

# Models for Predicting the Impact of Transportation Policies on Retail Activity

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Comprehensive urban land-use models designed in the past to predict the effects of large, capital-intensive transportation facilities on the spatial distribution of urban activities are not well suited for predicting the impacts of newer policies to control and manage existing facilities. This paper describes a case study that develops two alternative models with a much sharper, policy-oriented focus and substantially reduced requirements for data and computational resources. The case selected for study involves the hypothetical adoption of transportation control measures to improve air quality in the Denver central business district and the potential impact of controls on retail activity. The two models are a cross-section, lagged-adjustment regression that identifies determinants of aggregate sales at any location and a set of disaggregate travel demand models that predicts the equilibrium between shopping trips and retail activity. The forecasts of both models are consistent in predicting substantial declines in retail activity in response to restrictions on automobile access and negligible offsetting effects of improvements in transit service. It is concluded that compensatory nontransportation measures that enhance downtown amenities or the uniqueness of downtown retail opportunities may offset the negative influence of reduced accessibility.

With the increasing importance of transportation system management and transportation control plan strategies, transportation planners have been called on to forecast the impacts of new policy options that existing planning tools are ill-suited to simulate. Comprehensive urban activity, or land-use, models present an obvious example of this deficiency. Because they attempt to forecast the spatial distribution of all urban activities for the entire metropolitan area, these models invariably require a large data base, a major calibration effort, and substantial computational resources. Moreover, because of their generality, they often do not specify carefully the behavioral structure that lies behind observed location patterns. As a result, important determinants of location decisions are omitted from the models, and policies that affect these determinants cannot be accurately represented.

This paper demonstrates an alternative approach to activity system modeling that focuses more narrowly on a specific set of policies and a specific activity of interest. The policy selected for study is the adoption of transportation control measures to improve air quality in the central business district (CBD). The impact of interest is the response of retail activity in the CBD, an issue that has raised widespread concern among retailers who fear that transportation controls will undermine their competitive position. The potential impact of transportation controls on retail activity is examined in a case study of Denver, Colorado. [A more detailed discussion of this issue and of the Denver case is available elsewhere (1).] By sacrificing comprehensiveness and focusing on a single policy issue, the modeling strategy described in this paper requires substantially fewer data and resources, incorporates a much richer description of the determinants of retail activity, and forecasts the impacts of a wider range of policies than do comprehensive activity allocation models.

Two separate models have been developed to illus-

trate this approach. The first, termed the aggregate model, is a cross-section, lagged-adjustment regression that identifies determinants of aggregate sales at any location. It was constructed to make use of statistical skills and data sources that should be commonly available to local planning agencies. The second, the disaggregate model, predicts the destination, mode, and frequency of individual shopping trips and uses these predictions to determine retail activity at any site. Its purpose is to provide a particularly detailed representation of the behavior that underlies shopping travel. The models are first described in detail, and then model predictions for the Denver case are presented.

## AGGREGATE MODEL

The aggregate model is a cross-section regression that specifies retail sales at different locations as a function of market characteristics such as access to households, household income, and noon-hour shopping by nearby workers. An important objective of this methodology is the use of data that are readily available to the local planning agency. In Denver, the best such data are those originally assembled for the calibration of the Denver EMPIRIC activity allocation model. All data used to estimate the aggregate model have therefore been taken from this source. Each observation in the regression model is one of the geographic zones for which the EMPIRIC data are reported. The level of retail sales in each zone, which is not reported directly in the EMPIRIC data set, has been estimated from EMPIRIC information on retail employment by applying sales per employee ratios computed from the Census of Retailing. In cities where information equivalent to the Denver EMPIRIC data has not already been collected, all the information required for the aggregate model could be assembled from readily available sources such as census publications and local transportation network information.

The focus of the model is on estimating the sensitivity of sales to the access of stores to customers. For each retail zone, accessibility to households throughout the metropolitan area is defined for the EMPIRIC model as

$$A_i = \sum_{j=1}^N H_j \cdot f(t_{ij}) \quad (1)$$

where

- $A_i$  = accessibility of stores in zone  $i$  to households,
- $N$  = total number of zones,
- $H_j$  = number of households in zone  $j$ ,
- $t_{ij}$  = travel time between  $i$  and  $j$ , and
- $f(\ )$  = a travel impedance function.

