# Optimization of Pavement Rehabilitation and Maintenance by Use of Integer Programming 

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#### Abstract

An integer programming technique has been used to develop an operating computer program called RAMS, which determines optimal maintenance strategies for pavements by maximizing the overall maintenance effectiveness for all highway segments considered. The program can use numerous maintenance strategies, resources, and feasibility constraints to obtain solutions. An example problem that contains actual field data on 15 highway segments located in one Texas highway district was used to demonstrate typical program input and output. This example revealed that, for 9 of the 15 pavement segments studied, maintenance strategies selected by the computer program were essentially identical to those selected by district personnel of the Texas State Department of Highways and Public Transportation. Both RAMS cases presented are optimal with respect to department selections.


Optimization techniques are being applied to the problem of allocating highway rehabilitation and maintenance funds because of their established record in industry of saving around 10 to 25 percent of equipment maintenance budgets. If this kind of record can even be approached in the area of highway maintenance, very significant savings can be realized nationally.

This paper describes the solution to such a problem that has been achieved by using an operating computer program called RAMS (rehabilitation and maintenance strategies). The program uses an integer programming technique that is based on a mathematical model of the optimization process formulated by Lu and Lytton (1). The program is part of a methodology currently being developed by the Texas Transportation Institute for the Texas State Department of Highways and Public Transportation (TSDHPT).

The approach described here is different from what has been tried elsewhere. The University of California at Berkeley has developed an optimization computer program called CALMS 1 that uses a Markov process for describing the transition of a pavement from one condition to another (2). Two kinds of pavement conditions are considered-roughness and cracking-and these conditions are treated by means of three major alternative strategies-thin, medium, and heavy overlays. In a similar development, the Washington State Department of Highways has developed an optimization procedure for their highway system (3).

The method discussed hēre recognizes that many maintenance and rehabilitation strategies are in fact used by all transportation agencies; they range from seal coating to patching and overlaying to complete reconstruction of the pavement. The problem described in this paper recognized five types of distress and six maintenance and rehabilitation strategies. The program is written flexibly so that either more or fewer distress types and maintenance strategies may be used.

The purpose of this paper is to describe how optimal maintenance solutions for highway segments are obtained by using the RAMS program and to show the results of the solution of an actual problem for a group of highway segments in Texas as well as a general description of the required inputs.

## OPTIMIZATION METHODOLOGY

The mathematical model for maximizing the overall effectiveness of maintenance activities as applied to highways may be formulated in terms of $0-1$ integer programming, which may be written as follows (units of measurement used in the program are formulated in U.S. customary units, and in these cases no SI units are given):
$\operatorname{Maximize} \sum_{\mathrm{i}=1}^{\mathrm{N}_{H}} \sum_{\mathrm{j}=1}^{N_{S}} \sum_{\mathrm{k}=1}^{\mathrm{N}_{\mathrm{D}}} \sum_{\mathrm{t}=1}^{\mathrm{N}_{\mathrm{T}}} \mathrm{L}_{\mathrm{ij}} \mathrm{L}_{2 \mathrm{i}} \mathrm{d}_{\mathrm{ijk}} \mathrm{P}_{\mathrm{ijkt}} \mathrm{X}_{\mathrm{ij}}$
subject to the following constraints-(a) decision variable,
$\sum_{j=1}^{N_{S}} x_{i j} \leqslant 1 \quad i=1,2, \ldots, N_{H}$
(b) available supplies,
$\sum_{i=1}^{N_{H}} \sum_{j=1}^{N_{S}} s_{i j E} L_{1 i} L_{2 i} x_{i j} \leqslant S_{g} \quad g=1,2, \ldots, N_{G}$
(c) available equipment,
$\sum_{i=1}^{N_{H}} \sum_{j=1}^{N_{S}} e_{i j f} L_{1 i} L_{2 i} x_{i j} \leqslant E_{f} \quad f=1,2, \ldots, N_{F}$
(d) available manpower,
$\sum_{k=1}^{N_{H}} \sum_{j=1}^{N_{S}} h_{i j q} L_{1 i} L_{2 i} x_{i j} \leqslant H_{q} \quad q=1,2, \ldots, N_{Q}$
(e) available budget,
$\sum_{i=1}^{N_{H}} \sum_{j=1}^{N_{S}} C_{i j} L_{1 i} L_{2 i} x_{i j} \leqslant C$
(f) minimum rating for each type of distress,
$I_{i k}+\sum_{j=1}^{N_{S}} d_{i j k} P_{i j k t} X_{i j} \geqslant R_{i k t} \quad \begin{aligned} & i=1,2, \ldots, N_{H} \\ & k=1,2, \ldots, N_{D} \\ & t=0,1, \ldots, N_{T}\end{aligned}$
and (g) minimum overall pavement rating score,
$\sum_{k=1}^{N_{D}}\left(r_{i k}+\sum_{j=1}^{N_{S}} d_{i k} P_{i j k t} x_{i j}\right) \geqslant W_{i t} \quad i=1,2, \ldots, N_{H}$
where

