Using Functional Measurement to Identify the Form of Utility Functions in Travel Demand Models

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Existing statistical procedures for model estimation that use data on observed behavior focus principally on the problem of estimating the parameters of the model, given the functional form. In contrast, methods for measuring function, which are psychological measurement procedures that use laboratory or interview data, provide a powerful set of tools for diagnosing the functional form of behavioral relationships. This paper explores the potential of a synthesis of these approaches in which functional measurement is used to guide travel demand model specification. A case study on choice of residential location by rural workers provides evidence that a model form based on functional measurement gives a better specification than the typical alternative functional forms used in travel demand models. Although it is relatively limited in scope, the case study strongly suggests that functional measurement methods can improve demand model specification.

Estimating a travel demand model can be viewed as two interrelated problems: first, the development of a specification, or functional form, that describes the process of interest; second, the estimation of the parameters of that function. However, as Gaver and Geisel (1) point out, the existing literature on model estimation is oriented heavily toward the problem of estimating a set of parameters given a functional specification and offers only limited insight into how to select the appropriate specification in the first place. Available statistical approaches such as those suggested by Box and Cox (2) and those reviewed by Ramsey (3) rely either on statistical tests of goodness of fit or on a more general functional form that has other, simpler forms as restricted versions.

For example, Box and Cox suggest a transformation of the form

$$f(y) = \begin{cases} (y^{\lambda} - 1)/\lambda, \text{ if } \lambda \neq 0 \\ \log y, \text{ if } \lambda = 0 \end{cases} \quad (1)$$

One can show that in the case of $\lambda = 0$, the expression $(y\lambda - 1)/\lambda$ converges to log y. Clearly, the case $\lambda = 1$ corresponds to a linear transformation. By estimating λ (as well as the other parameters of the model), one can obtain insight into whether linear, logarithmic, or other transformations are appropriate and test whether the value of λ is significantly different from any given value.

Unfortunately, these transformations usually complicate the computational problems of model estimation greatly, and it is generally infeasible to test the functional form of every single variable and all possible combinations of variables. Most travel demand models, because of the availability of relatively inexpensive model estimation procedures, have been restricted to specifications that have linear parameters, i.e., models of the form

$$\begin{aligned} f(x) &= x\beta \\ &= x_1\beta_1 + x_2\beta_2 + x_3\beta_3 + \ldots \end{aligned} \quad (2)$$

This restriction is not particularly burdensome if one already knows that a particular nonlinear specification is appropriate, since, by judicious use of piece-wise linear forms and nonlinear transformations of the dependent and independent variables, one can approximate most nonlinear parameter functions fairly well. However, lacking guidance as to the appropriate functional form, and given, with existing techniques, the virtually infinite number of candidate transformations, choosing among specifications on goodness-of-fit criteria is far more likely to lead to one of the numerous incorrect specifications than to the correct one.

In contrast to the existing literature in econometrics, work in the areas of functional measurement, information integration theory, conjoint analysis, and direct utility assessment has been deeply concerned with questions of functional form. Studies that rely on these theories typically use laboratory-based experiments in which subjects are asked to make judgments about hypothetical alternatives. For example, subjects are confronted with a number of possible modes, each with an associated travel time, travel cost, etc. They are then asked to select a most preferred alternative, to rank the alternatives, or to associate some level of utility with each option.

Because a given subject can be asked to make a fairly large number of judgments in a single interview, the designers of these experiments can explore how individuals' responses are affected as a single independent variable changes while all other variables are held constant. This capability allows for a much more detailed assessment of the functional form of peoples' preferences, since, in an intuitive sense, the designer of the experiment can trace the shape of peoples' responses along each variable.

The fairly extensive experience with one of these psychological techniques, functional measurement, indicates that for any particular decision the functional form of peoples' preferences tends to be fairly stable across the population, even though the parameters of the function may vary widely (4, 5, 6, 7, 8). A key unresolved question, however, is whether the functional form derived from laboratory-based experiments is also a relevant model for actual decision making. If this is the case, then it would seem reasonable to develop a demand model estimation strategy that synthesizes the best features of both econometric estimation of revealed preferences (actual behavior) and function measurement (or some other, related technique) of laboratory or interview data. One could begin any demand model estimation by first performing a functional measurement experiment on a small sample; then, by using the resulting functional form as the starting point, one could estimate a demand model on data from real-world decisions.

This paper explores the potential of this two-phase demand model development technique in a small but

fairly suggestive case study. The next section describes the general methodology of the case study, followed by a discussion of the theory of functional measurement and its application in the case study respectively. The next two sections are parallel to the second and third in that they describe theory and empirical results respectively, except that they examine models estimated on revealed preference data. Since most readers are likely to be far more familiar with travel demand modeling methods based on revealed preferences than functional measurement, the theory of the latter technique is developed in far greater detail than that of the former.

The conclusion is an evaluation of the implications of the study and discusses the potential of the two-phase model-building technique for improving travel demand models.

