nance effectiveness obtained by using Equation 1 for the three maintenance programs is as follows: TSDHPT, 359 412; RAMS (case 1), 425 106; and RAMS (case 2), 451 318.

Comparing the TSDHPT and RAMS case 1 selections shows that use of the computer program increased the effectiveness of maintenance by 18 percent and resulted in a 2 percent budget savings. But case 1 selections did exclude one pavement segment that needed maintenance. Case 2 selections met this need and resulted in an increase in maintenance effectiveness of 26 percent over TSDHPT selections. The RAMS program accomplished this by using a budget approximately 6 percent larger than that used by TSDHPT.

SUMMARY

This paper has examined an operating computer program that uses integer programming to determine optimal maintenance strategies for pavements. The program uses the current pavement condition, potential gain of rating, and survivor matrixes as input to maximize overall effectiveness of maintenance for any group of highway segments. The program can use numerous maintenance strategies, resources, and feasibility constraints in determining optimal solutions. The required inputs can be expanded or reduced as necessary.

Fifteen highway segments located in one highway district in Texas were used to demonstrate the program. Based on these actual field data, a comparison of the computer program and TSDHPT selected maintenance strategies revealed similar selections and some notable exceptions. It was shown that, by using the RAMS program with the same budget as that used by TSDHPT, the effectiveness of the selected maintenance strategies could be increased by 18 percent over TSDHPT selections. The effectiveness of maintenance was increased by 26 percent with a 6 percent increase in the available budget. Although the example problem represented maintenance strategies planned for accomplishment by contract, the computer program also has the capability to optimize in-house district maintenance efforts.

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Selecting the Optimum Number, Size, and Location of Highway Maintenance Yards

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The basic characteristics of highway maintenance and their effect on the location, number, and size of maintenance yards were analyzed. The study dealt exclusively with the management unit that is directly responsible for all maintenance operations in a given area where all activities initiate and terminate at the yards on a daily basis. The yards were assumed to be of unlimited capacity and used for storage of materials and equipment. Variable cost functions for maintenance travel and maintenance yards were developed analytically for the special case of an unbounded area with uniform distribution of maintenance requirements. Both functions were found to be nonlinear and unimodal with respect to travel time. They also showed that travel time, used as a measure of distance, and a limit on daily work hours were the most critical factors in the maintenance yard problem. In the optimization process, a new, unique criterion was established. For any potential yard site, there ex-

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isted a specific maximum travel time that defined the conditions for minimizing the variable unit maintenance cost for any and all maintenance requirements served by that site. Furthermore, it was proved that optimization results based on either this criterion or total variable cost for the total given area were the same. It was also found that the "fixed" maintenance requirement of any highway segment was a variable that depended on travel time between the segment and the yard serving it. Similar cost functions were developed for the general case of highway maintenance in a bounded area that has nonuniform and discrete maintenance requirements. The use of travel time as a measure of distance showed that the maintenance yard problem was independent of boundary conditions. Use of the new optimization criterion for individual yard sites also made the problem independent of the magnitude of the bounded area.

Highway maintenance encompasses many areas that are only beginning to be developed and as such are worthy of research. Work measurement, scheduling, use of resources, work standards, work planning and control, plant layout and materials handling, cost analysis, and inventory are good examples. This investigation is addressed to a relatively limited area in that complex field.

The highway maintenance yard problem can be outlined as follows:

1. A bounded geopolitical area is given.
2. Also given in this area is a highway network made up of a collection of several classes of highways that are different in their location and age as well as in their physical, geometric, and other characteristics and thus are different in their maintenance needs.
3. The location and the characteristics of any segment of this highway network are accurately known.
4. Several types of highway maintenance activities exist that are different in their resource requirements and methods of execution as well as in the time and frequency of their occurrence.
5. Highway maintenance activities are executed basically by an operational or management unit of labor, equipment, and material. This unit starts its daily work at a prescribed time at a maintenance yard. Its crews travel to certain job locations on the highway network, finish required work, and keep moving to new locations, if time allows, returning to the maintenance yard at a later, prescribed time the same day.
6. The cost of maintenance work includes the cost of labor, equipment, and material related to or expended during the actual execution of the work itself and the cost of travel between the maintenance yard and the job location or between job locations.
7. The cost of maintenance crew travel from a maintenance yard to a job location has two components: (a) a direct transportation cost proportional to the distance traveled and (b) an indirect cost, the value of which is measured by the loss of productivity resulting from travel time and is also proportional to the distance traveled.
8. As the number of maintenance yards in the given area increases, the average travel distance of a maintenance crew and, thus, transportation cost decrease. At the same time the cost of land acquisition and construction, maintenance, and operation of the maintenance yards increases.

The basic objective here is the development of a model for selecting the optimum number, size, and location of maintenance yards in a given area so as to minimize the total combined cost of work travel for highway maintenance management units and the establishment, operation, and upkeep of their yards over a specified planning period. The model does not attempt to reflect the needs or detailed characteristics of any particular highway network in a particular geographical area but rather deals with a general and typical situation.

HIGHWAY MAINTENANCE YARDS AND THEIR LOCATION

The highway maintenance yard is defined here as an installation that (a) is used by maintenance crews as a major base of activities, (b) has substantial indoor space, (c) serves as a materials source, and (d) is used for storing equipment units. Documentation of past methods of selecting the number, size, and location of highway maintenance yards is scarce, and available sources indicate that this problem is far from being completely analyzed or solved (1, 2, 3, 4). The findings of the Iowa State Highway Maintenance Study concluded that "garages, stockpiles and other facilities were not always located strategically with respect to the area served" and that "lost time due to additional travel was substantial" (1, p. 36). These facilities were also found to be inadequate in size and inefficient in layout. Later work indicated that a major need still existed in 1968 for "adequate maintenance field headquarters in many locations" (3). In recent years, a new tendency has emerged in some areas to separate the maintenance of the Interstate highway system from that of the rest of the highway network and to establish special maintenance yards for this purpose. But the only available source on this is the statement that "maintenance buildings and storage areas usually were provided for each 25 to 30 centerline miles of highway", were located "near the interstate route in the vicinity of an interchange", and that they were between 1.2 and 4 hm^2 (3 and 10 acres) in size (2, p. 11). No mention is made of the rationale behind these general criteria.