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    \(\mathrm{N}_{\mathrm{H}}=\) number of highway segments in the analysis;
    \(\mathrm{N}_{\mathrm{s}}=\) number of maintenance strategies;
    \(\mathrm{N}_{\mathrm{D}}=\) number of distress types;
    \(\mathrm{N}_{\mathrm{T}}=\) number of years in the analysis period;
    \(L_{1 i}=\) pavement length of highway segment \(i\) (miles);
    \(L_{2 i}=\) pavement width of highway segment \(i(f t) ;\)
    \(\mathrm{d}_{\mathrm{ijk}}=\) potential gains of pavement rating of highway
        segment \(i\), maintenance strategy \(j\), and dis-
        tress k;
\(\mathbf{P}_{\mathrm{ijkt}}=\) pavement survival probability of highway seg-
        ment \(i\), maintenance strategy \(j\), and distress
        type \(k\) at time \(t ;\)
    \(\mathrm{x}_{\mathrm{ij}}=\mathrm{a}\) decision variable that will be 1 if mainte-
        nance strategy \(j\) is selected for highway seg-
        ment i and 0 otherwise;
    \(S_{\mathrm{ijg}}=\) amount of material (or supply) of type g per
        unit surface area 1 mile long and 1 ft wide re-
        quired for highway segment \(i\) if maintenance
        strategy \(j\) is selected;
    \(\mathrm{S}_{\mathrm{B}}=\) total amount of available material (or supply)
        of type g ;
    \(\mathrm{N}_{\mathrm{G}}=\) number of different material (or supply) types;
\(\mathrm{e}_{\mathrm{ijf}}=\) amount of equipment of type f required for
    highway segment \(i\) if maintenance strategy \(j\)
    is selected (equipment days per 1 -mile-long
    and 1 -ft-wide surface area);
    \(\mathrm{E}_{\mathrm{f}}=\) total amount of equipment of type f available
        (equipment days);
    \(N_{F}=\) number of different equipment types;
\(\mathrm{h}_{\mathrm{j} \mathrm{jq}}=\) amount of manpower of type \(q\) required for
    highway segment \(i\) if maintenance strategy \(j\)
    is selected (person days per unit 1 -mile-long
    and 1 -ft-wide surface area);
    \(\mathrm{H}_{\mathrm{q}}=\) total amount of manpower of type \(q\) available
        (person days);
    \(N_{Q}=\) number of different manpower types;
    \(\mathrm{C}_{\mathrm{ij}}=\) cost required for highway segment i if main-
        tenance strategy \(j\) is selected (dollars per unit
        1 -mile-long and 1 -ft-wide surface area);
    \(\mathrm{C}=\) total budget available (dollars);
\(r_{i k}=\) current pavement rating of highway segment \(i\)
        and distress type k ;
\(\mathrm{R}_{\mathrm{ikt}}=\) minimum required pavement rating of highway
        segment i and distress type k at time t ; and
\(\mathrm{W}_{\mathrm{it}}=\) minimum required pavement rating of highway
        segment \(i\) of all types of distress at time \(t\).
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## Solution Procedure

Optimizing the maintenance strategies for a large number of highway segments for which there are many strategies, resources, and feasibility constraints exceeds the capacity of current mathematical integer programming techniques to achieve exact optimal solutions. The problem formulated by the use of integer programming is solved by the effective-gradient method of Senju and Toyoda (4), which achieves near optimal solutions.

Table 1. Resource requirements for five highway segments.

| Highway | Percentage Total | Percentage Total |  |
| :---: | :---: | :---: | :---: |
|  | Available Budget | Available Material | Maintenance |
| Segment | Resource Used | Resource Used | Effectiveness |
| $\mathrm{H}_{1}$ | 74 | 70 | 6507 |
| $\mathrm{H}_{2}$ | 46 | 45 | 4072 |
| $\mathrm{H}_{3}$ | 76 | 70 | 3863 |
| $\mathrm{H}_{4}$ | 42 | 40 | 78109 |
| $\mathrm{H}_{5}$ | 47 | 50 | 78355 |
| Total | $285^{\circ}$ | 275 | 170906 |
| Note: Total available budget $=\$ 300000$. |  |  |  |
| "Extra resource required $=185$ percent. <br> ${ }^{\text {b }}$ Extra resource required $=175$ percent. |  |  |  |

## Effective-Gradient Method

A simple example uses five highway segments. The data for these highway segments come from a larger, more realistic problem that is discussed later. The goal of this short example is to demonstrate by use of the effective-gradient method how the five segments can be maintained optimally. For simplicity, it is assumed that only one maintenance strategy and two resources are needed. The maintenance strategy chosen is reconstruction, and the two resources are the amount of the budget and the materials available to accomplish the work. (The RAMS problem presented later actually considers six maintenance strategies and the resources of materials, equipment, manpower, and budget.)

Table 1 gives a listing of the five highway segments and the percentage of the total resources used for each. (These segments correspond to the last five segments given in Table 4.) The maintenance strategy that is considered is reconstruction with a total available budget of $\$ 300000$. The cost to reconstruct each segment was obtained by multiplying the length and width by the cost per unit area. The percentage of materials required by each segment was assumed to approximate the percentage of the budget consumed. The total required for each resource is shown and is the sum of the individual percentages for each highway segment. For the budget resource, the total required is larger than the available budget by a factor of 2.85 . A similar situation occurs for the materials resource.

