# Aggregate Prediction With Disaggregate Models: Behavior of the Aggregation Bias

Uzi Landau, Transportation Research Institute, Technion-Israel Institute of Technology, Haifa

**Disaggregate travel demand models are being increasingly applied to predictions of aggregate demand. This is usually done by applying these models to zonal aggregated data, but it causes a bias (the aggregation bias) in the predictions obtained. The purpose of this paper is to study empirically the characteristics of the bias for a variety of conditions. The empirical analysis focuses on the bias in aggregate predictions of mode choice for work trips in Washington, D.C. Two main factors of bias are identified as the geographic aggregation level and the level of detail by which the distribution of explanatory variables is represented. Both the magnitude and the behavior of the aggregation bias are examined for a wide range of geographic aggregation levels, for several approximate representations of the distribution of explanatory variables, and for two different transportation options. The simplest aggregate prediction method uses average zonal variable values in the disaggregate model. The results of this study indicate that, by applying this method, substantially biased predictions may result. Applying more accurate distribution representation reduces this bias significantly but does not ensure its complete elimination. A residual bias of significant magnitude still remained in many of the situations examined. The implication is that sophisticated methods for bias reduction should be developed in order to make aggregate predictions with disaggregate models a more reliable analysis tool.**

The prediction of aggregate travel behavior is an indispensable element of the transportation planning process. Over the past 20 years demand for travel has been estimated by aggregate models, but, due to their limitations (1, 2), more research efforts have been directed to the development of methods for applying disaggregate models of individual choice behavior to aggregate travel predictions. These models have a number of advantages over the aggregate models: they are more policy sensitive; they require relatively little data; and they are more likely to be transferable (3). A detailed analysis of the disaggregate models is given by Ben-Akiva (4) and Charles River Associates (1).

As is the case with all models, certain errors are also involved with the disaggregate model aggregate predictions. Koppelman (5, 6, 7) presents a comprehensive analysis of the sources of these errors. This paper focuses on the behavior and magnitude of a specific source of error—the aggregation bias—which usually appears in aggregate predictions made by disaggregate models.

## APPROXIMATE AGGREGATION PROCEDURES

The process of predicting aggregate behavior with disaggregate models consists of three components (7): (a) disaggregate choice model; (b) representation of the distribution of explanatory variables of this disaggregate model; and (c) aggregation procedure, which operates on the above two components to obtain the aggregate prediction.

Several aggregation procedures have been discussed in the literature (6, 8, 9, 10, 11); some are more accurate than others. The more accurate ones apply the disaggregate models directly to disaggregate data or to the exact joint probability distribution of the explanatory variables. Other procedures that apply the disaggregate models to forms of aggregated data are less accurate (7). However, the more accurate procedures are usually less practical, while the less accurate ones are more convenient and in fact more widely used (12, 13, 14).

The aggregation bias is one of several contributors to total aggregate prediction error, which, in certain situations discussed later, is of considerable magnitude. This paper concentrates on those characteristics of the bias associated with the more popular procedures such as estimating future aggregate demand by applying the disaggregate model in its exact functional form to zonal aggregate data.

## MAIN CAUSES OF THE AGGREGATION BIAS

Aggregation is grouping individuals into zones and representing them as one group with common characteristics. The data of the new zone system are an aggregate representation of the real underlying distribution of the data from the detailed (individual) level. There are several ways to represent the aggregate zonal data.

The most common way is the representation of the data with their means, such as average zonal income and mean travel time. However, every aggregate representation of the underlying detailed data results in a loss of information. The specific nature of data variation at the detailed level is lost in the process of aggregation.

If the disaggregate model is nonlinear, then this loss of information causes a bias in the predictions obtained that is known as the aggregation bias (7). The main factors that contribute to it are the following.

1. Geographic aggregation level (GAL): Aggregation of zones simply increases within-group variability in the zonal and interzonal distributions of the data. Going to higher levels of zone aggregation is identical to saying that the zonal and interzonal related data distributions have larger variances. Representing these distributions with few measures (traditionally, only one measure, the mean) results in a loss of some variational characteristics of the data at the considered aggregation level. Therefore, the more we aggregate, the more information we lose, and hence the greater the aggregation bias.
2. Distribution representation method (DRM): The representation of the underlying distribution of explanatory variables at the considered level of aggregation is termed here "distribution representation method" (DRM). The more we aggregate, the greater is the information loss and, consequently, the greater the aggregation error. However, given a certain aggregation level, the more accurate the distribution representation method, the smaller the loss and, hence, the better the aggregate predictions.