The first and only known attempt toward "maintenance station location through operations research" was made by Hayman and Howard for the Wyoming State Highway Department (4). Their effort concentrated on sanding and ploughing operations for snow removal and resulted in the development and solution of linear models for the selection of the optimum location (and number) of needed maintenance stations for each operation. They recommended that "optimization techniques should also be applied to other maintenance functions such as sealing and mowing" (4, p. 30).

These limited sources of accessible formal data on highway maintenance yards, supplemented by informal investigation, resulted in the formulation of the following general features of the maintenance yard problem:

1. Most highway maintenance yards were established before the addition of the Interstate highway system, the dramatic expansion of the total highway network in the last two decades, and some of the changes related to the responsibilities of the highway maintenance program.
2. All highway maintenance yards were located adjacent to a segment of the highway network and frequently near an intersection of two highways.
3. The number of highway maintenance yards was found to be dependent on the organizational structure and, as such, directly related to the number of departments responsible for the maintenance program, the boundaries and size of their administrative units, and the policies related to equipment rental or ownership.
4. The location of highway maintenance yards was influenced by different factors at different times. Some of these factors included proximity to urban or political centers, the price of available land for potential sites, and proximity to the geographical center of the administered area. The number of potential sites was always

limited, and there was no indication that the location of all highway segments and their generated maintenance load were seriously considered in the selection of the final locations.

5. Although many aspects of highway maintenance improved over the years, maintenance yards were rarely abandoned or relocated as service requirements and techniques changed.

In general, it is not unreasonable to conclude that the selection of the number, size, and location of highway maintenance yards has been a direct result of many forces that have through the years been external to the maintenance program and beyond the control of any single agency or plan. It can also be concluded that the continued retention of these locations is only justified now on the basis of historical momentum and the implied attitude of decision makers that no benefits from better locations can compensate for lost investment in the old ones. The basic weakness of this attitude is that it has always been based on personal judgment and that no attempt has been made to subject the problem to the rigorous techniques that have achieved positive results in other areas.

Several models and optimization techniques exist that deal with location-allocation problems similar to the problem of the highway maintenance yard (5, 6, 7, 8, 9, 10, 11, 12, 13). The basic structure of these models as well as their relation to the problem at hand is summarized in my research on the subject (14). These models contain several common characteristics that represent an interesting contrast in comparison with the maintenance yard problem:

1. All approaches of existing models tend to distort the problem to fit known techniques. They also require some trade-off with realism in terms of simplifying assumptions and, when the data used are more realistic, in terms of closeness to an optimal solution (13). Two basic features are critical to this discussion—(a) the general adoption of linear approximation of cost functions and (b) the use of Euclidean distance, or minor variations of it, to measure spatial separation in the given area. Both of these features can be a source of several types of error in real-life applications.
2. All existing approaches use total cost as the criterion for optimization.
3. Solution techniques rely exclusively on enumeration, simulation, heuristic models, and mathematical programming by use of branch and bound methods. These techniques, though significantly improved, are still not effective for large problems and depend on simplifying assumptions to decrease computer computations and storage requirements to a reasonable level (13).

The model used in this study is intended to deviate from past efforts in several respects:

1. It is intended to be independent of Euclidean geometry.
2. It is committed to the analytical development of realistic cost functions that reflect the essential characteristics of highway maintenance instead of adopting the arbitrary assumption of linearity in all costs.
3. It is committed to the search for new criteria of optimization that are capable of at least improving the efficiency of available solution techniques and considering all locations in a given area as potential yard sites.

FORMULATION OF THE MODEL

The highway maintenance yard problem can assume widely varying degrees of complexity depending on its input assumptions, parameters, and variables. Some of the factors that can affect it include the location and travel cost associated with materials sources, offices of other management levels, residences of workers, and equipment shops. The problem can also be affected by the jurisdictional boundaries of maintenance responsibility, the pattern of distribution of the given highway network, the variation in maintenance requirements of the different maintenance functions and individual highway segments, seasonal variation in maintenance requirements for individual maintenance functions, and the variation in travel time and cost for different types of equipment. Therefore, certain simplifications are made to help reveal the general and basic characteristics of the problem and the relations of its variables. The future relaxation of these simplifications depends on the accumulation, correct interpretation, and accuracy of the data.

Estimation of Highway Maintenance Requirements

Accurate and realistic estimation of highway maintenance requirements is a critical and demanding element in the development of the model because it affects the model structure and the accuracy of its solutions. In addition, it should have the ability to convert highway maintenance needs and their resource requirements to an input that can be accurately and consistently manipulated in the mathematical expressions of the model.

The method used for estimating maintenance requirements of the Interstate highway system meets the above two requirements and can thus be adopted (provided the proper additional data are collected) for all the highway systems in any given problem (2). In this method, the significant variables that affect a particular maintenance activity on a particular highway segment are converted into annual maintenance requirements by using a regression model. These requirements are expressed in terms of an index number called "maintenance requirement units," which in turn can be transformed into resource requirements, i.e., labor, equipment, and materials in annual worker hours, equipment hours, and dollars respectively. The total annual maintenance requirements for the highway segment are the sum of all the regression models representing all required highway maintenance activities.