METHOD OF ANALYSIS

In order to test the two-phase demand model formulation procedure, it was necessary to obtain two sets of data, one in which the behavior of interest was examined in a controlled experimental format and a second in which the corresponding real-world behavior was measured.

The data used represent the spatial choices of nonlocal workers in the rural West. The actual real-world behavior data are on nonlocal workers who take employment in the rural West and must select some town near the place of employment as a place of residence. These data were taken from surveys of power plant employees at six sites in Wyoming, North Dakota, and Montana. In all but one case, a complete record of each employee's residence was available; in the one exception, a 20 percent sample of employees was used. In total, approximately 9000-10 000 employees were included in the sample. The data used in the functional measurement experiment were based on surveys administered to a nonrepresentative sample of students, staff, and faculty at the University of Wyoming.

This case study examines a two-phase analysis in which (a) the functional form of a utility model is assessed in a controlled experiment, and (b) the parameters of this form are re-estimated by using data from actual decisions and are compared to more traditional linear parameter forms of logit choice models.

A major limitation of the study is that it deals with a relatively simple, two-variable model of behavior and a limited case study population. The results should be viewed, therefore, as a pilot test of a methodology rather than as a demonstration of a substantive result.

It is necessary to assume that individuals share a common form for their utility functions but that the parameters of those functions may vary widely. Empirical evidence available in psychological studies of judgment and decision making supports the existence of a common utility function for individual decision makers (4, 5, 6, 7, 8, 9, 10, 11, 12, 13). Thus, this assumption does not appear unreasonable. If such a function exists, it should be possible to infer (at least approximately) its functional form from relatively small, nonrepresentative samples. The parameters of these functions will differ, of course, from those of the population at large, but the form should be relatively invariant across subpopulations.

If the utility function derived from a laboratory study has any relevance to actual behavior, then a model estimated on data from actual decisions, or revealed preferences, should have the same functional form. In theory, if people behave as if they actually use the laboratory-derived functional form, then that specification should fit the actual data better than alternative specifications.

We shall demonstrate this to be approximately true in the case study to be detailed by showing that the functional measurement model is consistently superior, in terms of goodness of fit, to all functions usually assumed in existing choice models. A comparison of R^2-values adjusted for degrees of freedom and the standard errors of the models is made for alternative models.

While it would be better to compare alternative models based on how well they forecast behavior in an independent sample, data necessary for such a comparison were unavailable for this study.

OVERVIEW OF FUNCTIONAL MEASUREMENT

As used in this context, the term "functional measurement" describes an approach to modeling individual behavior that is characterized by two aspects: (a) functional measurement based on an explicit theory of how people reach decisions and (b) use of laboratory experimental measurement methods to estimate models rather than observations on peoples' revealed preferences.

Functional measurement is based on theoretical and empirical research in mathematical psychology and related fields, where there is extensive support for the following assumptions.

$$x_{ki} = f_{ki}(X_{ki}) \tag{3}$$

$$U_i = g_i(x_{1i}, x_{2i}, \ldots, x_{Ki}) \tag{4}$$

$$b = h(U) \tag{5}$$

where

- X_{ki} = physically measurable attributes of the alternative under study,
- x_{ki} = values of X_{ki} as perceived by individuals,
- U_i = some level of response (such as numerical judgments, rankings, or choices) observed in an experimental context for alternative i (for the purpose of this paper, we shall refer to this response as utility),
- U = vector $U_1, \ldots, U_I$,
- B = an actual choice or behavior in a nonexperimental situation,
- I = number of available alternatives, and
- K = number of variables.

In many cases, X_{ki} may include factors for which corresponding x_{ki} are difficult to measure or not well understood. For example, automobile safety may affect a person's choice of auto type, but its physical referrents are not well known. Such factors are treated in our theory as distinct, qualitative variables that are part of x_{ki}.

As developed above, this theory allows for responses, perceptions, and behavior over any set of discrete alternatives, indexed as i = 1, ..., I. For example, one might be interested in mode-choice behavior, in which there are different factors influencing the desirability of driving alone, carpooling, taking transit, etc. In many situations, however, the behavior of interest is continuous and involves only one alternative. In these instances, the theory can often be reduced to the case I = 1, and the i subscript can be deleted. However, because in this case study we are concerned with peoples' choices among discrete alternatives, we will retain the full notation except as noted.

Each of these assumptions is restated more formally below, and the case of additive and multiplicative utilities is explored in detail.

Assumption 1

For any observed travel behavior there exists a set of independent factors that are functionally connected to its occurrence or the magnitude of its occurrence. Each factor may be either quantitative or qualitative in nature. We shall denote the set of J quantitative factors by $S_i = (S_{1i}, S_{2i}, \ldots, S_j)$ and the set of L quantitative factors by $Q_i = (Q_{1i}, Q_{2i}, \ldots, Q_L)$; J + L = K. The entire vector X_i is simply S_i and Q_i.

