Spatial Aggregation of Disaggregate Choice Models: Areawide Urban Travel Demand Sketch-Planning Model

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This paper describes an aggregate urban travel demand model designed for areawide transportation policy evaluation with limited preparation of input data and fast response times. It does not include supply models but it can be used by itself for travel demand predictions with exogenously specified transportation level-of-service changes or it can be incorporated in the framework of the TRANS model. The methodology is generally applicable to urban transportation sketch-planning situations in which large geographic units are used. Aggregation is performed over spatial travel alternatives and spatially distributed individuals to produce required aggregate travel demand forecasts. An efficient solution method for spatial aggregation was developed that employs mathematical functions, expressed in terms of coordinates in the urban space, to describe the spatial choice process and to represent the geographic distribution of behavioral units, spatial alternatives, level-of-service characteristics, and locational attributes. This allows the spatial aggregation problem to be solved efficiently, by integrating the travel demand models over the urbanized area. Monte Carlo simulation techniques are employed and the procedure entails (a) generation of a sample of representative households distributed over the urban area using available census data, (b) generation of a sample of destinations for each trip purpose for each household, (c) computation of travel demand forecasts for each household based on the sampled destinations using a system of disaggregate travel demand models, and (d) accumulation and expansion of disaggregate predictions to produce aggregate forecasts.

This paper describes the methodology and application of an aggregate model of urban travel behavior. The model is designed to be applied at a high level of geographic aggregation—the entire urban area—for quick assessment of urban transportation policies. The underlying methodology is applicable to a wider range of sketch-planning analyses that are characterized by the use of readily available input data (for example, from the U.S. Census) and fast response times. These features are essential for a successful integration of technical analysis and transportation decision-making processes.

Typically, the travel demand models employed in existing sketch-planning packages offer little policy sensitivity and require separate calibration for different levels of spatial aggregation or zone sizes.

The basic premise of this study is that the use of disaggregate travel behavior models (1, 2, 3, 4) in sketch-planning applications with appropriate aggregation procedures would remove these shortcomings. Although disaggregate choice models have many advantages over conventional aggregate travel demand models in general, their distinct advantages from the sketch-planning standpoint are that once estimated they can be applied to any desired level of geographic aggregation and that they have the potential of being transferred from one urban area to another.

The aggregate demand model developed in this study represents an extreme level of spatial aggregation since it treats an entire urban area as a single analysis unit. It is suited to metropolitan transportation planning studies in which impacts on specific areas are not required. When incorporated into a complete supply-and-demand analysis system, it can be used to determine scale and composition of transportation investments on an areawide basis, involving approximate funding allocations to modes and facility types, to construction and maintenance expenditures, etc. It can also be used to analyze aggregate impacts of pricing and operating policies such as fuel price change, parking cost surcharge, and areawide transit improvements.

The model can be used in the framework of the multimodal national urban transportation policy planning model.
known as TRANS (5, 6, 7, 8), which is referred to as MIT-TRANS in this paper.

SPATIAL AGGREGATION OF DISAGGREGATE TRAVEL DEMAND MODELS

Aggregate forecasting cannot, in general, be performed by substitution of average values of the independent variables in a disaggregate model. Therefore, aggregate forecasting requires the application of an aggregation procedure that employs information about the distributions of the variables (9). An efficient aggregation procedure is particularly important in sketch-planning models designed for large spatial analysis units (10).

This section describes the basic concepts and methods of aggregating disaggregate travel demand models over individuals and spatially distributed alternatives.

Basic Definitions

Consider an origin zone as a group of individual behavioral units and a destination zone as a group of elemental spatial alternatives. The elemental spatial alternatives—such as housing units (in the choice of residential location) and jobs (in the choice of work place)—are assumed to be mutually exclusive in that one and only one of the available alternatives will be chosen.