There is always a need in the maintenance yard problem to reconcile the estimation of maintenance requirements on an annual basis with their seasonal variations and to express these maintenance requirements on a daily basis to incorporate the daily time limitation on highway maintenance activities. The resolution of this complication depends on adopted policies and solutions that govern equipment or materials storage and the scheduling and priority rating of maintenance work. This is outside the scope of this investigation, and therefore it is assumed that a set of transforming functions that relate annual, seasonal, and daily maintenance requirements are known and given. It is encouraging that available data on highway maintenance have revealed that the relative maintenance requirements among the different highway systems do not change seasonally nor change appreciably from year to year. This is expected to make the determination of the number and location of maintenance yards less sensitive to the variations given above and to the accuracy of the transforming functions that relate them. Only the determination of yard size is expected to be affected.

DESCRIPTION OF HIGHWAY NETWORK

Because of the obvious difficulties involved in finding the appropriate functions to describe the distribution of any given highway network and its maintenance requirements, an approximation is the only practical approach to use. The approximation adopted here is based on partitioning the given highway network into segments and representing every segment by a point. The point should ensure that total maintenance-related travel between any point in the highway network and that representing any segment is equal to the total maintenance-related travel between the point and all maintenance requirements along the segment.

Maintenance Travel Cost Function

Transportation or travel costs used in existing models for warehouse location problems are concerned exclusively with the direct costs of transporting a demanded item from a plant to a warehouse or from a warehouse to a customer. They often involve costs for different transportation modes, rate breaks, and less-than-carload or full-carload elements. Most of these models use linear approximations for these transportation costs, and the evidence indicates that this approach is reasonable (6, 8). This linear approximation is adopted here, with some modification, for direct travel cost in the maintenance yard problem. Let

T = available daily work time excluding the typical daily loading and waiting time at the yard;
t_{ij} = one-way "dead-haul" travel time between yard i and highway segment j, which is meant to exclude any travel time involved in loading or in the actual execution of a maintenance activity;
r_j = maintenance requirements of segment j, based on $t_{ij} = 0$, in maintenance requirement units[1] per day where r_j assumes positive values only;
e = equipment requirements in equipment hours per maintenance requirements unit (MRU) where no travel time is involved; and
c_1 = equipment cost per equipment hour.

Then, equipment requirements at segment j, in numbers of equipment, can be expressed as $[er_j/(T - 2t_{ij})]$, and the daily direct travel cost between yard i and segment j (DTC_{ij}) can be expressed as $DTC_{ij} = 2t_{ij}c_1 [er_j/(T - 2t_{ij})]$ $T > 2t_{ij}$, or

$$DTC_{ij} = c_1 er_j [2t_{ij}/(T - 2t_{ij})] \quad (1)$$

In addition to direct travel cost, another travel cost component is caused by travel time and is measured by the cost of additional equipment needed to compensate for the loss of working time. This additional equipment requirement, in numbers of equipment, can be shown to be equal to $[er_j/(T - 2t_{ij})] - er_j/T$, or $er_j[2t_{ij}/T(T - 2t_{ij})]$. This results in an additional equipment cost equal to $c_1Ter_j[2t_{ij}/T(T - 2t_{ij})]$ or $c_1er_j[2t_{ij}/(T - 2t_{ij})]$, which is identical to the direct travel cost. Similarly, it can be shown that the additional labor cost caused by travel time is equal to $c_2\ell r_j[2t_{ij}/(T - 2t_{ij})]$ where ℓ is labor requirements in worker hours per MRU where no travel time is involved and c_2 is labor cost per worker hour.

It should be noted that material requirements for the maintenance of a particular highway segment are not affected by any daily time limitation, by travel time, or even by the number, size, and location of maintenance yards. The sum of the above components of additional equipment and labor costs caused by travel time is designated the daily indirect travel cost (ITC_{ij}), which can be written

$$ITC_{ij} = r_j (c_1 e + c_2 \ell) [2t_{ij}/(T - 2t_{ij})] \quad (2)$$

This additional travel cost reflects a unique characteristic of highway maintenance where the demand or maintenance requirement of any segment changes with travel time instead of being a known constant.

The sum of Equations 1 and 2 is referred to as total travel cost (TTC_{ij}), which can be expressed as $TTC_{ij} = DTC_{ij} + ITC_{ij}$, or

$$TTC_{ij} = r_j(2c_1 e + c_2 \ell) [2t_{ij}/(T - 2t_{ij})] \quad (3)$$

If the unit travel cost is designated UTC_{ij}, it can be shown that $UTC_{ij} = TTC_{ij}/r_j$, or

$$UTC_{ij} = (2c_1 e + c_2 \ell) [2t_{ij}/(T - 2t_{ij})] \quad (4)$$

Let $C = 2c_1e + c_2\ell$, a constant; then Equation 3 becomes

$$TTC_{ij} = Cr_j [2t_{ij}/(T - 2t_{ij})] \quad (5)$$

and Equation 4 becomes

$$UTC_{ij} = C[2t_{ij}/(T - 2t_{ij})] \quad (6)$$

Equation 3 reveals the following relevant characteristics of highway maintenance, which will be shown to have direct effects on the optimum solution of any adopted model:

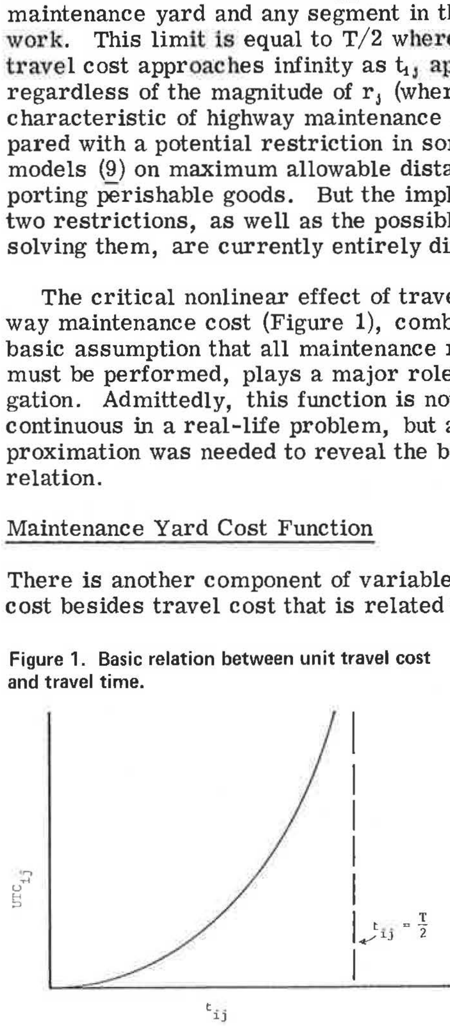
1. Travel cost is affected by c_1, c_2, e, ℓ, T;
2. Travel cost increases linearly with r_j and nonlinearly with t_{ij}; and
3. There is an absolute upper limit on t_{ij} between a maintenance yard and any segment in the highway network. This limit is equal to $T/2$ where maintenance travel cost approaches infinity as t_{ij} approaches $T/2$ regardless of the magnitude of r_j (where $r_j > 0$). This characteristic of highway maintenance may be compared with a potential restriction in some existing models (9) on maximum allowable distance for transporting perishable goods. But the implications of the two restrictions, as well as the possible means of resolving them, are currently entirely different.

The critical nonlinear effect of travel time on highway maintenance cost (Figure 1), combined with the basic assumption that all maintenance requirements must be performed, plays a major role in this investigation. Admittedly, this function is not expected to be continuous in a real-life problem, but a continuous approximation was needed to reveal the basic general relation.

Maintenance Yard Cost Function

There is another component of variable maintenance cost besides travel cost that is related to the estab-

Figure 1. Basic relation between unit travel cost and travel time.



lishment, maintenance, and operation of highway maintenance yards. Available data on these yards provide no clue to all the variables that affect their cost or to the proper form of their function and the values of their parameters. But data on warehouses reveal that these costs are composed of annual fixed costs of operating the warehouse—i.e., lease or depreciation charges, fixed payroll, and fixed indirect costs—and a variable cost associated with increased volume at the warehouse. These data also indicate that the relation between yard cost and radius of served area is likely to be discontinuous but that a continuous approximation is beneficial for revealing the underlying relation between the two. More important is the confirmation that "optimal sizing and locating of facilities are very sensitive to the shapes of the warehousing cost function" (10). My formulation of a maintenance yard cost function draws on these basic findings but includes proper modifications to reflect the characteristics of highway maintenance. Although the developed function is based on certain assumptions, it should be understood that its final form must be determined from the accumulation and analysis of relevant data.

Assuming that maintenance requirements are uniformly distributed in the given area, that yard cost is independent of yard location, and that the area served by any yard is circular and proposing that the dimensions of the area are expressed in travel time instead of distance, let

$\bar{r}$ = average daily maintenance requirement in MRU per unit area;
$R(t)$ = total daily maintenance requirements within radius t adjusted for varying t_{ij} with $0 \le t_{ij} \le t$;
K = given total fixed daily cost of one maintenance yard; and
k_t = daily fixed cost of one maintenance yard per MRU at t travel time from the yard.

All other variables remain as defined. It can be shown that

$$R(t) = \Pi T\bar{r} \{(T/2) \ln [T/(T-2t)] - t\} \tag{7}$$

and

$$k_t = \left(K/\Pi\bar{r}(T-2t)\{(T/2)\ln[T/(T-2t)] - t\}\right) \tag{8}$$

It can be seen from Equation 8 that k_t is not really a fixed cost and its variation is neither linear nor quadratic as might have been expected in warehouse models. This is again a result of the change in maintenance requirements caused by travel time.

In addition to the above fixed cost caused purely by the establishment and operation of a maintenance yard, the cost of any yard is increased by the load of maintenance requirements it is designed to serve. In considering one maintenance requirement unit located at distance t from its yard, it is easy to see that its daily equipment requirement is equal to $eT/(T - 2t)$, its labor requirements to $\ell T/(T - 2t)$, and its materials requirements to m in equipment hours, worker hours, and dollars respectively. Assuming that yard cost components related to the above requirements are additive and that the relation between yard cost and maintenance requirements is linear, this additional yard cost may be written

$$p_t = w_1 e[T/(T-2t)] + w_2 \ell [T/(T-2t)] + w_3 m - c_3 \tag{9}$$

where

p_t = additional daily yard cost per MRU located at t travel time where p_t assumes only positive values;
w_1, w_2, w_3 = cost factors relating additional yard cost and maintenance requirements for equipment, labor, and material respectively; and
c_3 = a constant that will adjust p_t for the fact that the mere establishment of any maintenance yard does enable it to serve a certain load of maintenance requirements without any additional cost.

Let $w_1 e + w_2 \ell = W$ and $w_3 m - c_3 = c_4$; then Equation 9 can be written

$$p_t = W[T/(T-2t)] + c_4 \tag{10}$$

Combining Equations 8 and 10 gives $UYC_t = k_t + p_t$, or

$$UYC_t = \left(K/\Pi\bar{r}(T-2t)\{(T/2)\ln[T/(T-2t)] - t\}\right) + [WT/(T-2t)] + c_4 \tag{11}$$

where UYC_t is the combined daily unit "processing" cost of a yard per MRU located at travel time t.

The only peculiar behavior revealed in the solution of Equation 11 about the yard unit processing cost function is the unexpected increase in R_t for the higher values of t. This is attributable to the steeper increase of maintenance requirements with higher values of travel time, a unique feature of the maintenance yard problem.