Assumption 2

Associated with each quantitative and qualitative factor is a corresponding value or quantity of its magnitude that may be obtained by one of several psychological measurement procedures. We shall let the utility of this quantity provided by one or a group of subjects be $(u_{1i}, u_{2i}, \ldots, u_{Ki})$. Because there may be K different values or corresponding utilities for each of the K factors, we may represent the utilities as u_{ki}. Formally, we postulate that

$$u_{ki} = f_{ki}(x_{ki}) \tag{6}$$

Assumption 3

In an experimental context we observe a response to a combination of $(S_{1i}, S_{2i}, \ldots, S_{Ji}, Q_{1i}, \ldots, Q_{Li})$ on a psychological measurement scale. We assume that this response measure is connected to the utility of the experimental factors according to some algebraic combination rule. If we agree to let U_i represent the response to the i th alternative,

$$U_i = g_i(x_{1i}, x_{2i}, \ldots, x_{Ki}) \tag{7}$$

The vector of responses (U) is connected to the observed travel behavior by means of some algebraic function. Hence, if we agree to call the observed behavior B, then we can write

$$B = h(U) \tag{8}$$

Then by substitution

$$\begin{aligned} B &= h(U) \\ &= h[g(x)] \\ &= h\{g[f(S,Q)]\} \end{aligned} \tag{9}$$

This is too general a formulation for modeling purposes. In a practical application, one must make explicit assumptions about f, g, and h and deduce their consequences. The results lead to a general paradigm for the analysis of travel behavior that has growing empirical support (14, 15).

THEORY DEVELOPMENT

The critical component of this theory for the purposes of developing appropriate functional forms for travel demand models is the specification of $U_i = g_i(x_{1i}, x_{2i}, \ldots, x_{ki})$. Analysis of variance provides a straightforward means of implementing the theory and diagnosing and/or testing alternative functional forms. In this study, we will consider both the linear and multiplicative cases.

There are two key conditions involved in the application of analysis of variance that must be satisfied. First, the pattern of the statistical significance (or nonsignificance) of the utility responses to various combinations of the independent variables must be of a specific nature so as to permit inference (diagnosis) or testing of model form. Second, corresponding graphical evidence must support the inference or test.

Consider the hypothesis that individuals in the experiment outlined above will trade off distance (or travel time) and town size (or amenities) independently of one another. That is, they will combine the effects of these two variables linearly. This hypothesis may be tested directly by an analysis of variance. If for clarity we suppress the subscript i and write

$$U_{mn} = U_m^1 + U_n^2 + \epsilon_{mn} \tag{10}$$

where

- U_m^1 = utility values assigned to the m th level of the first factor (say, distance) in a factorial experimental plan,
- U_n^2 = utility values assigned to the n th level of the second factor (say, town size),
- U_{mn} = overall utility assigned by individuals to combinations of levels of factors one and two, and
- ϵ_{mn} = random error term with zero mean.

The test for independence of the two effects (distance and town size) corresponds to the test of the significance of the interaction effect of $U_m^1 \times U_n^2$. In an analysis of variance, this is a global test for any and all interaction effects between distance and town size. If the interaction is not significant (i.e., the hypothesis that U_m^1 and U_n^2 combine linearly cannot be rejected), then the linear form may be accepted. If the interaction is significant, it signals that some form other than a simple linear combination is appropriate.

This test is accompanied by a graphical plot of the interaction. If the hypothesis of linearity is correct, the data should plot as a series of parallel lines when plotted against either U_m^1 or U_n^2 values on the abscissa.

To see why, assume the linear form to be correct and consider the effect of subtracting level 1 from level 2 of the first factor. This yields

$$\begin{aligned} U_{2n} - U_{1n} &= (U_2^1 + U_n^2) - (U_1^1 + U_n^2) + (\epsilon_{2n} - \epsilon_{1n}) \\ &= U_2^1 - U_1^1 + (\epsilon_{2n} - \epsilon_{1n}) \end{aligned} \tag{11}$$

where U_1^1 and U_2^1 are the utility values assigned to levels 1 and 2 of factor one, respectively. Thus, the difference between the points when U_n^2 takes on any value is always a constant $U_2^1 - U_1^1$ (except for disturbances). Hence, the graph should yield a series of parallel lines.

Note that this is true regardless of the forms we assume for the marginal relationships [i.e., $U_m^1 = f_1(X_m)$ and $U_n^2 = f_2(Z_n)$]. It can be demonstrated that a measure of the average effect or utility (the so-called marginal utilities) of each of the two variables is given by their marginal means. We now demonstrate that this is true for any multilinear utility model, thereby confirming that it holds for any more restricted form such as simple addition or multiplication.

If the data were obtained from a factorial design in which factor one is the row factor (subscripted m) and factor two is the column factor (subscripted n), we may write the most general multilinear form as

$$U_{mn} = k_0 + k_1 U_m^1 + k_2 U_n^2 + k_3 U_m^1 \times U_n^2 + \epsilon_{mn} \tag{12}$$

where all terms are as defined previously and k are scaling constants. Additional factors simply add additional one-way, two-way, three-way, and higher terms. Now, if we average the factorial data over the second subscript n (i.e., the column factor), we would have

$$U_{m.} = k_0 + k_1 U^1_m + k_2 \overline{U}^2_n + k_3 U^1_m \times \overline{U}^2_n + \epsilon_{m.} \quad (13)$$

where $\overline{U}^2_n$ is the average over-the-column factor. Thus, Equation 13 reduces to

$$U_{m.} = K_0 + K_1 U^1_m + \epsilon_{m.} \quad (14)$$

where K is collected terms.