Ideally, we would like to represent the data by their multivariate joint distribution and apply summation-integration procedures (7). In this case, no information is lost, and zone aggregation would not cause a bias. However, establishing this joint distribution is an intractable problem. Therefore, less accurate methods are used to represent the data (6, 8). They are imperfect compared to the multivariate distribution representation, but they are a more accurate representation compared to the means. They bring about a reduction in the aggregation bias, but some error still remains.

The traditional DRM in aggregate analysis is the use of weighted averages. That is, the aggregate prediction is made by making each variable in the disaggregate model equal to the weighted average of its disaggregate values.

Classification procedures are more accurate DRMs than the weighted averages method. They consist of approximating the distribution of a variable with a histogram of few classes. The expected demand for each class is estimated by using average values of all variables for this class. The overall demand is determined as a weighted sum of the individual classes.

## ANALYSIS OF BIAS IN AGGREGATE PREDICTION: PURPOSE AND METHODOLOGY

The purpose of the analysis is to study empirically the behavior of the aggregation bias, specifically to find

1. How the bias changes over a wide range of different GALs,
2. What the magnitude of the bias caused by using weighted average procedures is,
3. How well approximate DRMs of the explanatory variables reduce the bias, and
4. Whether different transportation alternatives produce similar (or different) biases.

A method for identifying the value of the aggregation bias was developed and tested in an applied prediction context. The method is illustrated by an empirical study of mode choice by work trip makers in the Washington, D.C., metropolitan area. The approach is to make multiple aggregate predictions of choice shares with a single disaggregate choice model for two different transportation options, for several GALs, and for several DRMs, each applied to all GALs and transportation options.

The predictions made with the disaggregate model by the complete enumeration procedure involve no bias (i.e., estimating expected shares by averaging the choice probabilities, calculated for each member of the population). They serve here as the no-bias reference level. The aggregation bias of the predictions obtained by applying the disaggregate model to a selected DRM, GAL, and transportation option, is determined by comparing these predictions with those obtained by complete enumeration.

## DEMAND MODEL, DATA, AND TRANSPORTATION OPTIONS

The demand model chosen for the analysis is of the N-dimensional logit form, developed by Peat, Marwick, Mitchell and Company (PMM) for San Diego (15). The model is developed to forecast central business district-(CBD-) oriented work trip makers' choice among three modes: transit passenger, automobile driver, and automobile passenger. The experiment focuses on the error in the aggregate prediction of transit share. The following variables appear in the model.

IN = household income,
TRNT = transit travel time, which is in-vehicle time plus transit transfer time,
TEXS = transit excess time, which is the walk to and from transit time plus first wait for transit,
FARE = transit fare,
HWTT = auto highway driving time,
COST = auto operating cost,
PARK = half of the auto parking cost, and
AEXS = auto excess time, which is the walk to and from the auto.

The transit share for individual t, $v_t$, is

$$v_t = [\exp(u_1)]/[\exp(u_1) + \exp(u_2 + G)] \quad (1)$$

where

$$u_1 = 1.1635 - 0.05625 \times \text{TRNT} - 0.09157 \times \text{TEXS} - 0.0106 \times \text{FARE},$$
$$u_2 = -1.4809 - 0.05625 \times \text{HWTT} - 0.09157 \times \text{AEXS} - 0.01062\,(\text{COST} + \text{PARK}), \text{ and}$$
$$G = 2.952\,[1 - \exp(-0.035 \times \text{IN})].$$

The model was reported to fit the San Diego data very well. Its predictions were also tested by PMM in the San Francisco and Boston transportation systems and were found to be satisfactory (15).

Three reasons underlie the choice of this particular model. First, it seemed desirable to test a realistic demand model. Similar models in terms of variables and value of parameters are likely to occur in many cases. Second, the model is sensitive to several level-of-service variables. Third, the model is nonlinear not only in its general structure but also in its utility function. Such a complex nonlinear form is potentially a major contributor to the aggregation bias (6).

The PMM model was applied to the Washington, D.C., data for predictions. The traffic corridor from the CBD northbound, through Silver Spring, Maryland, and beyond I-495 (the Capital Beltway) was chosen. It is a 144-traffic-zone segment of the entire metropolitan area (1207 traffic zones). This 144-zone system serves as our entire disaggregate population (144 × 144 pairs of transit share predictions).