Maintenance effectiveness is also given in Table 1 and is computed from the objective function in Equation 1. Thus, the effectiveness of maintenance is obtained by multiplying length, width, gain of rating for each distress, and the sum of the survival probabilities (gain of rating and survival probabilities are discussed in more detail later in this paper). The effectiveness of maintenance would be greater for highly distressed pavements than for nondistressed pavements of equal length and width. Highway segment $\mathrm{H}_{5}$ can be used to demonstrate how maintenance effectiveness is computed. For $\mathrm{H}_{5}$,

1. Length $=11.980 \mathrm{~km}$ ( 7.444 miles),
2. Width $=6.1 \mathrm{~m}(20 \mathrm{ft})$,
3. Gain-of-rating points for reconstruction for types of distress present in the roadway are as follows:

| Type of Distress | Maximum Points Available | Current Condition Rating | Gain of <br> Rating |
| :---: | :---: | :---: | :---: |
| Rutting | $=15$ | - 10 | $=5$ |
| Alligator cracking | $=25$ | - 10 | $=15$ |
| Longitudinal cracking | = 25 | - 10 | $=15$ |
| Transverse cracking | $=20$ | - 8 | $=12$ |
| Failures per mile | $=40$ | - 20 | 20 |

4. Probabilities of survival for reconstruction summed over 10 years for types of distress present in the roadway are as follows:

| Type of Distress |  | Probability of Survival |
| :--- | :--- | :--- |
| Rutting |  | 7.97 |
| Alligator cracking |  | 6.86 |
| Longitudinal cracking | 9.25 |  |
| Transverse cracking | 9.25 |  |
| Failures per mile | 6.69 |  |

5. Maintenance effectiveness can be calculated as

$$
\begin{align*}
\text { Maintenance effectiveness }= & \mathrm{L}_{15} \mathrm{~L}_{25} \sum_{\mathrm{i}=1}^{1} \sum_{\mathrm{j}=1}^{1} \sum_{\mathrm{k}=1}^{5} \sum_{\mathrm{i}=1}^{10} \mathrm{~d}_{\mathrm{ijk}} \mathrm{P}_{\mathrm{ijkt}} \\
= & (7.444)(20)[(5)(7.97)+(15)(6.86) \\
& +(15)(9.25)+(12)(9.25) \\
& +(20)(6.69)]=78355 \tag{9}
\end{align*}
$$

In Figure 1, the vectors $\overline{\mathrm{H}}_{1}, \overline{\mathrm{H}}_{2}, \ldots, \overline{\mathrm{H}}_{5}$ are plotted as a function of the required resources for each highway segment; i.e., $\overline{\mathrm{H}}_{1}$ denotes the amount of budget and materials required if reconstruction is done on this segment. The following vectors are defined: $\overline{\mathrm{R}}=$ resultant vector of all highway segments $=\bar{H}_{1}+\overline{\mathrm{H}}_{2}+\overline{\mathrm{H}}_{3}+$ $\overline{\mathrm{H}}_{4}+\overline{\mathrm{H}}_{5} ; \overline{\mathrm{L}}=$ limiting resource vector $=(100,100$ in example); and $\overline{\mathrm{E}}=$ excess vector $=\overline{\mathrm{R}}-\overline{\mathrm{L}}(285,275)-$ $(100,100)=(185,175)$.

If enough resources are available to reconstruct all five highway segments, that is what should be done. Of course, that situation will rarely occur. Resources are generally so scarce that maintenance cannot be applied to all the highway segments being considered. The

Figure 1. Vector sum of resource requirements for each highway segment.


Figure 2. Effective reduced length for highway segment 5.


Budget Resource
(Percent of Available)
maintenance should be applied to that combination of highway segments that maximizes the overall effectiveness of maintenance and satisfies the available resource restraints. Thus, some method must be used to determine which segments are dropped from consideration.

Figure 2 shows highway segment $\mathrm{H}_{5}$ being dropped. This caused the point $R$ to move in the general direction of $L$ and 78355 units of maintenance effectiveness to be lost. The contribution of highway segment $\mathrm{H}_{5}$ to the movement back toward $L$ (to satisfy the resource availability constraint requirement) is expressed by the projected length of vector $\bar{H}_{5}$ on the excess vector $\overline{\mathrm{E}}$ (denoted by $\overline{A^{\prime} R}$ ). The decision to drop a highway segment should be based on a comparison between maintenance effectiveness and the projected length on the vector $\overline{\mathrm{E}}$. This comparison determines the effective gradient and is taken as the ratio of maintenance effectiveness for a highway segment to the projected length $\overline{\mathrm{A}^{\prime} R}$ for that highway segment. Phrased another way, effective gradient indicates which highway segments show the greatest effectiveness of maintenance for the smallest amount of resources. Highway segments with small effective gradients are less desirable to schedule for maintenance than are segments with large effective gradients. Therefore, the effective gradient for each segment is calculated, and those segments that have the smallest gradients are dropped until the availability resource constraints are satisfied.