Equation 14 demonstrates that the marginal row means (in general, the marginal means for any subscript) are equal to the marginal utilities up to a linear transformation. Hence, they are as good as any other estimate measured on an interval scale. Equation 14 is important because it demonstrates that an estimate of the marginal utility for any factor may be obtained by manipulating that factor as part of a factorial or fractional factorial design so long as any multilinear utility function can be assumed to have generated the data.

Returning to the reduced, strictly additive form, it may also be demonstrated that these marginal means relate to the overall utility value of cell m,n as follows (5, 6, 7, 8):

$$U_{mn} = U_{m.} + U_{.n} - U_{..} + \epsilon_{mn} \quad (15)$$

where $U_{..}$ is the grand average utility (mean). Similarly, for a strictly multiplicative form, it may be demonstrated that the following is true (5, 6, 7, 8).

$$U_{mn} = k + [(U_{m.} - k)(U_{.n} - k)/(U_{..} - k)] + \epsilon_{mn} \quad (16)$$

where all terms are as defined in Equation 15 except k, which is a scaling constant that represents the arbitrary zero point on the utility scale.

Now, on the assumption that Equation 14 is true, we may write the following expressions by assigning levels of distance to the rows and levels of town size to the columns.

$$U_{m.} = f_1(\text{distance}_m) \quad (17)$$

$$U_{.n} = f_2(\text{town size}_n) \quad (18)$$

because the only source of variation in $U_{m.}$ and $U_{.n}$ is that due to the levels of distance and town size and error. Thus,

$$U_{mn} = f_1(\text{distance}_m) + f_2(\text{town size}_n) - U_{..} + \epsilon_{mn} \quad (19)$$

if the two factors combine additively, or

$$U_{mn} = \{[f_1(\text{distance}_m) \times f_2(\text{town size}_n)]/U_{..}\} + \epsilon_{mn} \quad (20)$$

if the factors combine multiplicatively and we assume that k in Equation 16 equals zero as a first hypothesis.

Following our previous logic, Equation 20 is testable statistically and graphically. In particular, Equation 20 requires that all interaction effects be statistically significantly different from zero and that the graph of the interaction must consist of a series of diverging curves. An exact statistical test may be obtained by using the marginal means as the independent values, estimating k (usually done by iterative methods) and performing the following linear regression.

$$\ln(U_{mn} - k) = \ln(U_{m.} - k) + \ln(U_{.n} - k) - \ln(U_{..} - k) \quad (21)$$

If Equation 20 is true, the coefficients of the distance and town size terms should not be significantly different from 1.0.

Thus, we have demonstrated that an algebraic and statistical theory for diagnosing and testing any multilinear utility form does exist. In order to derive a model in the units of the original variables (e.g., miles, minutes, population), it is necessary to first diagnose the overall form of Equation 12 and then make assumptions about the functions in Equations 19 and 20 (or a more general form given by Equation 12, if appropriate).

In the next section we shall demonstrate the application of this theory and methodology to the problem of choosing a town in which to live, given that one has taken a job at a plant or a mine located in a rural area. We then demonstrate that knowledge of the functional form of the utility expression provides quite accurate recovery of real-world data in an analogous choice situation. We then compare the derived function form to a large number of linear parameter forms that might typically be fit in a logit analysis.

RESULTS OF FUNCTIONAL MEASUREMENT EXPERIMENT FOR TOWN CHOICE

In order to develop a specification for a utility function for town choice, a functional measurement experiment designed to reflect the employment and residence situations for isolated plants in Wyoming, North Dakota, and Montana was developed. To maintain a fairly simple structure, only two variables, distance to work and size of town, were considered.