Six superzone systems, each of a different GAL, were defined along the study corridor. The basis for the definition of a GAL in this experiment is the number of traffic zones in one superzone. The superzones were defined such that, for a given GAL, each had exactly the same number of traffic zones as the others. Consequently, the units by which the level of aggregation is quantitatively expressed are elementary (traffic) zones per superzone, in shorter notation EZ/SZ. The six aggregate systems have 72, 36, 18, 9, 3, and 1 superzones. The number of EZ/SZ in each is 2, 4, 8, 16, 48, and 144, respectively. The one-superzone system, for example, is simply the total study corridor, treated as one big zone.

Data files were prepared to represent the variables by their DRMs for the six GALs. The DRMs were

1. Weighted averages: that is, representation of the distribution of each variable by one measure (= one class), the mean;
2. Classification with 2, 3, and 4 classes: that is, approximating the distribution of a variable with a frequency histogram of 2, 3, and 4 classes (for example, in the two-class case the distribution is split at its median and is represented by the means of the two sec-

tions, each given a weight of ½; in the three-class case the distribution is divided at its tertiles and is represented by the means of the three classes, each given a weight of ⅓; the four-class approximation is analogous); and

3. Marginal distributions: that is, each variable is represented by its exact marginal frequency histogram.

The approximate distributions applied consisted of the representation of few variables (one, two, or three) with their marginal distribution or approximated frequency histogram (i.e., classification), while all the rest of the variables were at their means.

Two transport alternatives were chosen for the analysis. Both concerned the routing of the public transit vehicles. The first, alternative 1, consisted of the bus services, as provided in 1968, where transfers were made by individuals for certain origin-destination pairs. The second alternative, alternative 2, cut all the transfers for the same trip interchanges assuming 5 min/transfer.

The PMM disaggregate model was applied to 292 and 128 different combinations of GAL and DRM for alternatives 1 and 2, respectively. The behavior of the aggregation bias was examined for all these combinations. The numerical results illustrated in Figures 1, 2, 3, 4, and 5 provide a representative sample of those obtained in the entire study. A more detailed description of the analysis and results is given elsewhere (8).

## MEASURES OF THE AGGREGATION BIAS

The aggregation bias is expressed by the following three error measures: average interzonal percentage error, total area percentage error, and root-mean-square error.

### Average Interzonal Percentage Error

For average interzonal percentage error ($P_{AE}$), let $p_{ij}$ be the basic error measure in transit share prediction between superzones i and j, such that

$$p_{ij} = (V'_{ij} - V_{ij})/V_{ij} \tag{2}$$

where

$V_{ij}$ = unbiased reference level aggregate prediction for superzone pair (i, j), and
$V'_{ij}$ = aggregate prediction by approximate aggregation procedure for superzone pair (i, j).

Equation 2 expresses the magnitude of the bias as a proportion of the unbiased prediction.

The average level of the interzonal prediction error is calculated by averaging out all interzonal errors:

$$P_{AE} = \sum_{i,j} w_{ij} p_{ij} \tag{3}$$

where $w_{ij}$ is the weight of trips of superzone pair (i, j) with respect to all superzone pair trips.

### Total Area Percentage Error

The purpose of total area percentage error ($P_{total}$) is to capture the bias of the entire study area prediction for transit ridership. It is analogous to $P_{AE}$ and is defined by

$$P_{total} = (V'_{..} - V_{..})/V_{..} \tag{4}$$

where $V_{..}$ and $V'_{..}$ are the unbiased and approximate entire area aggregate predictions for transit share, respectively.

### Root-Mean-Square Error

Root-mean-square error (RMSE) is defined by

$$RMSE = \left[ \sum_{i,j} w_{ij} (V'_{ij} - V_{ij})^2 \right]^{1/2} \tag{5}$$

## ANALYSIS OF RESULTS

Several characteristics of the aggregation bias were examined, as they varied for several GALs and DRMs. The bias was estimated by a variety of measures, all of which indicated similar bias behavior. Four more measures have been applied (8) with similar results. The empirical findings follow.

### Magnitude of the Aggregation Bias

The results obtained in this study show that the use of weighted averages in the disaggregate model causes significant bias. The values of the biases ranged from 16 to 40 percent error, depending on the GAL (Figures 1 and 3). These are much greater biases, compared to the 7 percent average bias reported by Koppelman (7).

This means that the application of the weighted averages method for aggregate prediction with disaggregate models may result in substantial biases, unless special procedures for bias reduction are applied.