Maintenance Combined Cost Function

Consider a circular area of uniformly distributed $\bar{r}$ served by one yard i located at its center. Let t be the maximum travel time, i.e., time between the yard and a point on the circumference, and t_{ij} be the travel time between the yard and any point j in the area. Combining travel cost and yard cost for one MRU at the circumference of the circular area gives

$$UMC_t = UTC_t + UYC_t \tag{12}$$

and substituting from Equations 6 and 11 gives

$$UMC_t = C[2t/(T-2t)] + \left(K/\Pi\bar{r}(T-2t)\{(T/2)\ln[T/(T-2t)] - t\}\right) + [WT/(T-2t)] + c_4 \tag{13}$$

where UMC_t is the daily variable unit maintenance cost of a highway section at maximum travel time t. This is the same as daily variable combined maintenance cost per MRU located at maximum travel time t. Similarly,

$$UMC_{ij}(t) = c[2t_{ij}/(T-2t_{ij})] + \left(K/\Pi\bar{r}(T-2t_{ij}) \times \{(T/2)\ln[T/(T-2t)] - t\}\right) + [WT/(T-2t_{ij})] + c_4 \tag{14}$$

where $UMC_{ij}(t)$ is the daily variable unit maintenance cost of a highway section located at j within the circular area assigned to yard i and having a maximum travel time t.

Figure 2 shows the general shape of the UMC_t function as expressed in Equation 13. Note that T/2 is no longer the upper limit for maximum travel time t; rather, the limit is t at which UYC_t assumes its minimum value. This was shown to be less than T/2. This narrows the area of search for the optimum t that will minimize the combined UMC_t unit maintenance cost.

Model for a Special Case

The basic emphasis in the previous discussion has been on the definition of variable maintenance cost based on one maintenance requirement unit or the maintenance requirements of one highway segment. Because it is recognized that the minimization of total cost remains a more legitimate and reasonable criterion for optimization, proof that the two approaches are equivalent was established. The adopted approach has the advantage of simplifying the expressions of the model, reducing needed computations, and increasing the capacity of the model to solve larger problems more efficiently. It also has the unique advantage of enabling one to analyze the maintenance cost associated with individual yard locations without solving for the entire given area.

The proof resulted in the development of the following total variable combined maintenance cost [TMC(A)] for area A:

Figure 2. General relation between maximum travel time and daily variable unit maintenance cost (including travel and yard cost components).

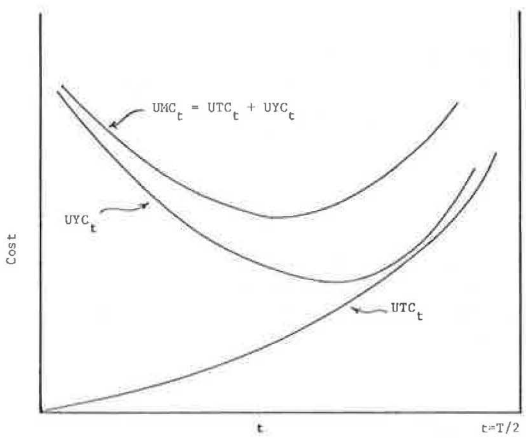
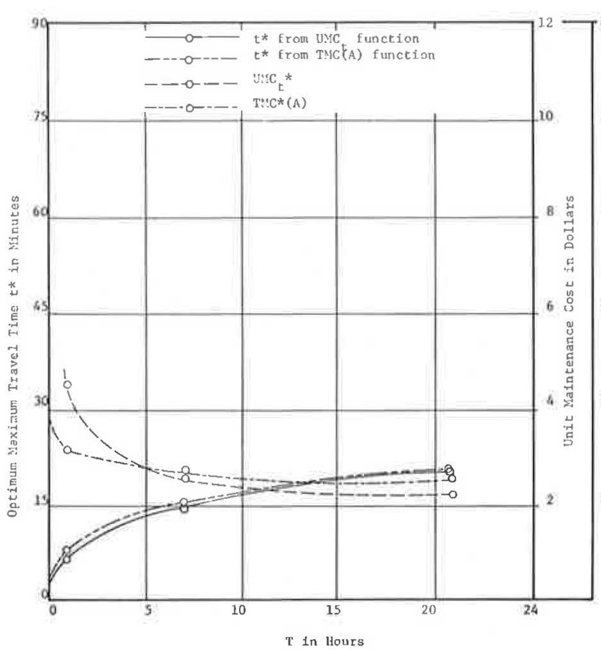


Figure 3. Theoretical variation with T of optimum maximum travel time, unit maintenance cost, and total maintenance cost÷A (all other parameters held constant at C = 3, K = 1000, $\bar{r}$ = 25 000, and W = 2).



$$TMC(A) = A(K/3.47t^2) + \bar{r}(C + W)\,\{(1/2)(T/t)^2 \ln \times [T/(T - 2t)] - (T/t)\} \quad (15)$$

This expression assumes that area A is divided into hexagons that contain tangent circles of radius t, which results in the number of yards being $A/(3.47t^2)$. Solving Equations 13 and 15 for the same wide range of parameters and travel time, it was found that optimum maximum travel times t* determined by both expressions were identical or almost identical. Figures 3 through 7 show the results of these solutions.

Figure 4. Theoretical variation with K of optimum maximum travel time, unit maintenance cost, and total maintenance cost÷A (all other parameters held constant at T = 7, C = 3, $\bar{r}$ = 25 000, and W = 2).