The effective gradient for each highway segment is given below:

| Proposed Order <br> of Segments | Effective <br> Gradient |  |
| :--- | ---: | ---: |
| $\mathrm{H}_{3}$ |  | 37 |
| $\mathrm{H}_{2}$ | 63 |  |
| $\mathrm{H}_{1}$ | 64 |  |
| $\mathrm{H}_{5}$ | 1144 |  |
| $\mathrm{H}_{4}$ |  | 1347 |

To demonstrate how the effective gradient is calculated, let $\overline{\mathrm{U}}$ stand for a unit vector parallel to $\overline{\mathrm{E}}$ and with the same sense:
$\overline{\mathrm{U}}=\overline{\mathrm{E}} /|\overline{\mathrm{E}}|$
and from the example,
$\overline{\mathrm{U}}=\left[185 /\left(185^{2}+175^{2}\right)^{1 / 2}, 175 /\left(185^{2}+175^{2}\right)^{1 / 2}\right]$
Let $\mathrm{U}_{5}=$ projection of vector $-\overline{\mathrm{H}}_{5}$ on vector $-\overline{\mathrm{U}}$ where $\mathrm{U}_{5}$ is given by the scalar product of vectors $-\bar{H}_{5}$ and - $\bar{U}$ :

$$
\begin{align*}
\mathrm{U}_{5}= & -\overline{\mathrm{H}}_{5} \times-\overline{\mathrm{U}}=(47)\left[185 /\left(185^{2}+175^{2}\right)^{1 / 2}\right] \\
& +(50)\left[175 /\left(185^{2}+175^{2}\right)^{3 / 2}\right]=68.5 \tag{12}
\end{align*}
$$

Let
$\mathrm{G}_{5}=$ effective gradient of maintenance effectiveness
$=$ maintenance effectiveness $/ \mathrm{U}_{5}=78355 / 68.5=1144$
The effective gradients for the other four highway segments were similarly computed.

By using the ranked effective gradients, a choice can be made as to the highway segments to be dropped:

| Condition | Budget Resource (\%) | Material Resource (\%) |
| :---: | :---: | :---: |
| Initial excess resource requirements | 185 | 175 |
| Subtract $\mathrm{H}_{3}$ | 109 | 105 |
| Subtract $\mathrm{H}_{2}$ | 63 | 60 |
| Subtract $\mathrm{H}_{1}$ | -11 | -10 |

It can be seen in the table that, after highway segments $\mathrm{H}_{3}, \mathrm{H}_{2}$, and $\mathrm{H}_{1}$ are dropped, 11 percent of the budget and 10 percent of the materials are not used. The overall result is that only segments $\mathrm{H}_{4}$ and $\mathrm{H}_{5}$ can be reconstructed and represent the optimal solution.

The problem of determining optimal maintenance strategies intensifies rapidly when additional strategies, resources, and distress considerations are added. The RAMS program treats this kind of problem.

## Program Steps

The RAMS program considers the following steps in obtaining optimal maintenance solutions:

1. Find the feasible maintenance strategies for each highway segment according to the minimum rating for each distress constraint (Equation 7) and the overall pavement rating constraint (Equation 8).
2. Rank the feasible strategies for each highway segment according to the ratio of maintenance effectiveness to resource requirement. The ranking criterion is computed as follows:
$r_{i j}=M_{i j} / \sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{a}_{\mathrm{ij} 1}$
where

$$
\begin{aligned}
r_{1 j}= & \text { ranking ratio for highway segment } i \text { and strat- } \\
& \text { egy } j, \\
M_{1 j}= & \text { maintenance effectiveness if strategy } j \text { is ap- } \\
& \text { plied to highway segment } i, \text { and } \\
a_{1,11}= & \text { percent of lth type of resource needed if strat- } \\
& \text { egy } j \text { is applied to highway segment } i .
\end{aligned}
$$

For each highway segment, the feasible strategies are ranked according to the highest value of the ranking ratio.
3. Select the best ranked feasible strategy for each highway segment and calculate the effective gradient.
4. Sort the effective gradients for all highway segments.
5. Select the highway segment with the smallest effective gradient and exchange its currently considered strategy with the next best available. This highway segment with its exchanged strategy and the remaining highway segments with their current strategies are used to recalculate the effective gradients for all highway segments. The program then switches back to step 4 un-
less all the available feasible strategies for this highway segment are exhausted, in which case the program goes to step 6 .
6. Make one of two possible decisions: (a) If any of the constraints are exceeded, drop the highway segment from the solution, subtract the resources required for the segment from the excess resource vector, recalculate the effective gradients for the remaining highway segments with their current strategies, and return to step 4; or (b) if all of the constraints are satisfied, there is no need to drop more highway segments, so go on to step 7.
7. The remaining highway segments with their corresponding strategies constitute the optimal solution set. If additional or "slack" capacity is available in the resource constraints, additional highway segments may be added back to eliminate or reduce this capacity.

## EXAMPLE PROBLEM

A larger example problem can be used to compare the maintenance strategies selected by TSDHPT personnel with those selected by the RAMS program. The problem was prepared by using field data obtained from 15 highway segments in TSDHPT district 17 in east-central Texas. Eleven of the segments selected were scheduled for various kinds of contracted highway maintenance or rehabilitation within the next several months. The highway department has actually scheduled these segments for either a seal coat, asphalt concrete overlay, or reconstruction. Four additional segments were added to the initial 11 because they were considered to be in excellent condition and, as such, to require no significant maintenance. Although the intent of the RAMS methodology was not to optimize maintenance on segments that required none, it was felt that adding the 4 segments would demonstrate that the program could distinguish a segment that needs rehabilitation from one that does not.