The form of the impedance function is the gamma function,

$$f(t_{ij}) = (b^a/\gamma)\Gamma_{ij}^{a-1}e^{-bt_{ij}} \quad (2)$$

where  $a$ ,  $b$ , and  $\gamma$  are empirical parameters that have been estimated for the Denver EMPIRIC model to be  $a = 3.434$ ,  $b = 0.314$ , and  $\gamma = 3.0922$ .

The stores in zone  $i$  do not operate in isolation but in competition with retailers in all other zones in the metropolitan region. Their competitive position depends not only on their accessibility to potential customers but also on the access of competitors to this market. It is useful, therefore, to introduce a slightly modified accessibility measure for retailers in each zone. This modified measure, which has been named competitive accessibility, is simply the access to customers of stores in zone  $i$  divided by the average of access measures for all competitive zones:

$$A_i/\bar{A}_j = \left\{ A_i / [1/(N-1)] \sum_{j \neq i}^N A_j \right\} \quad (3)$$

Because of arbitrary variations in zonal retail sales caused by variations in zone size, the dependent variable has been defined as sales per zone acre (the models described here were calibrated in U.S. customary units of measurement, and therefore no SI units are given). The estimated relation between sales and competitive access takes the form

$$S_i^* = \alpha_0 (A_i/\bar{A}_j)^{\alpha_1} \quad (4)$$

where  $S_i^*$  is sales in zone  $i$  per zone acre. If all zones were equally accessible, sales in each would be  $\alpha_0$ . The coefficient  $\alpha_1$  measures the percentage change in sales with respect to a 1 percent change in competitive accessibility.

Equation 4 implicitly assumes full adjustment of sales to current levels of accessibility. To satisfy this assumption, retail centers must expand in areas where access to households has recently increased and contract where access has recently declined. Households must adjust shopping patterns in favor of centers that have become more accessible to them and those whose growth has created greater shopping opportunities. Such adjustments typically take many years. New stores are not opened immediately nor are existing ones closed in response to changes in demand. Households continue to be governed by shopping habits acquired in the past. These lags in adjustment can be explicitly modeled so that

$$S_i^t = (S_i^*/S_i^{t-1})^\theta \cdot S_i^{t-1} \quad (5)$$

where

$S_i^t$  = current sales,  
 $S_i^*$  = fully adjusted sales, and  
 $S_i^{t-1}$  = sales in the past period.

Current sales depend on the relative size of fully adjusted sales and last-period sales and on  $\theta$ , which measures the fraction of the disparity that is closed in the current period. If, for example,  $\theta = 0.4$  and fully adjusted sales are 10 percent above those achieved in the past, current sales will rise by 4 percent.

Substituting Equation 4 into Equation 5 and taking natural logs yield

$$\ln S_i^t = \theta \ln \alpha_0 + \theta \alpha_1 \ln(A_i/\bar{A}_j) + (1 - \theta) \ln S_i^{t-1} \quad (6)$$

Equation 6 shows the basic form of the regression model, which can be easily estimated by linear regression techniques. In the central role accorded accessibility as a determinant of retail sales, this model resembles earlier work by Lakshmanan (2) and Lakshmanan and Hansen (3, 4). However, the specification employed here is quite dissimilar.

The model actually estimated for Denver is somewhat more complex. To measure the separate effects of automobile and transit accessibility on retail sales, measures of each are included separately as explanatory variables in the regression equation. Both of the accessibility variables are of the form just described. They differ from one another in numerical value because of differences in travel times by automobile and by bus.

As indicated at the beginning of this section, several variables other than access to households are likely to be important in explaining variations in retail sales. These variables are included in the regression to control roughly for forces that operate simultaneously with accessibility to determine retail sales rather than to provide precise measures of their impacts. Therefore, the form of their appearance in the regression has been determined by the requirement that they be easily available from the EMPIRIC data set and that they be compatible with the functional form in Equation 6. Careful specification of functional form is not as important for these variables as for accessibility, for which precise measures of impact are the principal objective of the study.

Because sales depend not only on accessibility to households but also on levels of income in the most accessible markets, income in these markets is the first control variable. For each retail zone, it is measured by the fraction of households in the zone with incomes in the lowest 15 percent of the regional income distribution. The second control, which accounts for the impact of noon-hour shopping by white-collar workers, is the number of service, government, finance, insurance, and real estate workers in the zone. Because this number can vary arbitrarily with zone size, the variable has been defined as white-collar workers per zone acre.