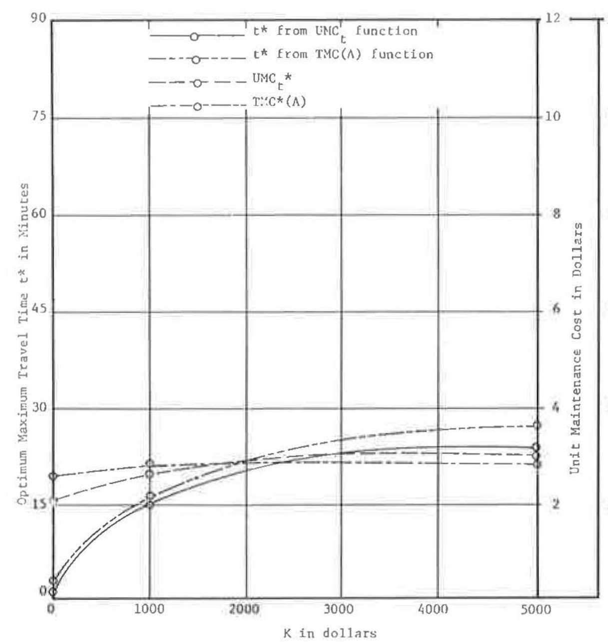


Figure 5. Theoretical variation with C of optimum maximum travel time, unit maintenance cost, and total maintenance cost÷A (all other parameters held constant at T = 7, K = 1000, $\bar{r}$ = 25 000, and W = 2).

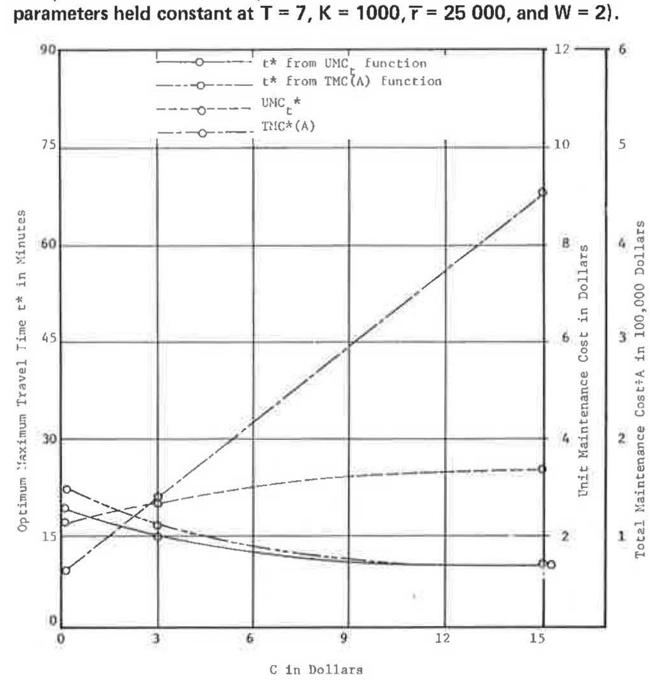


Figure 6. Theoretical variation with $\bar{r}$ of optimum maximum travel time, unit maintenance cost, and total maintenance cost÷A (all other parameters held constant at T = 7, C = 3, K = 1000, and W = 2).

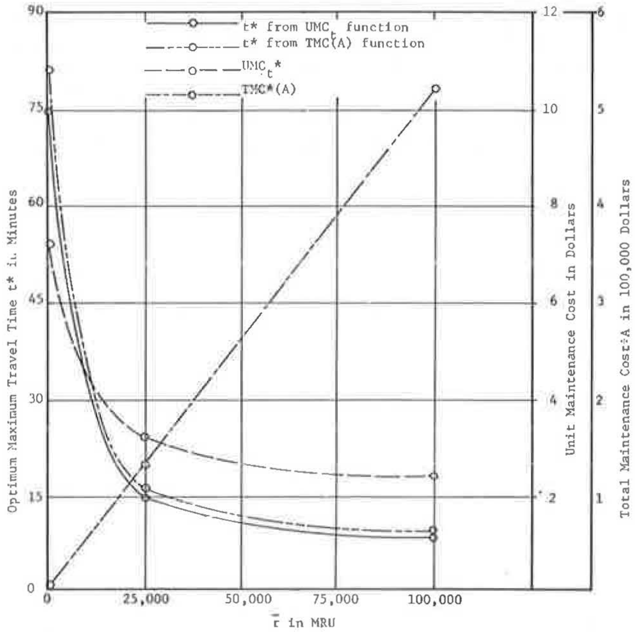
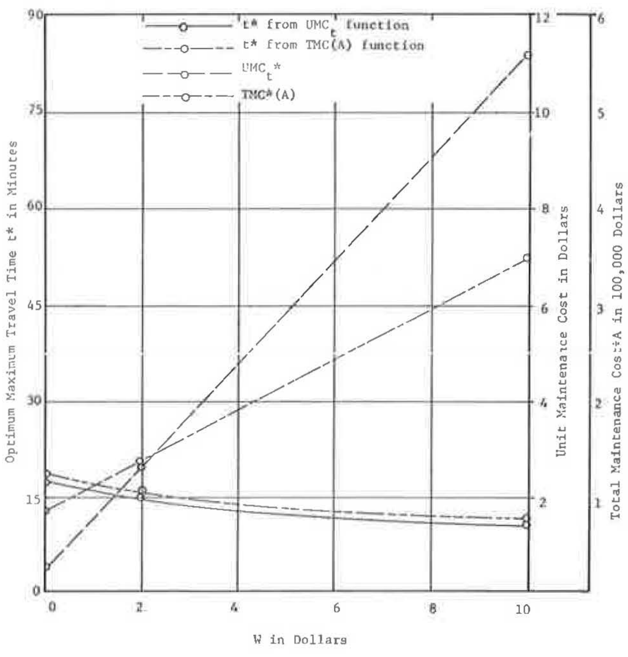


Figure 7. Theoretical variation with W of optimum maximum travel time, unit maintenance cost, and total maintenance cost÷A (all other parameters held constant at T = 7, C = 3, K = 1000, and $\bar{r}$ = 25 000).