Consider a prediction of the number of trips (given purpose) from an origin i to a destination j (both i and j are equal to the entire urbanized area as used for the level of aggregation in the MIT-TRANS model). First, for each individual t in the origin i predict the probability that the individual will choose each spatial alternative e in destination j. Denote this probability P_t(e). Then sum the probability for all spatial alternatives in the destination to produce the probabilities for all spatial alternatives in the destination and call this step aggregation of alternatives. Thus

\[ P_j = \sum_{t} P_t(e) \]  

(1)

Finally, sum the probabilities for all individuals t in origin i to get the expected number of trips from origin i to destination j and call this step aggregation of individuals.

\[ T_{ij} = \sum_{t} P_t(e) = \sum_{t} \sum_{e} P_t(e) \]  

(2)

To illustrate the relationship between spatial and nonspatial alternatives, let \( P_t(m|s) \) be the probability of trip maker t choosing mode m given that the trip maker has chosen spatial alternative s. By similar argument, the expected number of trips from origin i to destination j by mode m is given by

\[ T_{imj} = \sum_{s} \sum_{e} P_t(m|s) P_t(e) \]  

(3)

The above example illustrates the basic idea of spatial aggregation that consists of two basic steps, the aggregation of spatial alternatives for each individual and the aggregation of spatially distributed individuals. These steps are conceptually applicable to any desired traffic zone size and to both intrazonal and interzonal trips.

Spatial Aggregation Using Continuous Functions

Obviously, the discrete summation form used in the above example is too microscopic for actual prediction, since complete enumeration, as the example implies, would require astronomical amounts of data and computation. Furthermore, even if computational costs were not a barrier, it is still infeasible to describe the detailed characteristics of each spatial alternative and behavioral unit. In order to develop an operational spatial aggregation procedure, some degree of abstraction of spatial alternatives and behavioral units is necessary.

There are many possible ways to represent spatial distributions, depending on the level of detail desired. To generalize the definition of spatial aggregation, assume that mathematical functions expressed in terms of two-dimensional coordinates represent the spatial distributions. This type of representation can be used as a basis for comparing different spatial aggregation methods (9, 11, 12, 13, 14, 15).

Suppose that the attributes of spatial alternatives and the distributions of spatial alternatives and behavioral units could be expressed in terms of coordinates of the urban space. Define a spatial choice density function, denoted \( G_k(p, q; x, y) \), as the probability of a behavioral unit type k located at point \((x, y)\) choosing a spatial alternative located at point \((p, q)\) for a specific purpose such as shopping destination (16). This is a unique surface for individual type k located at \((x, y)\) that is also a function of the distribution of attributes of spatial opportunities and their transportation level of service for origin point \((x, y)\) and socioeconomic variables of individual type k. Define spatial distribution functions for spatial alternatives and behavioral units as \( M(p, q) \) equals the number of elemental spatial alternatives per unit area at point \((p, q)\), and \( H_k(x, y) \) equals the number of behavioral units type k per unit area at point \((x, y)\). The number of trips from zone i to zone j can now be derived as (a) aggregation over spatial alternatives to obtain the probability of behavioral unit type k located at \((x, y)\) choosing an alternative in zone j:

\[ P_k(j|x,y) = \iint_{\text{zone } j} G_k(p, q; x, y) M(p, q) dpdq \]  

(4)

or (b) aggregation over behavioral units to obtain the expected number of behavioral units type k located in zone i who travel to zone j:

\[ T_{ki} = \iint_{\text{zone } i} P_k(j|x,y) H_k(x,y) dx dy \]  

(5)

The total number of trips is

\[ T_{ij} = \sum_k T_{ki} \]

(6)

It is also possible to repeat the above steps to derive other travel demand predictions. For example, let \( D(p, q; x, y) \) be the distance traveled between points \((p, q)\) and \((x, y)\). Then, the expected kilometers of travel for zone pair \((i, j)\) is given by

\[ MT_{ij} = \sum_k \iint_{\text{zone } j} \int_{\text{zone } i} D(p, q; x, y) G_k(p, q; x, y) M(p, q) dx dy \]

(7)
Numerical Techniques for Spatial Aggregation

The spatial choice density function can be expressed in terms of spatial coordinates via the distribution over space of the independent variables of a spatial choice model. The independent variables that enter the utility functions of choice models of travel behavior are

\[ L(p, q; x, y) = \text{transportation level-of-service attributes by different modes, times, and facilities between origin point (x, y) and destination point (p, q)} \]

\[ A(p, q) = \text{location attributes, or attraction variables, of the relevant elemental spatial alternatives at point (p, q)} \]

\[ S_k = \text{socioeconomic characteristics of an individual of type k}. \]

Thus, the required input data include spatial distributions for transportation level of service variables (L), locational attributes (A), and spatial density functions for elemental spatial alternatives (M) and behavioral units (H) of different types.

