

therefore, decided to temporarily defer its use in tunnels in favor of radar.

In our opinion, radar shows qualitative distinctions in the conditions of the brick tunnel. However, because the brick tunnel is a more complex system than the roadway in terms of the data, we have decided to direct our present efforts to the interpretation of the bridge-deck and roadway data.

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#### Abridgment

## Dynamic Properties of Skewed Beam-Slab Highway Bridges

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The dynamic response and vibrational characteristics of railroad and highway bridges are of concern to bridge designers. This is due to the fact that, through the prediction of the dynamic response, the amplification of the static live-load stresses and deformations can be estimated and this, in turn, may require the re-dimensioning of the superstructure. It has been shown that the amplification of the static response through the use of an impact factor can lead to erroneous results (1, 2). The vibrational characteristics (more specifically, the periods of vibration) of the bridges are usually used in conjunction with the definition of sensitivity to earthquake forces and the human perception of the vibration of the superstructure.

The natural periods of vibration of highway bridges can be used in the discrimination of the dynamic characteristics. The development of simple empirical formulas that can predict the natural periods of vibration would be the optimal solution. Analytical studies have resulted in sufficient information on the natural periods of vibration of simple-span beam-slab highway bridges that have reinforced concrete decks and prestressed concrete I-beams and may or may not have skew (1, 3, 4, 5, 6). This paper summarizes the results of research carried out to predict the natural periods of vibration of these types of bridges that have skew.

#### DESIGN AND ANALYSIS OF BRIDGES

The paper focuses attention on simple-span beam-slab bridges that have prestressed concrete I-beams. To be representative of the variety of bridges encountered in

the field, 33 right bridges were designed by using current engineering practices (7). Their span lengths, curb-to-curb widths, and beam spacings varied from 12.20 to 27.44 m (40 to 90 ft), 7.32 to 18.30 m (24 to 60 ft), and 1.46 to 2.61 m (4 ft 9 in to 8 ft 7 in) respectively. A detailed description of the bridges is available elsewhere (3, 5). The configurations considered cover a wide range of variation in the design parameters.

In the definition of the dynamic characteristics of the bridges, the superstructures were simulated by using the finite-element method. The deck slab was modeled via plate-bending elements and the beams as beam-bending elements. The analysis was performed by using the computer program SAP IV (8). The first three fundamental periods of vibration of the superstructures, both those that had and those that did not have skew, are available elsewhere (3, 5). The empirical formulas for the prediction of the periods of vibration of right bridges that have not already been computed are also available (5). The analysis assumed that no vehicle is on the bridge. The differences between these unloaded and loaded periods have been presented previously (1).

#### SKEWED BRIDGES

In the parametric study, the design parameters of the 33 right bridges were retained intact, but the geometries of the bridges were changed for skew angles of 60° and 45° (90° skew being the right bridge). The inclusion of the new bridges has resulted in the consideration of a total of 99 bridges. Through the application of the finite-

Table 1. Nondimensionalized natural periods of vibration.

Bridge No.	$t_1^{90}$	$t_1^{45}$	$t_2^{60}$	$t_2^{45}$	$t_3^{90}$	$t_3^{45}$
1	0.96	0.90	0.94	0.85	0.88	0.73
2	0.96	0.91	0.94	0.86	0.88	0.72
3	0.93	0.85	0.93	0.83	0.87	0.70
4	0.95	0.88	0.94	0.86	0.92	0.81
5	0.95	0.89	0.95	0.87	0.91	0.78
6	0.93	0.84	0.92	0.82	0.90	0.77
7	0.89	0.83	0.92	0.87	0.94	0.84
8	0.95	0.89	0.95	0.89	0.93	0.83
9	0.93	0.85	0.92	0.82	0.91	0.77
10	0.96	0.91	0.94	0.85	0.87	0.70
11	0.96	0.90	0.95	0.84	0.91	0.75
12	0.96	0.91	0.96	0.90	0.93	0.84
13	0.97	0.92	0.94	0.84	0.87	0.70
14	0.97	0.93	0.95	0.87	0.87	0.70
15	0.96	0.91	0.94	0.86	0.87	0.70
16	0.97	0.92	0.95	0.87	0.91	0.78
17	0.96	0.92	0.96	0.90	0.91	0.79
18	0.96	0.90	0.95	0.87	0.91	0.79
19	0.96	0.92	0.95	0.89	0.93	0.82
20	0.96	0.92	0.96	0.91	0.94	0.84
21	0.95	0.90	0.94	0.88	0.94	0.85
22	0.97	0.93	0.97	0.93	0.96	0.92
23	0.96	0.92	0.96	0.93	0.96	0.92
24	0.96	0.92	0.96	0.93	0.96	0.92
25	0.98	0.94	0.94	0.86	0.86	0.68
26	0.98	0.95	0.95	0.88	0.87	0.70
27	0.97	0.93	0.95	0.87	0.85	0.68
28	0.97	0.94	0.96	0.90	0.91	0.79
29	0.97	0.94	0.97	0.91	0.92	0.80
30	0.97	0.92	0.96	0.90	0.91	0.79
31	0.97	0.93	0.97	0.91	0.94	0.84
32	0.97	0.94	0.97	0.92	0.94	0.86
33	0.96	0.92	0.96	0.90	0.94	0.85

Note: Subscripts indicate period of vibration, and superscripts indicate skew angle.

element simulation of the superstructure and the use of the SAP IV program, as had been done for the right bridges, the natural periods of vibration of skewed bridges were computed. Table 1 gives the nondimensionalized values of the periods of vibration (i.e., the period of vibration of the skewed bridge is divided by the corresponding period of the right bridge). This table shows that, for a given skew angle and nondimensionalized period, the values tend to be similar (i.e., within each column, the values are similar). The results of the statistical analysis of these values for bridges that have 60° skew are given below.

Value	Mean	SD	Standard Error of Mean
$t_1$	0.958	0.017	0.003
$t_2$	0.949	0.014	0.002
$t_3$	0.910	0.032	0.005

In the computation of these values, it is assumed that the corresponding nondimensionalized periods of vibration for right bridges are equal to 1.000.

The results of the statistical analysis of the periods of vibration of bridges that have 45° skew are given below.

Value	Mean	SD	Standard Error of Mean
$t_1$	0.908	0.030	0.005
$t_2$	0.878	0.031	0.005
$t_3$	0.787	0.071	0.012

The relatively small magnitudes of the SD and the standard error of the mean indicate the consistency of the mean values given here. Therefore, if the natural period of vibration of a right bridge is known, then the period of vibration of a skewed bridge can easily be approximated through the use of the appropriate value

given in this paper. Another observation that can be made is in regard to the relatively small changes in the mean periods of vibration for different skew angles. For example, the first period of vibration has values of 1.000, 0.958, and 0.908 for skew angles of 90°, 60°, and 45°. This close proximity permits interpolation for the prediction of the natural periods of bridges that have skew angles between 90° and 45°. However, any extrapolation beyond 45° may lead to erroneous results because the variation of the periods beyond 45° has not been determined.

## CONCLUSIONS

The dynamic and vibrational characteristics of bridge superstructures can be predicted through the use of predominant natural periods. It has been shown that the natural periods of vibration of skewed simple-span beam-slab bridge superstructures can be computed through the use of the appropriate multipliers given here, when the periods of vibration of the equivalent right bridge are known. The natural periods of vibration of bridges that have skew angles up to 45° have been computed. On the average, the reduction in the first fundamental period is at the most 10 percent and, for the second and third periods, this reduction can be up to 21 percent.

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