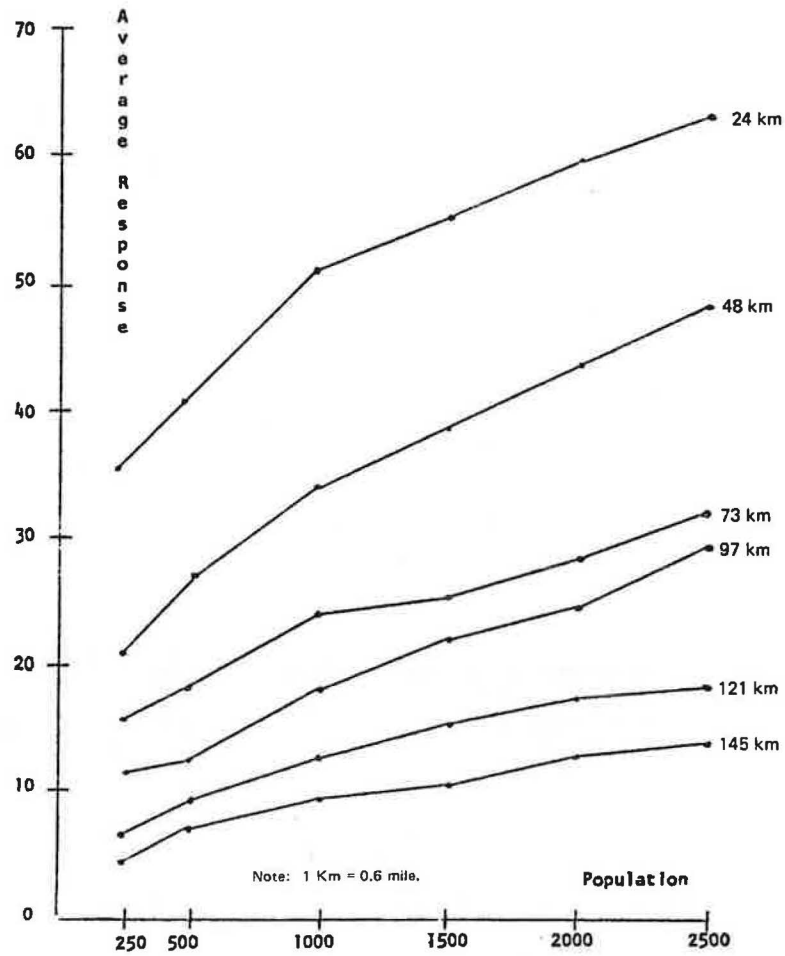
Sets of hypothetical classes of towns were constructed by developing linear regression functions relating population to the number of each of ten types of facilities such as bars, grocery stores, restaurants, and churches. The number of expected functions in each class was predicted from population sizes of 250, 500, 1000, 1500, 2000, and 2500.

This procedure is based on empirical research in central place theory (16). Thus, there are six levels of the composite stimulus (town size and facilities). The six levels of driving distance, chosen by examining actual commuting distances of plant workers (17), were 24, 48, 73, 97, 121, and 145 km (15, 30, 45, 60, 75, and 90 miles). All combinations of towns and distances yield a 6 × 6 factorial design. This design was printed in five different random orders. In addition, four filler combinations more extreme than the design combinations were inserted. Thus, the experiment involved 40 distinct combinations that were presented to respondents on sheets.

Filler combinations are used to transfer response bias away from the experimental combination extremes. Subjects respond more extremely to the fillers that they quickly learn are the best and worst combinations in the design. To test the effects of the order of the combinations in the questionnaires, five different orders were prepared by random draw. Sixty usable questionnaires were obtained from students, faculty, and staff at the University of Wyoming who volunteered to participate. These subjects were randomly assigned to the five order conditions. Thus the experiment is a 6 × 6 × 5 × 12 factorial design (town times distances times orders times subjects). It should be recalled that this sample should be as good as any other for estimating functional form, although the parameter values will be biased for the population as a whole.

Subjects were asked to estimate a numerical value for their degree of preference for each combination by assigning a number (interpreted as a utility measure) between zero (absolutely the worst combination imaginable) and 100 (the best imaginable). Subjects were shown combinations that pretesting had revealed to be very undesirable and very desirable. They were told that all items to be evaluated were considered less ex-

Figure 1. Utility responses by population and distance.



treme than these and that they were to use these combinations to anchor their numerical judgments of preference. All subjects, therefore, completed all combinations.

Data were first analyzed by means of analysis of variance. The results showed that the effect of order was not significant, while that for distance and town size was, as was their interaction. The traditional gravity model of trip distribution would indicate a multiplicative relationship between these factors that would yield a significant interaction; this is confirmed by the analysis. The form of this interaction, however, is critical. Hence, graphical evidence of multiplication is necessary to bolster the statistical evidence of a significant interaction.

As discussed earlier, if the data are graphed as a function of increasing (or decreasing) column values, the difference between each row must increase (or decrease). Thus, the data plot as divergent (convergent) curves. This is approximately true in Figure 1. We can therefore tentatively accept a multiplicative combination rule as a reasonable approximation to the decision process for this experiment.

As demonstrated in Equations 13 and 14, the marginal row and column means are interval scale estimates of the utility values corresponding to the levels of the experimental factors, measured in the units of the dependent variable. These effects or utilities can only arise from variation in the experimental values. Hence,

$$U_m^1 = f_1(\text{town size}) + \epsilon_m \tag{22}$$

$$U_n^2 = f_2(\text{distance}) + \epsilon_n \tag{23}$$

We may assume specific function forms for f_1 and f_2; two likely candidates consistent with both psychological and utility theory are

$$U_m^1 = a_1 + b_1 \text{ town size}_m^{c_1} + \epsilon_m \tag{24}$$

$$U_n^2 = a_2 e^{-b_2 \text{ distance}_n} + \epsilon_n \tag{25}$$

where U_m is measured by the marginal row mean and U_n is measured by the marginal column mean derived from the factorial design. Because these are sums or averages of random variables, it is reasonable to assume them to be normally distributed. The experimental factor values are fixed, so we have a classical fixed-effects regression case and can estimate the desired parameters via least-squares.