There are two broad approaches in which the spatial distribution of the input data can be applied to carry the spatial aggregation—a direct and an indirect approach.

Direct Approach

The mathematical functions \( L(p, q; x, y) \), \( A(p, q) \), \( M(p, q) \), and \( H_k(x, y) \) are expressed explicitly in terms of the coordinates \((p, q)\) and \((x, y)\). Furthermore, the spatial integration is also carried directly.

Define the following probability density function:

\[
PDF_{kj}(p, q; x, y) = \begin{cases} 
  \frac{M(p, q)H_k(x, y)}{M_iH_{ki}} & \text{for } (x, y) \in \text{zone } i \\
  0, & \text{otherwise}
\end{cases}
\]

(8)

where

\[ M_i = \text{the number of elemental alternatives in zone } j, \text{ or } \int M(p, q)dpdq \text{ and } \]

\[ H_{ki} = \text{the number of behavioral units type k in zone } i, \text{ or } \int H_k(x, y)dx. \]

The integral for the number of trips by individuals type k from i to j, as an example, can now be rewritten as

\[ T_{ki} = \int \int \int C_k(p, q; x, y)PDF_{kj}(p, y; x, y)dpdqdx \]

\[ = \frac{M_jH_{ij}}{M_{ij}} \cdot G_{ij} \]

(9)

where \( \bar{G}_{ij} \) is the expected value of the spatial choice density function for individuals type k in zone i and elemental spatial alternatives in zone j. Thus, the expectation of the spatial choice density function is taken over the distribution of (p, q) and (x, y) defined by function PDF_{kj}.

This is the approach taken in the development of the MIT-TRANS model. Two broad classes of numerical integration methods are possible: mechanical or approximate quadrature techniques such as described in Davis and Rabinowitz (17) and Monte Carlo simulation techniques such as described in Hammersley and Handscomb (18).

If the functions \( G_k(p, q; x, y) \), \( L(p, q; x, y) \), \( A(p, q) \), \( M(p, q) \), and \( H_k(x, y) \) are relatively well behaved (e.g., having a continuous first derivative) and if zones i and j assume a simple shape (e.g., a circle or rectangle), mechanical quadrature techniques seem appropriate. Otherwise, Monte Carlo simulation seems to be the only feasible approach.

In the MIT-TRANS model, because of the difficulty in setting the bounds of the integrals for the complex shape of the urban area, mechanical quadrature techniques were ruled out as infeasible, and the Monte Carlo approach was chosen. Although Monte Carlo methods are generally less accurate than mechanical quadrature methods, they are appropriate to travel demand forecasting applications because great precision is not required and we are interested in predicting changes.

In urban transportation planning applications, the Monte Carlo approach offers the following advantages:

1. No aggregation bias (in the context defined by Koppelman, 1975) is produced by Monte Carlo simulation;
2. Forecast error measures are available immediately as a by-product of the Monte Carlo simulation process;
3. Errors in Monte Carlo forecasts can be expressed as a function of sample size, which is directly proportional to computation effort; hence, the errors can be parametrically controlled by making a direct trade-off between accuracy and cost;
4. The Monte Carlo approach can be applied to any type of mathematical representation; and
5. It is possible to stratify Monte Carlo forecasts by socioeconomic or other groups. The prediction errors for each group can be controlled separately.