The application of more accurate representation of the distribution of explanatory variables reduced the bias significantly, especially for high GALs. This finding agrees with Koppelman's conclusion about the performance of classification procedures. However, the residual bias in this study (13-35 percent) is substantially greater than the 1.4 percent average bias reported by Koppelman (7).

### Flattening of Bias With Higher Levels of Aggregation

Although the bias monotonically increases with aggregation, it does so at a decreasing rate until it reaches a certain point at which the rate almost does not change. This characteristic is clearly illustrated by Figures 1 and 3.

An interesting phenomenon is that the bias increases very rapidly at very low levels of aggregation. Moving from the original system of 144 traffic zones (GAL = 1 EZ/SZ) to a 72-superzone system (GAL = 2 EZ/SZ) and using the weighted averages method, an error jump of 15.8 percent is made. The next move to the 36-superzone system (GAL = 4 EZ/SZ) produces an additional error of only 2.8 percent, which continues to decrease for higher levels of aggregation.

This characteristic has an important practical implication. Aggregation will have to be pursued in many analysis situations. Since most of the error is already made for low GALs, it may be cost effective to go into much higher levels of aggregation. The additional small error is traded off for large savings in computational costs.

**Figure 1. Aggregation bias for different GALs and DRMs for alternative 1.**

| GAL EZ/SZ | 2 | 4 | 8 | 16 | 48 | 144 |
|---|---|---|---|---|---|---|
| WEIGHTED AVERAGES | | | | | | |
| $P_{total}$ | -15.8 | -18.6 | -21.6 | -23.4 | -26.6 | -40.4 |
| RMSE | 4.2 | 4.8 | 5.3 | 5.6 | 6.3 | 9.4 |
| $P_{AE}$ | -19.9 | -23.6 | -27.2 | -29.6 | -32.3 | -40.4 |
| TEXS BY MARGINAL DISTRIBUTION (a) | | | | | | |
| $P_{total}$ | -14.8 | -16.8 | -19.2 | -20.7 | -23.2 | -34.2 |
| RMSE | 3.9 | 4.3 | 4.7 | 5.0 | 5.5 | 8.0 |
| $P_{AE}$ | -18.0 | -20.7 | -23.5 | -25.4 | -27.7 | -34.2 |
| TRNT BY MARGINAL DISTRUBUTION (a) | | | | | | |
| $P_{total}$ | -14.6 | -16.2 | -17.7 | -18.6 | -19.3 | -23.1 |
| RMSE | 3.8 | 4.2 | 4.4 | 4.5 | 4.6 | 5.4 |
| $P_{AE}$ | -17.8 | -19.6 | -21.4 | -22.3 | -22.2 | -23.1 |
| TEXS AND TRNT BY MARGINAL DISTRIBUTION (a) | | | | | | |
| $P_{total}$ | -13.6 | -14.6 | -15.4 | -15.9 | -16.1 | -18.1 |
| RMSE | 3.6 | 3.8 | 3.9 | 3.9 | 3.9 | 4.2 |
| $P_{AE}$ | -15.8 | -16.6 | -17.5 | -18.0 | -17.5 | -18.1 |
| TEXS AND TRNT EACH BY 3 CLASSES (a) | | | | | | |
| $P_{total}$ | -14.1 | -15.4 | -16.5 | -17.1 | -17.9 | -21.4 |
| RMSE | 3.7 | 4.0 | 4.1 | 4.2 | 4.30 | 5.0 |
| $P_{AE}$ | -16.7 | -18.3 | -19.5 | -19.9 | -20.1 | -21.4 |

(a) All other variables are at their means.

**Figure 2. Bias for two transportation alternatives in total area percentage error.**

| GAL (a) / ALTERNATIVE | 2 | 4 | 8 | 16 | 48 | 144 |
|---|---|---|---|---|---|---|
| WEIGHTED AVERAGES | | | | | | |
| 1 | -15.8 | -18.6 | -21.6 | -23.4 | -26.6 | -40.4 |
| 2 | -14.7 | -16.8 | -18.8 | -20.0 | -22.6 | -33.9 |
| TEXS BY MARGINAL DISTRIBUTION (b) | | | | | | |
| 1 | -14.8 | -16.8 | -19.2 | -20.7 | -23.2 | -34.2 |
| 2 | -13.7 | -15.1 | -16.5 | -17.3 | -19.2 | -27.9 |
| TRNT BY MARGINAL DISTRIBUTION (b) | | | | | | |
| 1 | -14.6 | -16.2 | -17.7 | -18.6 | -19.3 | -23.1 |
| 2 | -14.0 | -15.4 | -16.5 | -17.0 | -17.9 | -21.3 |
| TEXS AND TRNT BY MARGINAL DISTRIBUTION (b) | | | | | | |
| 1 | -13.6 | -14.6 | -15.4 | -15.9 | -16.1 | -18.1 |
| 2 | -13.0 | -13.7 | -14.2 | -14.4 | -14.8 | -16.6 |
| TEXS AND TRNT BY 3 CLASS HISTOGRAM (b) | | | | | | |
| 1 | -14.1 | -15.4 | -16.5 | -17.1 | -17.9 | -21.4 |
| 2 | -13.4 | -14.4 | -15.1 | -15.3 | - | - |