The sales equation used for empirical analysis is

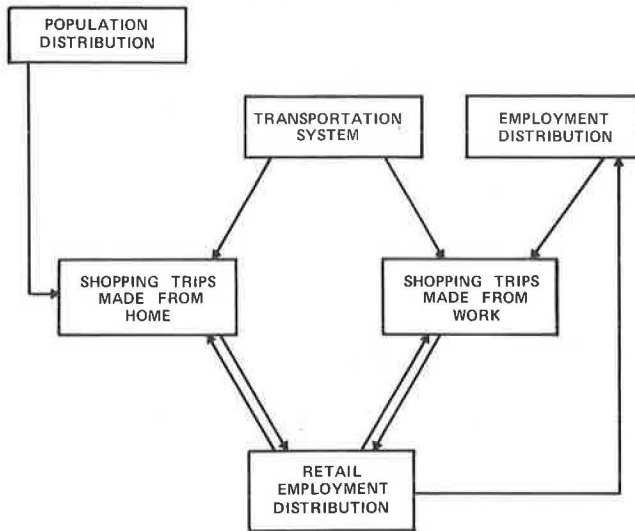
$$\ln S_i^t = \theta \ln \alpha_0 + \theta \alpha_1 \ln(C_i/\bar{C}_j) + \theta \alpha_2 \ln(T_i/\bar{T}_j) + \delta_1 \ln Y_i + \delta_2 \ln E_i + (1 - \theta) \ln S_i^{t-1} + e \quad (7)$$

where

$C_i/\bar{C}_j$  = competitive access to shoppers traveling by automobile,  
 $T_i/\bar{T}_j$  = competitive transit access,  
 $Y_i$  = the proportion of households in  $i$  with incomes in the lowest 15 percent of the regional income distribution,  
 $E_i$  = number of white-collar workers in  $i$  per zone acre,  
 $e$  = a random, standard normal error, and  
 $\alpha_1$ ,  $\delta_1$ , and  $\theta$  = parameters.

Empirical results are given in the table below. All coefficients are significantly different from zero (0.01 level) with the expected signs except for transit access, which is both insignificant and incorrect in sign. Because there are no plausible circumstances in which improved transit access should reduce retail sales, the combination of incorrect sign and statistical insignificance argues strongly that the true effect of transit on sales is zero. Therefore, the equation has been re-estimated without transit access as an explanatory vari-

Figure 1. Structure of disaggregate model system.



able, a procedure that constrains its coefficient to be zero. These results are also given in the table:

Variable	With Transit Access Variable		Without Transit Access Variable	
	Coefficient	t-Ratio	Coefficient	t-Ratio
Automobile access (1970)	0.327	3.09	0.271	2.82
Transit access (1970)	-0.081	-1.25	—	—
Low-income residents (1970)	-0.330	-2.92	-0.351	-3.15
White-collar workers (1970)	0.209	3.82	0.200	3.68
Last-period sales (1960)	0.598	13.34	0.585	13.38
Constant	3.424	—	3.238	—

$R^2$  for the model results with and without the transit access variable was 0.834 and 0.833 respectively. The number of observations was 153 in both cases.

The reestimated results can be used to measure the responsiveness of sales in any zone to changes in the travel time for shoppers arriving by automobile. First, note from Equation 7 that the estimated speed of adjustment ( $\theta$ ) can be determined by subtracting the coefficient of last period sales from 1. According to the reestimated estimates given in the table, therefore, if current market conditions imply equilibrium sales 100 percent greater than those of 10 years earlier, 41.5 percent of the full adjustment will have occurred over the decade. This observation implies that the measured coefficient of automobile access, as given in the reestimated results in the table above, represents only 41.5 percent of the true responsiveness of sales to access ( $\alpha_1$ ), a conclusion confirmed by Equation 7. Therefore, a 100 percent increase in automobile access leads to a fully adjusted rise in retail sales of  $0.271/0.415 = 0.653$  or 65.3 percent. Finally, by using the relation between access and travel time shown in Equations 1 and 2, substituting into Equation 2 the values of the  $a$ ,  $b$ , and  $\gamma$  parameters estimated for the Denver EMPIRIC model, and using the actual 1970 travel times of shoppers driving to the CBD, it can be shown that an increase in travel time of 10 percent leads to a 15.6 percent reduction in the automobile access variable. This relation in turn implies that a 10 percent increase in travel time to the CBD will cause an ultimate decline in CBD sales of  $0.156 \times 0.653 = 0.102$  or 10.2 percent. Analogous methods can be used to estimate the sales impact of any policy that restricts automobile access to the CBD.