The major disadvantage of the Monte Carlo approach is that, for a given Monte Carlo procedure, the magnitude of random error is inversely proportional to the square root of sample size. Thus, to reduce the magnitude of error by one-half, the sample size must be quadrupled. Although this weakness can be serious in applications where a high degree of accuracy is required, it is not so in travel demand forecasting applications, since relative errors in the range of 10-20 percent or even greater are normally quite acceptable for decision-making purposes. Furthermore, a number of techniques, such as the stratified sampling and importance sampling used in the MIT-TRANS model, can be employed to substantially reduce error without increasing the sample size.

Indirect Approach

The distribution of the coordinates \((p, q)\) and \((x, y)\) is first transformed into that of the level-of-service and attraction variables. Integration is then performed over the distribution of the latter variables.

The spatial choice density function is a function of the level-of-service variables (L) and the locational attributes (A), which are, in turn, functions of \((p, q)\) and \((x, y)\). Therefore, \( G_{i,j} \) can also be expressed as the expectation of \( G(L, A) \) over distribution of \( L \) and \( A \):

\[
G_{ij} = \int \int G(L, A)PDF_{ij}(L, A)dl \ dA
\]

(10)

Therefore, the key problem is to find the distribution of
from (9):

1. Parametric distribution functions with analytical or numerical (including Monte Carlo) integration techniques (11, 12);
2. Moments of distribution with the statistical differentials method that expresses the aggregate quantity in terms of moments of the distributions using a Taylor series expansion (11) (this approach proved to be unstable and is not employed in this study but is included for completeness);
3. Classification (or categorical representation of the distributions) with each class, which is a group of individuals or a group of alternatives, represented by its average values and the aggregate quantity, and is a weighted average of the choice probabilities for the classes; if overall average values are used for the independent variables—a one-class discrete distribution—it becomes the so-called "naive method" that ignores the aggregation problem entirely (9); and
4. Sample enumeration where aggregation is carried by summation of choice probabilities for a sample of individuals and a sample of elemental or homogeneous groups of spatial alternatives for each individual (this approach is identical to Monte Carlo integration except that the samples are drawn from actual observed data instead of from parametric distributions).

In the MIT-TRANS model a sample enumeration approach is used for the socioeconomic characteristics of households. The spatial distribution of behavioral units \( H(x, y) \) is represented by classification and parametric functions within each class. All other distribution information is represented by parametric functions in terms of coordinates.

In conventional urban transportation model systems, the basic method used is classification. The population and spatial alternatives are classified into traffic zones with size varying according to the urban area and the purpose of the analysis. For each zone only average values are generally given.

The short-range travel demand prediction system developed by Ben-Akiva and Atherton (23) employs a sample enumeration approach to represent behavioral units, and a sample of aggregate traffic zones is used to represent destinations.

The method of statistical differentials was developed by Talvitie (11) and applied by Difiglio and Reed (24), Liou (25), and Dunbar (20).

**BASIC OPERATIONS OF THE MIT-TRANS MODEL**

MIT-TRANS represents an extreme form of test for the feasibility and validity of the spatial aggregation methodology developed in this study, since it treats an entire urban area as a single traffic zone with almost all trips being internal. The model is based on the application of Monte Carlo simulation using synthetic socioeconomic and land-use distribution data to forecast trip generation, distribution, and modal split, with a system of aggregate travel demand models. There are seven disaggregate models for both work and nonwork trips that have been linked together; i.e., outputs from one model become inputs to lower-hierarchy models (13). Examples of predictions are the number of trips made, mode shares, person-kilometers of travel, vehicle-kilometers, average vehicle occupancy rates for work and nonwork trips, number of automobiles per family, and so on. These predictions are policy sensitive as reflected in the elastic travel demand models for the choices of work place, auto ownership, mode to work, nonwork travel frequency, destination, and mode.

It should be noted that the MIT-TRANS model in its present form represents only the demand component of the TRANS overall policy evaluation package, which also includes the supply component and evaluation procedures. Future extensions of the MIT-TRANS model will include the development of network abstract transportation supply and traffic assignment models and the integration of these models and the aggregation procedure into an equilibrium framework. In lieu of a complete supply-demand equilibrium framework, a set of level-of-service relationships describing spatial distribution of the equilibrium conditions of an existing transportation system with externally specified parameters is being used in the current MIT-TRANS model.