## Description of Highway Segments

Table 2 gives general information for each highway segment used. It includes a general description of each segment and the maintenance strategies scheduled by TSDHPT. The average serviceability index (SI) given for each segment was obtained by use of the Mays road meter. As can be seen in the table, a mixture of ü.s., state, and îarm-to-market ( $\mathbf{F i v i}$ ) highways were used. Pavement length and width for each highway were direct inputs into the computer program.

Table 2. Data for highway segments used in example problem.

| Segment | Highway | County | Length (km) | Width (m) | Avg <br> SI | TSDHPT Scheduled Maintenance |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | US-79 | Milam | 7.282 | 7.9 | 2.7 |  |
| 2 | US-77 | Milam | 19.821 | 8.5 | 2.5 | $2.5-\mathrm{cm}$ HMAC overlay |
| 3 | US-190 | Milam | 5.821 | 7.9 | 2.1 | $3.8-\mathrm{cm} \mathrm{HMAC} \mathrm{level-up} \mathrm{overlay}$ |
| 4 | State* | Madison | 11.265 | 6.1 | 2.3 | Seal coat |
| 5 | State ${ }^{\text {a }}$ | Madison | 3.632 | 6.7 | 1.9 | Seal coat |
| 6 | FM-1696 | Walker | 22.215 | 6.1 | 1.9 | Seal coat |
| 7 | FM-1791 | Walker | 19.914 | 6.7 | 0.8 | Seal coat |
| 8 | FM-2821 | Walker | 5.370 | 7.3 | 2.1 | Seal coat |
| 9 | Tex-30 | Walker | 11.885 | 7.9 | 3.4 | Seal coat |
| 10 | Tex-36 | Burleson | 19.346 | 7.9 | 3.9 | None |
| 11 | US-290 | Washington | 14.515 | 7.9 | 3.9 | None |
| 12 | US-79 | Milam | 9.083 | 7.9 | 4.5 | None |
| 13 | Tex-36 | Burleson | 15.001 | 7.9 | 4.7 | None |
| 14 | State ${ }^{\text {a }}$ | Brazos | 10.729 | 6.1 | 0.9 | Reconditioning of base and surfacing |
| 15 | FM-908 | Milam | 11.980 | 6.1 | 1.5 | Reconditioning of base and surfacing |

Notes: $\begin{aligned} 1 \mathrm{~km}=0,62 \text { mile; } 1 \mathrm{~m}=3.3 \mathrm{ft} \text {; and } 1 \mathrm{~cm}=0.39 \mathrm{in} \text {, } \\ \text { HMAC }=\text { hot-mix asphalt concrete. }\end{aligned}$
HMAC $=$ hot-mix asphalt concrete.
${ }^{-}$Old Spanish road,

## Pavement Condition

The pavement condition rating system that was used is the one currently being implemented in Texas $(\underline{5}, \underline{6})$ with slight modification. This system is based on evaluating the quantity and severity of nine different distress manifestations. For reasons that will be explained later, only five distress types were used in this example problem.

Each distress type is assigned a certain number of "points" up to a maximum amount. The points determine the current pavement rating of highway segment i and distress type k . The more points assigned to a certain highway segment and distress type, the less distress is present. The summation of available points for the individual distress types for a given highway segment will determine the overall rating. Table 3 gives the information on current condition rating that was used as input to the computer program. Note that the maximum overall rating score taken over the five distress types is $125-$ not 100 as in many other rating systems (7). The percentage of total is taken as the ratio of the overall rating to the maximum rating and is equivalent to a pavement score based on a 0 to 100 scale.

## Gain-of-Rating Matrix

The gain-of-rating matrix represents the $\mathrm{d}_{\mathrm{ijk}}$ input for the RAMS program. The gain-of-rating points are the same kind of points used in determining the pavement condition for the highway segments.

Three kinds of ratings (points) are used to generate the gain-of-rating matrix. These are maximum points available for a given type of distress (Table 3), maximum gain-of-rating points for a given maintenance strategy and type of distress (Table 4), and current pavement rating for a given highway segment and distress type (Table 3). The maximum points available for a distress type indicate what number of points constitutes a perfect rating (no distress condition). The maximum gain-of-rating points indicate the maximum gain that can be expected from using a given kind of maintenance strategy to treat a specific distress.

These three types of ratings are used by the RAMS program to generate the gain-of-rating points ( $\mathrm{d}_{\mathrm{ij}}$ ) for each highway segment (i), maintenance strategy (j), and distress type (k) by one of two possible procedures. If the maximum gain-of-rating points and the points for current pavement rating add up to less than the maximum points available for a given highway segment and distress type, then the maximum of gain-of-rating points is used as the $\mathrm{d}_{\mathrm{ijk}}$ input. If that sum of points is greater than the maximum points available, then the difference between the maximum points available and current pavement rating points is used as the $\mathrm{d}_{\mathrm{jk}}$ input. For example, if a moderate overlay, a thick overlay, or a reconstruction maintenance strategy is used, the maximum gain-of-rating points for rutting $=15$. This indicates that, for a highway segment with a rutting distress rating of 0 (which is the severest rutting condition), application of one of these three strategies would completely eliminate the manifestation of distress immediately after the required work was performed. Some maintenance strategies may have negative gain-of-rating points for some types of distress, which indicates that they have accentuated the distress.