(a) In EZ/SZ

(b) All other variables are at their means.

Figure 3. Aggregation bias for different GALs for alternative 1.

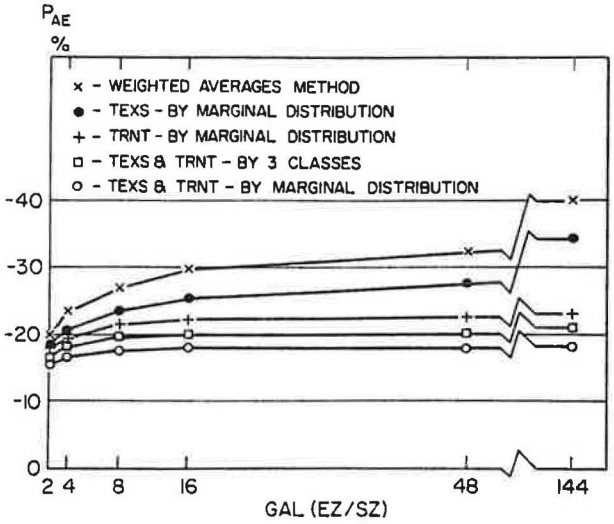
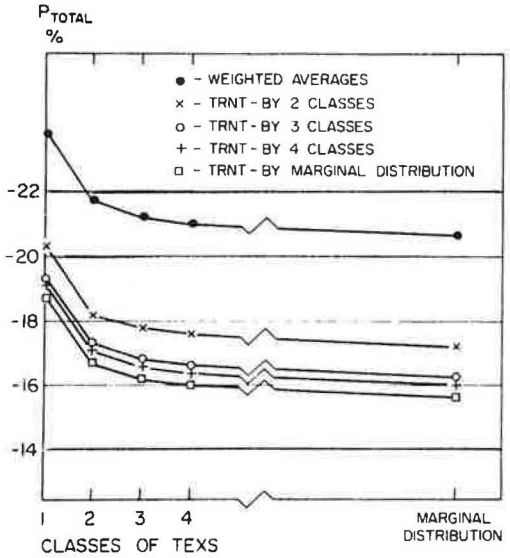



Figure 4. Areawide modal split percentage error for alternative 1.

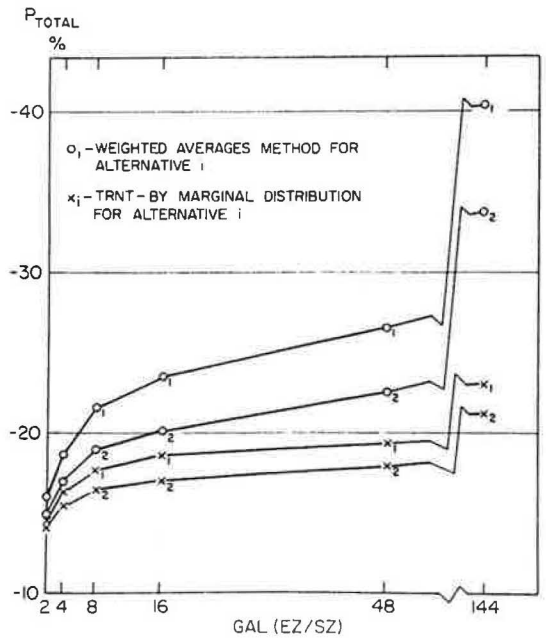



## Usefulness of More Accurate Variable Representation

Figure 4 illustrates the aggregation biases produced by transit excess time (TEXS) and travel time (TRNT), as they are represented by all possible combinations of pair classifications under the independence assumption (i.e., representing each by its marginal class histogram as if it were independent of the other variable). This figure indicates the pattern of diminishing marginal error with better representation. For every given representation of TRNT, the curve monotonically decreases as the accuracy of the representation of TEXS improves, but the rate of decrease is slower and levels off as its representation approaches the marginal distribution. These same results occur for all GALs, for all variables, and for all DRMs examined.