The existing MIT-TRANS model can be employed to analyze a broad range of area-wide transportation operating and pricing options—those policies that are not expected to significantly alter congestion on the transportation system. The results of some transportation policy alternatives are reported in the summary of empirical tests.

As summarized schematically in Figure 1, the operations of the MIT-TRANS model require three sets of inputs: (a) the aggregate city geometry and land-use
distribution parameters, (b) the urban area's socioeconomic characteristics, and (c) the specifications of a transportation policy alternative. These policy specifications are used to modify the level-of-service relationships that have been calibrated for the base conditions. The aggregation procedure, a Monte Carlo simulation, operates on these inputs, the disaggregate choice models, and the modified level-of-service relationship to produce aggregate travel demand forecasts for the urban area. The forecasts can be disaggregated by market segment, such as by income group.

In the context of the spatial aggregation concept, the MIT-TRANS model uses the direct approach to the integration problem. The urban area is modeled as a quasi-circular shape with the origins (home ends of trips) and destinations (nonhome ends) defined by sets of coordinates (\(R, \lambda\)), and (\(r, \ell\)) or (\(L, \Theta\)\(R, \lambda\)), respectively, as depicted in Figure 2.

For each of three income classes the household density function is assumed to have a negative exponential shape. The spatial alternatives—jobs, shopping destinations, and social recreational facilities—are also represented by negative exponential employment density functions and functions describing location characteristics. The parameters of these density functions can be easily obtained from total counts of population and employment for an inner ring and the entire urbanized area. The transportation level-of-service functions by mode and time of day are expressed in terms of trip geometry variables, which are, in turn, functions of the coordinates of the trip ends.

MIT-TRANS also includes a procedure similar to the one used by Duguay, Jung, and McFadden (26) to obtain the distribution of socioeconomic characteristics of the urban area population by generating a sample of households from available data. The procedure is based on a sample of disaggregate observations from the U.S. Census public-use sample, or any other household survey, and available aggregate data from surveys or published sources for past years or from forecasts or staged scenarios for future years.

Monte Carlo simulation techniques are employed in all steps of the aggregation process: aggregation of spatial alternatives for a behavioral unit and aggregation of spatially distributed behavioral units. The operations of the Monte Carlo aggregation procedure include the following basic steps:

1. Determining household sample size,
2. Generating sample of households for forecast year [each household characterized by (\(L, \Theta\)\(R, \lambda\)) and a set of socioeconomic attributes],
3. Determining sample size of spatial alternatives by purpose,
4. Generating sample of spatial alternatives by purpose for each household in the sample [each destination defined by (\(r, \ell\)) coordinates],
5. Modifying appropriate attributes of the alternatives for policy analysis,
6. Applying linked demand models for each household in the sample,
7. Expanding sample forecasts to population market segments, and
8. Comparing forecasts against base case for policy analysis.
SUMMARY OF EMPIRICAL TESTS

The MIT-TRANS model was programmed in Fortran for an IBM370/168 computer. It requires about 0.6 min of computer-use time per policy run with a standard error of about 1 percent of predicted average passenger kilometers of travel. This demonstrates the computational feasibility of the Monte Carlo simulation approach for urban transportation sketch-planning applications.

The model was calibrated for the 1968 Metropolitan Washington, D.C., area and was then used to forecast 1975 conditions as a validation test. Between 1968 and 1975 there were substantial changes in this area. The main differences in land use included a large increase in the fraction of high-income households and a corresponding decline in low-income households, plus a large population growth in the suburbs together with a sharp decline in population in the inner ring. Household income increased as did the number of workers, while the average household size declined. The transit system experienced almost no change at all, while, for drivers, auto ownership cost decreased 15 percent and auto running cost increased 9 percent in real terms.

The combination of population growth in the suburbs and almost no change in transit coverage meant fewer households had available transit in 1975. The major changes in travel behavior between 1968 and 1975 included increased auto ownership, increased vehicle-kilometers of travel per household, and decreased transit patronage. All the forecast changes given below agree with these trends.