The six maintenance strategies used in this example problem are considered to be typical of the maintenance performed on TSDHPT district 17 pavements. The only maintenance strategies that require additional description are light-duty and heavy-duty reconstruction. Lightduty reconstruction is generally used on low-traffic highways and consists of scarifying the existing surface and base, recompacting, and then applying a one-course surface treatment. Heavy-duty reconstruction is generally used on higher traffic highways and consists of scarifying the existing surface and base, adding additional flexible (unstabilized) base, recompacting, and applying a thin asphalt concrete surface $[\leqslant 3.8 \mathrm{~cm}(\leqslant 1.5$ in)].

The maximum gain-of-rating points associated with each maintenance strategy and type of distress were obtained from subjective ratings by Texas Transportation Institute personnel and are expected to change slightly as TSDHPT personnel begin to use the computer program.

Table 3. Current pavement condition ratings for highway segments.

| Type of Distress | Rating by Highway Segment |  |  |  |  |  |  |  |  |  |  |  |  |  |  | Maximum Points Available |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |  |
| Rutting | 10 | 10 | 10 | 10 | 10 | 10 | 8 | 10 | 15 | 15 | 15 | 15 | 13 | 8 | 10 | 15 |
| Cracking |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Alligator | 5 | 15 | 10 | 20 | 25 | 25 | 0 | 15 | 25 | 25 | 25 | 25 | 25 | 5 | 10 | 25 |
| Longitudinal | 20 | 25 | 15 | 20 | 25 | 25 | 10 | 25 | 5 | 25 | 25 | 25 | 25 | 0 | 10 | 25 |
| Transverse | 17 | 20 | 13 | 20 | 20 | 20 | 20 | 20 | 5 | 20 | 17 | 17 | 20 | 17 | 8 | 20 |
| Failures per mile | 20 | 40 | 40 | 40 | 40 | 40 | 10 | 20 | 40 | 40 | 40 | 40 | 40 | 20 | $\underline{20}$ | 40 |
| Total points (overall rating) | 72 | 110 | 88 | 110 | 120 | 120 | 48 | 90 | 90 | 125 | 122 | 122 | 123 | 50 | 58 | 125 |
| Percentage of total | 58 | 88 | 70 | 88 | 96 | 96 | 38 | 72 | 72 | 100 | 98 | 98 | 98 | 40 | 46 | 100 |

Table 4. Maximum gain-of-rating matrix for all highway segments.

| Maintenance Strategy | Type of Distress |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Rutting | Alligator Cracking | Longitudinal Cracking | Transverse Cracking | Failures per Mile |
| Seal coat | 0 | 15 | 15 | 15 | 10 |
| Thin overlay ( $\leq 3.8 \mathrm{~cm}$ ) | 13 | 20 | 20 | 20 | 25 |
| Moderate overlay ( $>3.8$ to 7.6 cm ) | 15 | 25 | 25 | 20 | 30 |
| Thick overlay ( $>7.6 \mathrm{~cm}$ ) | 15 | 25 | 25 | 20 | 35 |
| Reconstruction |  |  |  |  |  |
| Light-duty | 15 | 25 | 25 | 20 | 40 |
| Heavy-duty | 15 | 25 | 25 | 20 | 40 |

Note: $1 \mathrm{~cm}=0,39 \mathrm{in}$.

Table 5. Pavement survival matrix for transverse cracking.

| Maintenance Strategy | Probability of Survival by Time After Maintenance |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ${ }_{\text {Year }}$ | $\begin{aligned} & 2 \\ & \text { Years } \end{aligned}$ | $\begin{aligned} & 3 \\ & \text { Years } \end{aligned}$ | $\begin{aligned} & 4 \\ & \text { Years } \end{aligned}$ | $\begin{aligned} & 5 \\ & \text { Years } \end{aligned}$ | $\begin{aligned} & 6 \\ & \text { Years } \end{aligned}$ | $\begin{aligned} & 7 \\ & \text { Years } \end{aligned}$ | $\begin{aligned} & 8 \\ & \text { Years } \end{aligned}$ | $\begin{aligned} & 9 \\ & \text { Years } \end{aligned}$ | $\begin{aligned} & 10 \\ & \text { Years } \end{aligned}$ |
| Seal coat | 1.00 | 0.92 | 0.86 | 0.85 | 0.67 | 0.38 | 0.33 | 0.18 | 0.09 | 0.06 |
| Thin overlay ( 53.8 cm ) | 1.00 | 1.00 | 0.94 | 0.94 | 0.43 | 0.18 | 0.18 | 0.14 | 0.06 | 0.01 |
| Moderate overlay ( $>3.8$ to 7.6 cm ) | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.63 | 0.26 | 0.22 | 0.11 | 0.04 |
| Thick overlay ( $>7.6 \mathrm{~cm}$ ) | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.33 | 0.33 | 0.28 | 0.17 | 0.17 |
| Reconstruction |  |  |  |  |  |  |  |  |  |  |
| Light-duty | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.65 | 0.60 |
| Heavy-duty | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.65 | 0.60 |

Note: $1 \mathrm{~cm}=0.39 \mathrm{in}$.