As we shift our attention to a pair representation of the variables, each with the same number of classes, the same behavior of diminishing marginal error continues to hold. There is a large decrease in error

Figure 5. Aggregation bias for two transportation alternatives.




(about 5.5 percent) from the weighted averages method to representing TEXS and TRNT with a two-class histogram for each; the next step (three-class histogram for each) improves the bias only by about 1.5 percent, and so on.

We have assumed that TEXS and TRNT are uncorrelated. Consequently, they were represented each by an approximated marginal distribution. The results, as expressed by Figure 4, do not contradict this assumption. The figure illustrates that, with every class added to represent any of the variables, (a) the overall error is reduced and (b) this reduction is made in quite a smooth manner, where the rate of reduction for each variable is unaffected by the DRM of the other variable.

As can be seen from Figure 4, the largest portion of the total bias that can potentially be corrected by representing a single variable with its marginal distribution is already achieved by a two-class representation (in our example, about two-thirds of the total error). In the case of TEXS, the two-class representation reduces the error by about 2 percent compared to 3 percent by marginal distribution, while the corresponding figures for TRNT are 3.5 and 5.2 percent. These results are consistent with those of Koppelman (6), who observed the same pattern for his model. As our attention shifts to pair representation, an analogous phenomenon occurs.

## Aggregation Bias and Transportation Alternatives

The aggregate prediction bias is a function of the transportation alternative under examination. As Figures 2 and 5 illustrate, the difference between the prediction bias of two transportation alternatives is not necessarily small, although the same model is used for both, and only one variable, TRNT (which expresses the change in policy), is changed.

In our example, the difference between the prediction biases increases with the level of aggregation for all DRMs, especially for the weighted averages method, and other DRMs that do not represent more accurately the variable indicating the change in policy.

## SUMMARY

The specific conclusions summarizing the findings about the behavior of the aggregation bias are

1. The application of a nonlinear disaggregate demand model to aggregate data may result in substantially biased predictions;
2. A large aggregation bias is likely to be introduced even in very low GALs;
3. As aggregation increases, the bias increases monotonically but with a diminishing marginal rate;
4. A representation of an explanatory variable (which is uncorrelated with others), even if only with a small number of classes, reduces its error contribution substantially;
5. The bias reduction is generally greater if a number of variables are represented with few classes (two or three), than if only one or two of them are represented very accurately;
6. The aggregation bias is a function of the transportation alternative; and
7. If only some variables are represented more accurately, while other variables are represented by their means, then a residual aggregation bias of a significant magnitude may occur.

The practical implications of these findings are

1. Aggregate predictions with disaggregate models are not meaningful unless the aggregation biases are explicitly considered;
2. Since the aggregation bias is systematic and significant, as in our case study, it is important to correct it;
3. Classification methods applied to correcting the bias are not always efficient (in view of the possible high residual value of this bias of 13-34 percent, sophisticated bias correction methods should be developed and applied); and
4. If it is desired to test a variety of transport options and if the variable reflecting them is changed over a wide range, then it is important to represent more accurately the distribution of this variable.

Indeed, the empirical analysis has focused on examining a specific model structure in a specific transportation system. However, the characteristics of the applied PMM demand model appear to be representative of many other nonlinear demand models in terms of the functional form (i.e., s-shaped curve), the number and nature of explanatory variables, and the value of the model's parameters (or elasticities). Similarly, the Washington, D.C., data seem to be representative of many urban areas in terms of their variety of socioeconomic characteristics, trip patterns, available transportation alternatives, and so on.

It is, of course, possible that the demand function will be of unusual nature (e.g., step function, many discontinuities), or that the analysis situation will have extreme characteristics (e.g., very high- and very low-income people living in the same blocks). But such cases are not very common. Assuming that the characteristics of our case study are representative of many analysis situations, then in view of preliminary results (6) and in the absence of any evidence to the contrary, the empirical results obtained here appear to indicate the general behavior of the aggregation bias.

## ACKNOWLEDGMENTS

This study was part of a project at the Massachusetts Institute of Technology, sponsored by the U.S. Department of Transportation for development of an aggregate model of urban travel behavior. The data were provided by the Metropolitan Washington Council of Governments.

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*Publication of this paper sponsored by Committee on Passenger Travel Demand Forecasting and Committee on Traveler Behavior and Values.*