<table>
<thead>
<tr>
<th>Household Transit Factor</th>
<th>Observed Changes (%)</th>
<th>Predicted Changes (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auto ownership per household</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 autos</td>
<td>-23.2</td>
<td>-23.6</td>
</tr>
<tr>
<td>1 auto</td>
<td>-14.0</td>
<td>4.8</td>
</tr>
<tr>
<td>2 autos</td>
<td>34.1</td>
<td>11.6</td>
</tr>
<tr>
<td>Average autos per household</td>
<td>15.1</td>
<td>8.9</td>
</tr>
<tr>
<td>Average transit trips per household</td>
<td>-16.7</td>
<td>-14.5</td>
</tr>
<tr>
<td>Average auto-kilometers per household</td>
<td>15.9</td>
<td>18.1</td>
</tr>
</tbody>
</table>

Apart from Washington, D.C., the model was also calibrated for the Minneapolis-St. Paul area, which has two central business districts. The elasticities produced by the model for both the Washington, D.C., and the Twin Cities areas are comparable to before-and-after empirical evidence and forecasts obtained from other studies.

The MIT-TRANS model's range of applications and policy sensitivity were tested in the following area-wide transportation pricing and operating policies.

1. Transit policies: 50 percent fare increase, free transit, 10 percent transit coverage reduction, 100 percent transit coverage;
2. Auto policies: doubling running cost (in testing auto pricing policies the existing value of kilometers per liter is taken as a constant), annual $400 tax on ownership;
3. Carpool policy: carpool-only highway lanes and preferential parking sufficient to achieve a 20 percent reduction in out-of-vehicle travel time and a 30 percent reduction in in-vehicle travel time.

Detailed results compiled in Watanatada and Ben-Akiva (15) show that auto ownership is affected most by policies that either reduce the need for an auto or make ownership more expensive. It is less affected by policies shifting the relative travel costs by transit and auto.

Auto-kilometers traveled is predicted to change by -13 percent for Washington and -11 percent for the Twin Cities because of the doubling of the price of gasoline. Smaller impacts (-7 percent for Washington) are predicted from providing transit within a kilometer of every household and business in the urban area. No other policy—including carpooling incentives—results in more than a 3 percent reduction in auto-kilometers traveled.

Both the number of trips and the mode shares of transit can be increased by either pricing or coverage policy. Reducing transit coverage by 10 percent has approximately half the impact of a 50 percent fare increase. Free transit increases transit trips more than complete coverage, where the impact on nonwork trips is very different, nonwork trips being much more price sensitive.

In addition to the policies shown and discussed, other
policy costs options can be analyzed by using the model. Parking costs may be varied throughout the city. In-vehicle and out-of-vehicle times for all three modes may be modified for both peak and nonpeak conditions.

Several Monte Carlo sampling experiments were conducted to investigate the statistical properties of the model. We found empirically that a small sample of destinations results in minimal bias and optimal efficiency. The results of sensitivity tests of major input parameters show the importance of the distribution of transit route coverage.

The empirical results have led to the basic conclusion supporting the applicability of disaggregate travel demand models and Monte Carlo aggregation for sketch planning. The travel demand forecasting methodology proposed operates with readily available aggregate input data while still maintaining the full degree of policy sensitivity available in recently developed systems of disaggregate models. The most important future extensions of the methodology are the incorporation of supply and traffic assignment models (15) and the development of a version of MIT-TRANS for multiple zones of varying sizes.

ACKNOWLEDGMENTS

The work reported in this paper was performed at the MIT Center for Transportation Studies as part of the project on the development of an aggregate model of urbanized area travel behavior for the Office of the Secretary of Transportation and the Federal Highway Administration. We acknowledge the significant contributions of Edward Weiner, Helen Doo, and David Gendell. Joseph Baily, Peter Furth, John Nordin, and Patrick O'Keefe have significantly contributed to the conduct of this study. In addition, we benefited from the advice of Frank Koppelman, Steve Lerman, Charles Manski, and Paul Roberts.

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