If we assume the multiplicative hypothesis encouraged by Figure 1 and the results of the analysis of variance, we can write (assuming Equations 16 and 20 and letting k = 0)

$$U_{mn} = (U_{m.})(U_{.n})/(U_{..}) + \epsilon_{mn} \tag{26}$$

where $U_{m.}$ and $U_{.n}$ are marginal means and $U_{..}$ is the grand mean. Equation 26 is always true if there is a true multiplicative rule underlying the data. Substituting Equations 24 and 25 into 26 yields

$$U_{mn} = [1/(U_{..})](a_1 + b_1 \text{ town size}_m^{c_1})(a_2 e^{-b_2 \text{ distance}_n}) + \epsilon_{mn} \tag{27}$$

By expanding Equation 27 and combining constants, we have the expectation of the following equation.

$$U_{mn} = k_0 + k_1 e^{-b2\ \text{distance}_n} + k_2(\text{town size}_m^{c1} \times e^{-b2\ \text{distance}_n}) + \epsilon_{mn}. \quad (28)$$

The parameters of this equation were estimated via an iterative, least-squares procedure that yielded the following utility expression for town size and distance.

$$U_{mn} = 24.76(e^{-0.021\,41\ \text{distance}_n}) + 1.313(\text{town size}_m^{0.562}) \times (e^{-0.021\,42\ \text{distance}_n}) + \epsilon_{mn} \quad (29)$$

This equation accounts for 98.6 percent of the variance in the experimental design cell means. Both terms are highly significant, as is the overall equation ($F = 1154.8$; $df = 2.33$). Hence, we tentatively retain Equation 29 as a reasonable approximation to the utility function employed in the experiment.

ESTIMATION OF CHOICE MODEL FROM REVEALED PREFERENCES

Given that the functional form developed in the previous section using functional measurement adequately describes respondents' expressed preferences under hypothetical conditions, the next logical step is to demonstrate that the same functional form usefully describes actual choice of residential location. In order to do so, we must first postulate a model of decision making. Because of measurement errors, omitted variables, and the use of proxy individuals that associate with choice alternatives, we cannot reasonably expect to describe choices perfectly. However, by assuming a distribution of the random elements, we can model the probability with which any alternative is selected.

For this study we have chosen the multinomial logit model. For the sake of brevity, we shall forego the derivation of the logit model (18, 19, 20, 21). In terms of the simple model of location decisions under consideration, this model can be expressed as

$$P(i|A_t) = [e^{V_t(P_i,D_{it})}] \Big/ \left[\sum_{j \in A_t} e^{V_t(P_j,D_{jt})}\right] \quad (30)$$

where

$P(i|A_t)$ = denotation of the probability that town i is selected by person t from some feasible set of towns A_t,
$V_t(P_i, D_{it})$ = denotation of the representative utility (i.e., the nonrandom portion of the total utility) for person t,
P_i = population of the i th town, and
D_{it} = distance between the i th town and person t's work place.

The parameters of this model are generally estimated by maximum likelihood. However, Berkson (22) and Theil (23) describe the use of least-squares estimation for binary and multinomial logit respectively when the observations have many repeated entries. For multinomial logit, this can be done by noting that

$$\{[P(i|A_t)]/[P(k|A_t)]\} = \{[e^{V_t(P_i,D_{it})}]/[\sum_j e^{V_t(P_j,D_{jt})}]\} \div \{[e^{V_t(P_k,D_{kt})}]/[\sum_j e^{V_t(P_j,D_{jt})}]\} = e^{V_t(P_i,D_{it}) - V_t(P_k,D_{kt})} \quad (31)$$

Hence

$$\ln[P(i|A_t)/P(k|A_t)] = V_t(P_i,D_{it}) - V_t(P_j,D_{jt}) \quad (32)$$

If we now impose the restriction that V_t is linear in its parameters, the above equation reduces to

$$\ln[P(i|A_t)/P(k|A_t)] = \beta(X_{it} - X_{kt}) \quad (33)$$

where β is a vector of parameters and X_{it} and X_{kt} are the vectors of variables characterizing alternatives i and k respectively.

In this technique, each observation represents the proportion of people with common values of X and common choice set A_t who choose alternative i relative to the proportion choosing the base alternative k for that group. Thus, while the actual sample may have a great number of responses, the data for estimation are grouped. For example, the original data used for this study were gathered from a 100 percent sample of workers at six power plants and from a 20 percent sample at one other. Since only population and distance are used as independent variables, each person at a single power plant has the same set of available towns, A_t, and the same values of the independent variables characterizing that set. The proportion of people at a power plant choosing a town i for a residence, denoted as $f(i|A_t)$, is a consistent, unbiased estimate for $P(i|A_t)$. Thus, $n[f(i|A_t)/f(k|A_t)]$ is a consistent (though biased) estimate for $\ln[P(i|A_t)/P(k|A_t)]$.