## Pavement Survivor Matrixes

Pavement survivor matrixes were developed for each combination of distress type and maintenance strategy. As an example, Table 5 gives the probability of survival for the six maintenance strategies obtained for transverse cracking conditions. The determination of the probabilities for each of the five types of distress used in this example problem is described in detail below. The maintenance strategies considered are (a) seal coat, (b) thin overlay, (c) moderate overlay, (d) thick overlay, (e) light-duty reconstruction, and (f) heavyduty reconstruction.

To determine the probability of survival for a given maintenance strategy, failure must first be defined. Sivazlian and Stanfel (9) define it as ". . . an event associated with a shift in the operating characteristics of a system from its permissible limits." Thus, pavement failure may occur when the SI for a given highway type reaches or falls below a preselected lower limit. Failure could also be defined as occurring when the highway develops a certain amount of a particular distress manifestation. But, for this problem, the time to failure for a given maintenance strategy is taken to be that time when some type of maintenance strategy must be accomplished that supersedes the previously applied maintenance.

The pavement survival matrixes are currently based on subjective "failure analysis" data obtained from TSDHPT district maintenance management personnel. These data were obtained from a diagnostic examination of pavement segments located in four separate areas in the state. The district personnel evaluated these highway segments for future maintenance and rehabilitation needs based on their visual ohservations of the payements and objectively measured data that were provided on traffic, skid, deflection, ride, and construction history.

From such information, time to failure was calculated for each maintenance strategy considered. For seal coats, time to failure was determined when any of the six maintenance strategies considered were rescheduled for application. For the three overlays and reconstruction, time to failure was determined only when one of these five maintenance strategies was rescheduled for application; i.e., seal coats were not considered as superseding any of these five.

The time-to-failure data obtained for each maintenance strategy were arranged into histograms. These histograms approximate the failure density distribution curve discussed in reliability theory (8, 9). Failure density distributions are similar to normal distributions of data in that the area under the curve is equal to one.

From these histograms or failure density distributions, the failure density function can be defined by $f(x)$ taken over the interval $0<x<\infty$ where x defines a time scale. The probability that a maintenance strategy will fail within a time interval ( $x, x+d x$ ) is given by $f(x) d x$.

The corresponding cumulative density function can be defined by $\mathrm{F}(\mathrm{x})$, also taken over the interval $0<\mathrm{x}<\infty$, and is the probability that a given maintenance strategy will fail on or before some time $t$. This can be expressed as follows:
Probability of failure on or before $t=F(t)=\int_{0}^{t} f(x) d x$
This expression assumes that a maintenance strategy that will survive past time $t$ is given by $R(t)$ and is expressed as
$R(\mathrm{t})=1-\mathrm{F}(\mathrm{t})=\int_{\mathrm{t}}^{\infty} \mathrm{f}(\mathrm{x}) \mathrm{dx}$
This expression can be adequately approximated for a given maintenance strategy by a cumulative frequency distribution that may be plotted from a histogram of time-to-failure data to result in a survival curve. Data from such curves are entered into the RAMS program in matrix form (Table 5).

The pavement survival matrixes currently being used will be updated in the near future. This will be accomplished by combining the subjectively obtained data just described with objective data from a pavement data base assembled for Texas pavements. It is planned to use Bayesian techniques to accomplish this task.

## Budget Resource

Four types of resource constraints are used in the program: (a) material and supply, (b) equipment, (c) manpower, and (d) cost. Each resource constraint has two major inputs: requirements and availability. The requirement input indicates how much of a given resource will be used by a maintenance strategy, and availability indicates how much of a given resource is available to be used. Of the four types of resource constraints, budget is the most significant one in this example problem.

The available budget used as input was essentially the same amount as the contract funds allocated for the TSDHPT selected maintenance strategies. This was an important constraint because it forced the computer program to consider maintenance decisions within approximately the same financial framework as that used by TSDHPT personnel. The table below gives cost requirements per unit area for each maintenance strategy ( $1 \mathrm{~m} \cdot \mathrm{~km}=2.04 \mathrm{ft}-\mathrm{miles}$ ):

| Maintenance Strategy | Cost per Unit Area (\$/m•km) |
| :---: | :---: |
| Seal coat | 436 |
| Thin overlay | 1886 |
| Moderate overlay | 4078 |
| Thick overlay | 7234 |
| Reconstruction |  |
| Light-duty | 1925 |
| Heavy-duty | 5301 |

The total available funds in this case $=\$ 1130000$. The costs generally increase as the maintenance strategies become more extensive. Notable exceptions to this are the two kinds of reconstruction.

## Comparison of TSDHPT and RAMS Selected Maintenance Strategies

Comparisons of the TSDHPT and RAMS selected maintenance strategies for the 15 highway segments in the example are given in Table 6. First, both the TSDHPT and RAMS (case 1) strategies selected use the same original TSDHPT budget amount. Another RAMS solution (case 2) was obtained by increasing the TSDHPT budget by approximately 6 percent. To facilitiate discussion of the comparisons, those highway segments that reveal little or no difference between the TSDHPT and RAMS (cases 1 and 2) selected maintenance strategies will not be examined.

A combination of highway types was used in this example, and the RAMS program treated all with equal priority except in applying the two kinds of reconstruction. For low-traffic segments (4, 5, 6, 7, 8, 14, and 15), the program was restricted to applying only the light-duty type of reconstruction (if required); for the remaining, higher traffic segments, only the heavyduty type of reconstruction could be used. Traffic and climate indexes can also be used as input to account for differences in highway types. In addition, groupings of similar highway types can be assembled and processed together if desired.