The model of this special case revealed that for any set of parameters of the maintenance yard problem there exists a unique maximum travel time t^* from any maintenance yard that can define a solution that minimizes the total variable maintenance cost. This optimum maximum travel time is unique to the maintenance yard problem and is capable of either defining the optimum solution or at least minimizing its area of search.

Furthermore, based on the assumptions used in developing maintenance cost Equations 13 and 15, the optimal solution to this special maintenance yard problem can now be defined:

1. From Equation 13 or Equation 15, optimum maximum travel time t^* is determined;
2. The optimum number of yards = $[A/3.47(t^*)^2]$;
3. The optimum location of yards is at the center of each of the hexagons dividing area A; and
4. The optimum size of any yard is determined by means of the following method.

The size of any maintenance yard is a function of maintenance requirements assigned to it as well as of the distribution of required labor, equipment, and materials. Maintenance requirements $R_{t^*}(h)$ in a hexagon that contains a circle of radius t^* can be obtained, with some modification, from Equation 7, which becomes

$$R_{t^*}(h) = 3.47\,\bar{r}\,\{(T/2)\,\ell n\,[T/(T - 2t^*)] - t^*\} \tag{16}$$

Assuming that the relation between the size of a yard and the required resources can be expressed as

$$v = G_1 e + G_2 \ell + G_3 m \tag{17}$$

where v = yard size per MRU and G_1, G_2, G_3 = size factors that relate yard size and resources requirements of equipment, labor, and materials respectively, then the optimum size V^* of any yard can be expressed as $V^* = R_{t^*}(h) \times v$ or

$$V^* = 3.47\,T\bar{r}\,\{(T/2)\ell n\,[T/(T - 2t^*)] - t^*\}\,(G_1 e + G_2 \ell + G_3 m) \tag{18}$$

In addition to providing a solution and a new optimization criterion through t^*, the model in its present form has revealed the general relations between optimum maximum travel time, maintenance cost, and the parameters of the maintenance yard problem. These relations are purely theoretical and are based on specific assumptions but nevertheless are indicative of the basic, and sometimes unique, characteristic of highway maintenance.

Figure 3 shows the relation between adopted T and optimum t^* as well as corresponding UMC_{t^*} and TMC*(A). That relation indicates that low values of T can result in excessive maintenance costs but that high values beyond a certain limit do not result in any significant savings. This finding indicates that low values of T (i.e., less than 7 h) should be avoided on the basis of cost and that adoption of relatively high values ought to be based mainly on emergency needs and human, legal, and safety limitations. This represents a highly critical issue, especially when it is viewed in the light of increasing demands from labor for changes in working hours.

Figure 4 shows the relation between K and t^* as well as UMC_{t^*} and TMC*(A). Note that t^* and maintenance costs increase with K. But, although the relative increase in t^* is noticeable, the change in maintenance costs is hardly detectable, especially at the higher range of K. This indicates that, although the change in K affects considerably the optimum number and size of maintenance yards, it has relatively little effect on total maintenance cost. This can only be explained by the existence of an almost complete balance between the changes in total maintenance yard fixed cost and all other variable costs in the optimum solution.

Figure 5 shows the effect of C on t^*, UMC_{t^*}, and TMC*(A). Of significance here is the extremely low rate of change of t^* at the higher values of C. This implies that the optimal number and size of maintenance yards do not change when C increases beyond a certain limit, which makes an optimum solution immune to the existing trend of continued increases in labor and equipment costs.

Figure 6 shows the effect of the intensity of $\bar{r}$ on t^*, UMC_{t^*}, and $TMC^*(A)$. Both t^* and UMC_t assume relatively high values at the lower level of $\bar{r}$, and $TMC^*(A)$ assumes relatively low values. In addition, while total cost in the optimum solutions increases linearly with $\bar{r}$, both t^* and UMC_{t^*} decrease nonlinearly at a decreasing rate. This again shows that, although the higher values of $\bar{r}$ increase the total maintenance cost of the optimum solution, the effect on the optimum number of yards is relatively negligible.

Figure 7 shows the effect of W on t^*, UMC_{t^*}, and $TMC^*(A)$. Both unit and total maintenance costs of the optimum solution increase linearly with the increase of W, and t^* decreases nonlinearly at a decreasing rate. An interesting point here is the radical difference between the effect of W and the effect of K on the optimum solution.

Both maintenance cost functions (Equations 13 and 15) were found to be unimodal with respect to maximum travel time t. This means that the optimum solution for any set of parameters is always unique, which eliminates the complications of local optima.

Model for a General Case

This model is based on the findings of the special case model and on the development of analogous expressions that apply to the new case.

Assume that the optimum solution to the given general case includes locating a maintenance yard at highway segment i to serve all highway segments within a maximum travel time t_i^*. Consider one MRU located at t_i^* from yard i and served by it; then

$$UMC_{t_i^*} = C[2t_i^*/(T-2t_i^*)] + \left(K_iT/(T-2t_i^*)\left\{\sum_j r_j[T/(T-2t_{ij})]\right\}\right) + W[T/(T-2t_i^*)] + c_4 \qquad (19)$$

for all segments j with $t_{ij} \le t_i^*$ and where all symbols are as previously defined but are expressed in terms of the particular yard location i and its optimum maximum travel time t_i^*.

In addition, consider any segment k with $t_{ik} \le t_i^*$ and all segments j with $t_{ij} \le t_i^*$; then the unit maintenance cost at k is

$$UMC_{ik}(t_i^*) = C[2t_{ik}/(T-2t_{ik})] + \left(K_iT/(T-2t_{ik})\left\{\sum_j r_j[T/(T-2t_{ij})]\right\}\right) + W[T/(T-2t_{ik})] + c_4 \qquad (20)$$

Determine t_i^* by solving for

$$UMC_{ik} = C[2t_{ik}/(T-2t_{ik})] + \left(K_iT/(T-2t_{ik})\left\{\sum_j r_j[T/(T-2t_{ij})]\right\}\right) + W[T/(T-2t_{ik})] + c_4 \qquad (21)$$

for any segment k in the total given area and all j with $t_{ij} \le t_{ik}$. The optimum maximum travel time t_i^* is equal to t_{ik} at which UMC_{ik} is minimized.