In a Monte Carlo simulation study Domencich and McFadden (21) demonstrated that, when the number of repetitions for each alternative is reasonably large, the bias in least-squares estimation is small and the Berkson-Theil procedure is to be preferred over maximum likelihood on the grounds of computational efficiency.

The use of the Berkson-Theil procedure requires selecting one alternative location to act as the base (denoted by k above) for each power plant. In the results reported below, the town with the median share of the workers was used as a base location for each of the groups of workers employed at the seven power plants.

In our data, the total number of towns used in the models was 46, although each power plant had a different subset of towns available. Some towns were eliminated because they had unusual characteristics that did not match the range of independent variables used in the functional measurement experiment. Given that a base alternative is used for each town, the actual number of observations as input to the estimation program was 39, the original 46 towns minus the 7 used as base alternatives.

Another way to view the base alternative is to recognize that the full set of probabilities $P(i|A_t)$ is not independent; the probabilities must sum to unity. Hence, if there are I alternatives in any choice set A_t, only I - 1 of them convey any information; the I th is redundant.

RESULTS OF ESTIMATION FROM REVEALED PREFERENCES

In order to test the usefulness of the functional form derived above, the same specification was estimated on the town choice data by nonlinear least squares. A large number of linear parameter specifications using ordinary least squares via the Berkson-Theil method then were estimated. If the functional form derived from the functional measurement experiments is indeed a useful description of actual decision making, then one would expect it to fit the available town choice data better than more commonly applied linear parameter forms.

The results of the estimation for the specification in Equation 29 are summarized below in Equation 34.

Figure 2. Summary of linear parameter models.

model #	# of parameters	P	D	$\ell_n D$	$\ell_n P$	1/P	1/D	ε^P	ε^D	D^2	P^2	PD	$\ell_n PD$	R^2	$\bar{R}^2$	s
1	2	X	X											.5225	.5096	1.014
2	2			X	X									.5915	.5805	.941
3	2					X	X							.2249	.2040	1.30
4	2							X	X					.3022	.2833	1.23
5	2				X				X					.5819	.5708	.952
6	2				X		X							.2267	.2058	1.29
7	2			X				X						.4292	.4138	1.11
8	2						X	X						.2082	.1868	1.31
9	2			X		X								.5339	.5213	1.01
10	2					X		X						.3862	.3696	1.15
11	3	X	X									X		.6207	.5996	.919
12	3			X	X								X	.6351	.6148	.902
13	4	X	X							X	X			.7221	.6983	.798
14	4	X	X	X	X									.7468	.7251	.762
15	4	X	X			X	X							.6356	.6044	.914
16	4	X	X					X	X					.6242	.5920	.928
17	4	X	X							X		X		.7127	.6881	.811
18	4	X	X							X			X	.5785	.5424	.983
19	4	X	X								X	X		.7220	.6982	.793
20	4	X	X								X		X	.7191	.6950	.802
21	5	X	X							X	X	X		.7447	.7147	.776
22	5	X	X							X	X		X	.7221	.6894	.810
23	5	X	X	X	X							X		.7581	.7296	.755
24	5	X	X	X	X								X	.7656	.7380	.744
25	6	X	X	X	X					X	X			.7592	.7248	.763
26	7	X	X	X	X					X	X	X		.7752	.7331	.751
27	8	X	X	X	X					X	X	X	X	.8042	.7600	.711

$$V_t(P_i,D_{it}) = \underset{(0.091)}{-0.141}\ \exp(\underset{(0.003\,66)}{-0.0128}.D_i) + \underset{(0.044)}{0.165} + \underset{(0.799)}{1.74}\ \exp(\underset{(0.003\,66)}{-0.0128}.D_i)\ P_{it} \qquad (34)$$

where

$R^2 = 0.8052$,
$\bar{R}^2 = 0.7885$, and
$s = 0.668$.

These estimates were derived by using a nonlinear least-squares procedure incorporated in Time Series Processor (TSP), an econometric software package developed at Massachusetts Institute of Technology. The numbers in parentheses below the parameter estimates are their estimated standard errors.

Three statistics also are reported. The first, R^2, is the percentage of explained variance; $\bar{R}^2$ is the value of R^2 corrected for the number of parameters estimated. The standard error of the regression, s, is an estimate of the standard error of the disturbance in the model. Note that in interpreting these measures, one must recall that the dependent variable is $\ln[P(i|A_t)/P(k|A_t)]$, not $P(i|A_t)$.

In order to determine whether the specification derived from the functional measurement was in any sense "better" than more usual functional forms, a series of linear parameter forms was estimated. The first set of these runs used distance and population in linear, exponential, inverse, and logarithmic form. These speci-

fications, however, have only two parameters, while the form in Equation 34 has four. Therefore, a wide range of combinations of the variables, including quadratic forms of the first two, was also estimated. Some of these forms were simply curve-fitting efforts; it is difficult to see how one would arrive at them from any behavioral argument. Others are extended forms of gravity models of spatial interaction.