Table 6 indicates that the selected strategies for segment 2 differ. TSDHPT selected a thin overlay, and the RAMS program (cases 1 and 2) selected a seal coat. The manifestations of pavement distress for this segment are composed of alligator cracking and extensive flushing (flushing is not considered in the RAMS program). All maintenance strategies are feasible as determined by the minimum and overall rating constraints, which allows the RAMS program to evaluate the appropriateness of five maintenance strategies (seal coat, thin overlay, moderate overlay, thick overlay, and heavy-duty reconstruction). For this segment, the maintenance effectiveness computed for a seal coat is about half that calculated for a thin overlay, but the cost for a thin overlay is four times as great. It can be seen in a subjective way that a seal coat is an attractive maintenance strat-
egy. The TSDHPT decision to use a thin overlay may also have been based on the rough ride and flushing present on this highway.

Segments 5 and 6 were scheduled for seal coats by the TSDHPT, and no strategies were scheduled by the RAMS program. Table 3 indicates that no manifestations of distress, with the exception of minor rutting, were present in these pavements. But, in fact, flushing was present (not given in Table 3) and may have been a consideration in the TSDHPT decision.

Segment 7, which has numerous and extensive manifestations of distress, is scheduled for a seal coat by the TSDHPT and a light-duty reconstruction strategy by RAMS. The feasible strategies allowed by the minimum and overall rating constraints given in Table 7 indicate that only a thick overlay or greater is allowable. A similar situation occurs in the case of segment 8.

For segment 9, TSDHPT scheduled a seal coat, but the RAMS program (case 1) scheduled no maintenance. This occurred because there was not enough budget to allow application of a thin overlay or greater to this segment. The inexpensive seal coat alternative was eliminated by the minimum and overall rating constraints. For the RAMS case 2 selection, the original TSDHPT budget was increased by approximately 6 percent. This small budget change allowed the segment to be scheduled for a suitable, cost-effective maintenance strategy (a thin overlay).

As shown by the case of segment 9, the RAMS program can also be used to help estimate required maintenance budgets. This can be accomplished by inputting all data as previously discussed but varying the budget amount. The budget could be selected where adequate maintenance is scheduled for all necessary segments.

Segments 11 and 12 are in excellent condition; both have only minor transverse cracking. The RAMS program in case 1 scheduled seal coats for these segments because some benefit could be obtained by using this strategy. This occurred because the program maximizes the maintenance effectiveness for the amount of budget available. In case 2 , the funds were more adequately used by slightly increasing the available budget; one result was that these two seal coats were eliminated.

A comparison of overall maintenance effectiveness resulting from the TSDHPT and RAMS case 1 and case 2 maintenance strategy selections provides an indication of the optimality of the computer solutions. The mainte-

Table 6. Comparison of TSDHPT and RAMS selected maintenance strategies.

nance effectiveness obtained by using Equation 1 for the three maintenance programs is as follows: TSDHPT, 359412 ; RAMS (case 1), 425 106; and RAMS (case 2), 451318.

Comparing the TSDHPT and RAMS case 1 selections shows that use of the computer program increased the effectiveness of maintenance by 18 percent and resulted in a 2 percent budget savings. But case 1 selections did exclude one pavement segment that needed maintenance. Case 2 selections met this need and resulted in an increase in maintenance effectiveness of 26 percent over TSDHPT selections. The RAMS program accomplished this by using a budget approximately 6 percent larger than that used by TSDHPT.

## SUMMARY

This paper has examined an operating computer program that uses integer programming to determine optimal maintenance strategies for pavements. The program uses the current pavement condition, potential gain of rating, and survivor matrixes as input to maximize overall effectiveness of maintenance for any group of highway segments. The program can use numerous maintenance strategies, resources, and feasibility constraints in determining optimal solutions. The required inputs can be expanded or reduced as necessary.

Fifteen highway segments located in one highway district in Texas were used to demonstrate the program. Based on these actual field data, a comparison of the computer program and TSDHPT selected maintenance strategies revealed similar selections and some notable exceptions. It was shown that, by using the RAMS program with the same budget as that used by TSDHPT, the effectiveness of the selected maintenance strategies could be increased by 18 percent over TSDHPT selections. The effectiveness of maintenance was increased by 26 percent with a 6 percent increase in the available budget. Although the example problem represented maintenance strategies planned for accomplishment by contract, the computer program also has the capability to optimize in-house district maintenance efforts.

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# Selecting the Optimum Number, Size, and Location of Highway Maintenance Yards 

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The basic characteristics of highway maintenance and their effect on the location, number, and size of maintenance yards were analyzed. The study dealt exclusively with the management unit that is directly responsible for all maintenance operations in a given area where all activities initiate and terminate at the yards on a daily basis. The yards were as-

[^0]sumed to be of unlimited capacity and used for storage of materials and equipment. Variable cost functions for maintenance travel and maintenance yards were developed analytically for the special case of an unbounded area with uniform distribution of maintenance requirements. Both functions were found to be nonlinear and unimodal with respect to travel time. They also showed that travel time, used as a measure of distance, and a limit on daily work hours were the most critical factors in the maintenance yard problem. In the optimization process, a new, unique criterion was established. For any potential yard site, there ex-


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