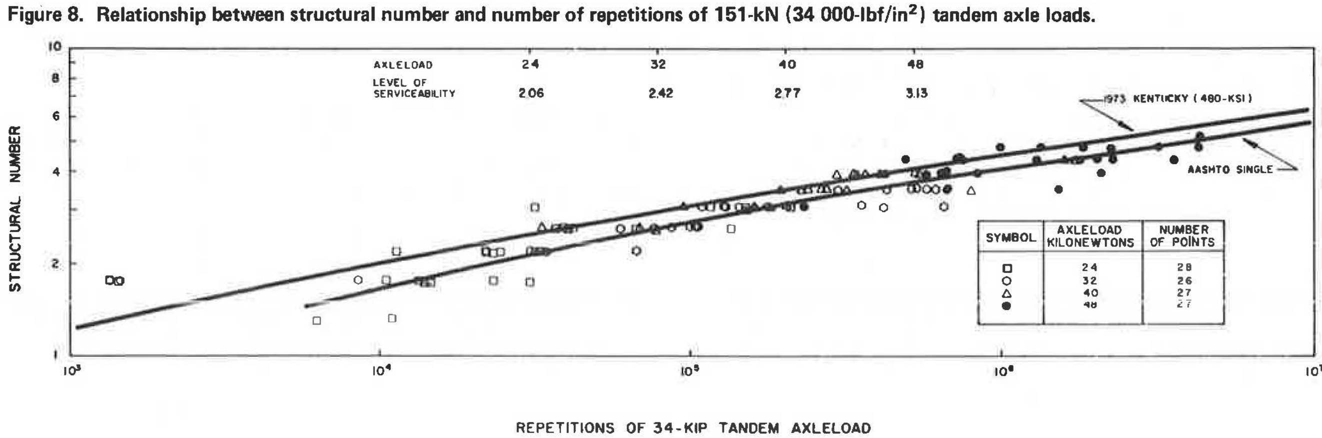


Figure 9. Relationship between structural number and number of repetitions of 80-kN (18 000-lbf/in²) single or 151-kN (34 000-lbf/in²) tandem axle loads.

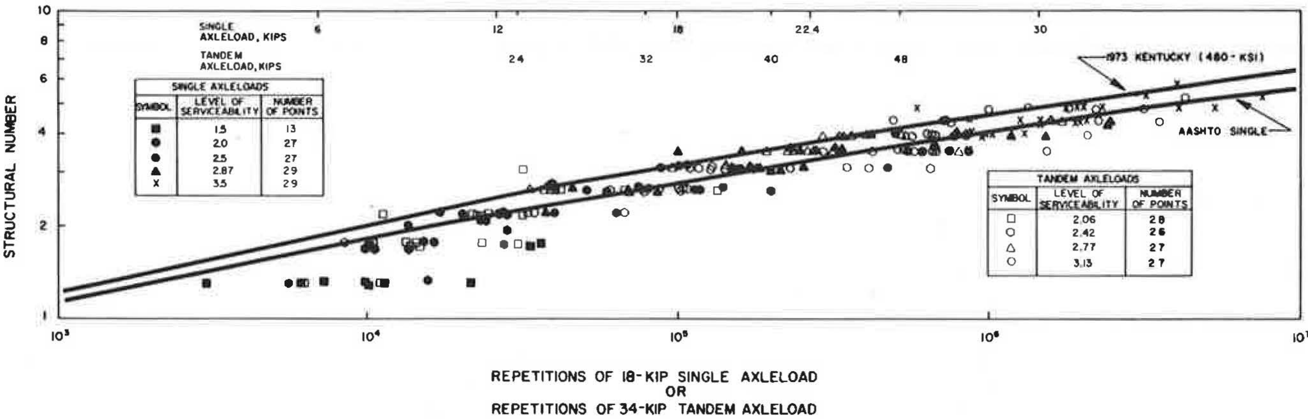
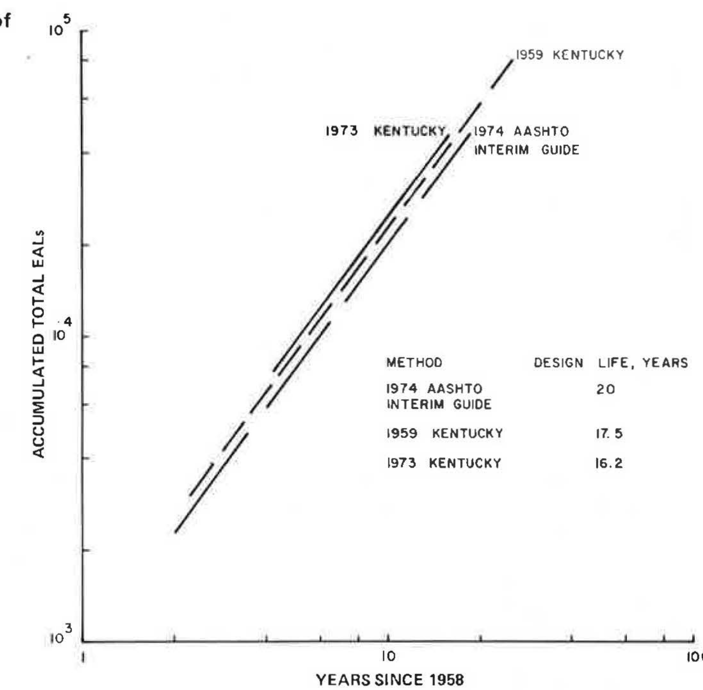


Figure 10. Relationship between accumulated total number of equivalent axle loads and time.



ADDITIONAL RESEARCH IN PROGRESS

Current efforts beyond the scope of this report use superposition principles, elastic theory, and strain-energy equations to analyze the AASHO Road Test loads and configurations of wheel loads applied to various pavement thicknesses to separate the fatigue effects caused by front axles from those caused by rear axles. Increased use of wide tires on the front axle and of trailers having three or four axles in one group will require additional analyses to determine the appropriate equivalent loads and associated damage factors.

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