Figure 2 summarizes these models and is structured so that the columns denote the various independent variables used and the rows correspond to different linear-in-parameters function forms. In each model, an x indicates that a particular variable was used. The figure also summarizes the number of parameters in the model and the values of R^2, $\bar{R}^2$, and s.

The most important feature of these results is that, even without correcting for the number of parameters estimated, none of the linear models fits these data better than the specification in Equation 34. The last regression, with eight parameters, comes close in terms of R^2-values to the form derived from the functional measurement experiment before adjustment, but it is significantly worse after the degrees of freedom are accounted for.

A second significant point is that the behavioral underpinnings for the form in Equation 34 are relatively clear (after the analysis is performed), but the specifications involving many parameters in Figure 2 are somewhat obscure. For this reason it is unlikely that someone using the revealed preference data would actually choose many of these models. Furthermore, many of the parameter estimates for the linear models with five or more coefficients are statistically insignificant.

A side result not reported in the figure is that many of the coefficients from at least the two-, three-, and four-parameter linear models were statistically more significant (as measured by their t-statistics) than those from the functional measurement form. It is difficult to interpret this result other than to note that, if the specification of either (or both) model(s) is incorrect, then the estimated coefficients and their corresponding t-statistics are in general inconsistent; the higher t-statistics in the linear form may be meaningless and may simply reflect the outcome of a particular form of misspecification.

CONCLUSION

It is interesting—at least as a mental exercise—to ask the question, What form would a reasonable modeler select if he or she were developing a model from the revealed preference data and the functional measurement experiment had been infeasible? Obviously the answer to such a question depends on the criteria for model selection used by the analyst, but reasonable answers might include simple two-parameter forms such as model 2 or 5 in Figure 2 or quadratic forms such as 13 or 23. It is most unlikely that the form in Equation 34 would ever be considered, particularly since its estimation requires the use of a fairly expensive nonlinear estimation procedure. Without some prior evidence, such as an equation developed in a functional measurement experiment, that such a functional form might be useful, most modelers would never even consider nonlinear forms unless no reasonable linear model could be developed.

Obviously, the empirical evidence presented in this paper is extremely limited, and it would be premature to suggest that functional measurement or some other, similar technique should be a major component of a demand model estimation strategy. However, the relatively low cost of laboratory experiments does make it appear to be an attractive way to analyze functional form. The two-step procedure proposed in this paper offers at least one feasible approach to improving travel demand models and may actually reduce the cost of model estimation by restricting the class of model specifications with which travel demand modelers need be concerned.

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Effects of Employment and Residential Location Choices on Urban Structure: A Dynamic Stochastic Simulation

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The pattern of home-to-work linkages in urban areas is affected by household mobility decisions. This paper describes a dynamic stochastic simulation model designed to illustrate the effects of mobility decisions on urban structure. The major feature of the model is the representation of household changes in employment or residential locations or both. The sequential process of the decision to move and the search for and selection of new location is specified. The role of accessibility in this process is an important consideration in the model, which allows quick execution of simulation experiments. Experiments consist of alternative input assumptions involving factors such as city size, numbers and locations of job centers and dwelling units, initial pattern of home-to-job linkages, moving rate, and importance of accessibility in selecting new locations. Two types of experiments are presented. The first examines the dynamic properties of the model by varying the initial pattern of home-to-job linkages and the mobility rate. The major conclusion is that there appears to be an equilibrium pattern of home-to-job linkages that is independent of the initial configuration and mobility rate. The second type of experiment involves the variation of the importance of accessibility in the mobility decision process. The results show that the pattern of home-to-job linkages varies in the expected way with changes in the decision process.

The pattern of linkages between residential and employment locations in urban areas and the changes in this pattern over time are the result of many complex economic and social processes. To isolate the contribution of each by looking at the total patterns through time series or cross-sectional analyses is difficult, if not impossible. An alternate approach is to study theoretical models that deal with a small number of processes, or a small part of the problem, at a time. Data and theory must finally agree, of course. In attempting to model the effects of the processes on urban structure, one must make simplifying assumptions in order to make the problem analytically tractable. The nature of the assumptions is dictated by the purposes of a particular modeling approach.

The purpose of the present modeling effort is to examine the effects of accessibility-based household decision rules on the changing structure of prototypical urban areas. The emphasis on accessibility is consistent with numerous previous studies and also facilitates an examination of the transportation requirements for urban areas. The model is a dynamic stochastic simulation model that processes household location decisions sequentially within a given time period. Each time period represents a fraction of the population making intraurban mobility decisions, i.e., residential location changes, employment location changes, or simultaneous home-job changes. An additional objective of the modeling effort is to generate a basis for conducting controlled theoretical experiments, the purpose of which is to examine questions such as stability, statistical variabilities, and observability of urban relationships. These issues are important for linking theoretical results with empirical data.

Much simplification of the socioeconomic details such as the distributions of household and dwelling unit types over the urban areas is allowed in order to sharpen the