Extending Equation 20 to all the highway segments served by yard i and summing their cost to find the total combined travel and yard cost $[MC(t_i^*)]$ of maintenance operations that originate from yard i give

$$MC(t_i^*) = \left\{C\sum_j r_j[2t_{ij}/(T-2t_{ij})]\right\} + K_i + \left\{W\sum_j r_j[T/(T-2t_{ij})]\right\} + c_4\sum_j r_j \qquad (22)$$

for all j with $t_{ij} \le t_i^*$.

Equation 19 has the distinct advantage of being independent of the boundary conditions of the given area, the spatial location and configuration of highway segments, and the magnitude of the area within the boundary. When Equation 19 is combined with the adaptability of maintenance yards to the assumption of their unlimited individual capacity, it represents a unique and powerful tool in the search for the optimum solution. When the matrixes $[t_{ij}]$, $[r_i]$, $[k_i]$ and the fixed parameters of the given problem (C, T, W, C_4) are known, Equations 19, 21, and 22 provide

1. Optimum maximum travel time t_i^* for all i;
2. $UMC_{t_i^*}$ and $UMC_{ij}(t_i^*)$ for all i and j; and
3. $MC(t_i^*)$ for all maintenance operations originating from any yard i in the given area.

These represent the basic tools needed for the selection of the optimum location, number, and size of maintenance yards in any real-life problem. Application of the model to hypothetical situations not included in this paper proved the merits of the developed model. It is anticipated that its application to real-life situations will provide better guidance in decisions that involve maintenance yards.

SUMMARY AND CONCLUSIONS

This investigation has succeeded, through a purely analytical approach, in accomplishing two basic objectives: (a) revealing the general characteristics of the highway maintenance yard problem and the relations between its variables and parameters and (b) developing a model capable of reflecting these relations and solving the given problem. Many of the findings were found to be unique in comparison with existing warehouse or location-allocation models. Among the factors that contributed to this, the most critical proved to be the existence of a limit on daily working hours. As a result, travel time between maintenance yards and work sites on the highway network became the most influential variable in all cost functions entering the model and resulted in the following unique characteristics of the problem:

1. There existed an absolute upper limit on travel time from any maintenance yard. This limit was found to be less than the travel time at which the yard fixed unit cost was minimized, which in turn was found to be strictly less than half the daily work period.
2. The magnitude of the given "fixed" maintenance requirements of any highway segment proved to be a variable that was dependent on the travel time between the segment and the yard serving it. This characteristic precluded the use of the traditional center-of-gravity approach for solving the problem because the maintenance requirements of a highway segment were subject to change during the search for the optimum solution.
3. The effect of travel time on the variable maintenance cost was found to be nonlinear but not quadratic as in some warehouse models.
4. A unique, new criterion for optimization was established: For any yard, there existed a specific maximum travel time that defined the conditions for minimizing the variable unit maintenance cost for any

and all highway segments served by that yard and consequently for minimizing the variable maintenance cost of the total given area. This criterion embodied some significant implications:

a. It gave the model the capability of analyzing the potential and cost of locating an individual yard at a certain location at a very early stage and before the optimum solution for the total given area was known. This capability proved to be completely independent of the total size of the given problem and provided an effective tool for evaluating the efficiency of existing maintenance yards that serve certain jurisdictions.

b. It gave the model the capability of stopping cost calculations for individual yards when the maximum optimum travel time was reached, which saved an appreciable amount of computation.

c. It gave the model the capability of stopping the search for the optimum solution at an early stage under certain conditions, i.e., $t_i^* = \max t_{ij}$ for all i and j in the given problem.

5. The yard cost function per unit maintenance requirement proved to be different from equivalent functions in existing models. Its value stopped decreasing as the number of yards decreased (as travel time increased) and started an upward trend at high values of travel time.

6. The model proved, through the adoption of travel time as a measure of separation, its independence of the area and boundary conditions of the given problem and of the configuration of the given highway network. Some existing models, through the adoption of distance as a measure of separation, proved to be dependent on all these factors, especially on the configuration of the given network (e.g., gridiron, radial, or loops).

Another factor that significantly increased the potential of the model was the extreme adaptability of individual maintenance yards to the assumption of unlimited capacity, which meant that any highway segment could be served optimally by one and only one yard. This increased both the efficiency of the model and the size of the problem that it could handle.

Past experience in highway maintenance indicates that the findings of the model have been intuitively recognized but not quantitatively defined or evaluated. Nevertheless, the validity of the model still needs to be tested and its efficiency in solving real-life problems verified. In designing an algorithm to search for and find the optimum solution, it is recommended that future study be directed to the following efforts:

1. Establish a relation, if any, between the number of yards in the optimum solution and both the given travel time between highway segments and the computed optimum maximum travel time for all potential yard sites in the given area.

2. Provide a criterion, or a set of criteria, to eliminate some highway segments as potential sites for maintenance yards at an early stage of the search.

3. Investigate the potential of reaching the optimum solution in two stages: (a) Start with a limited number of relatively long highway segments, each of which has a uniform fixed yard cost along its length, ensure that border segments are represented, and find the optimum solution; and (b) divide into shorter sections the segments at which the yards of the optimum solution were located, and find the best location among these sections.

It is anticipated that the development of probabilistic models for specific maintenance activities might prove effective in overcoming some of the difficulties encountered in this investigation or in the actual management of these activities.

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