## DISAGGREGATE MODEL SYSTEM

The disaggregate model system focuses on individual shopping trips and treats retail activity as simultaneously determined with people's shopping choices. It is derived from some of the basic Lowry model concepts but is based on a more sophisticated set of models than has been used previously. The number of shopping trips to each destination is predicted by separate submodels for home-based shoppers and for workers who shop during the noon hour. Shopping trips are linked to aggregate retail activity by two intuitively plausible relations. First, shoppers' choices of shopping destinations are influenced in part by the scale of retail activity at each destination as well as the level of service provided by the transportation system; and, second, the scale of retail activity at any location expands or contracts in accordance with the number of people who choose to shop there. The entire model system is shown in Figure 1.

### Home-Related Shopping Submodel

The home-related submodel used in the disaggregate model system is based on the short-range generalized policy (SRGP) model developed by Cambridge Systematics for the Metropolitan Transportation Commission in San Francisco. This model accepts as input a sample of households and predicts for each household (among other things) the expected frequency of shopping trips and the probability that each trip will go to every potential destination by both automobile and transit. These predictions are then appropriately factored to represent the population as a whole.

The destination-mode choice predictions are produced by a disaggregate choice model. Because such models are based on the decisions of individual households or travelers, they eliminate the need for aggregating various segments of the population either geographically or demographically. Model parameters, therefore, are in theory not subject to aggregation bias. Because they can be estimated by using very small samples, disaggregate choice models also offer the potential for significantly reducing the costs of data collection. However, most importantly, disaggregate choice models are based on a clear, credible, and consistent theory of how decision makers choose among available alternatives.

Choice theory is concerned with the behavior of an individual decision maker confronted with a mutually exclusive set of alternatives from which one and only one can be selected. The individual decision maker  $n$  associates some level of utility with each available alternative  $i$ . Denote this utility as  $U_{in}$ , and denote the set of alternatives available to individual  $n$  as  $D_n$ .

According to Lancaster (5), each alternative and decision maker can be characterized by a set of attributes. Thus, the utility of the  $i$ th feasible alternative to decision maker  $n$  can be expressed as follows:

$$U_{in} = U_{in}(V_i, W_n) \quad (8)$$

where

- $U_{in}$  = utility of alternative  $i$  to individual  $n$ ,
- $V_i$  = a vector of attributes describing alternative  $i$ , and
- $W_n$  = a vector of attributes describing decision maker  $n$ .

A more convenient expression for the utility function can be developed by defining a vector  $Z_{in} = g(V_i, W_n)$ ,

where  $g$  is some vector-valued function. Thus,  $U_{in} = U_{in}(Z_{in})$ .

Each decision maker is assumed to evaluate the attributes of every alternative and select the one yielding the greatest utility. However, since some of the attributes are unobserved, variables are improperly measured, or utility relations are misspecified, it is generally impossible for an observer to determine precisely which alternative any decision maker will select. However, by making suitable assumptions about the distribution of the unobserved elements in the utility function, it is possible to predict the probability with which any alternative will be selected. When each utility is a random variable, the probability that alternative  $i$  is selected from any set of alternatives  $D_n$  is

$$\Pr(i|D_n) = \Pr[U_{in}(Z_{in}) \geq U_{jn}(Z_{jn}) \text{ for all } j \in D_n] \quad (9)$$

Within the class of random utility model forms, the most generally applicable have been what Manski (6) defines as linear in the parameters with additive disturbances (LPAD). In this case, it is assumed that

$$U_{in} = \beta Z_{in} + \epsilon_{in} \quad (10)$$

where  $\beta$  is a vector of parameters and  $\epsilon_{in}$  is a random variable.

The LPAD form used in this study is the multinomial logit model. This model was chosen for a variety of practical and theoretical reasons including the lack of alternative methods for modeling decision problems with large choice sets and the substantial existing base of successful prior applications. The logit model relies on the assumption that the  $\epsilon_{in}$ 's are independently and identically distributed with the Gumbel distribution; i.e.,

$$\Pr(\epsilon < \omega) = \exp\{-[e^{-(\alpha + \omega)}]\} \quad (11)$$

By using this distribution, McFadden (7) demonstrates that

$$\Pr(i|D_n) = \left( \frac{e^{\beta Z_{in}}}{\sum_{j \in D_n} e^{\beta Z_{jn}}} \right) \quad (12)$$

The parameters of this model can be estimated by maximum likelihood. Such estimates are consistent, asymptotically normal, and asymptotically efficient. McFadden also demonstrates that under relatively weak conditions such estimates exist with probability approaching unity and are unique.

Note that the set of available alternatives  $D_n$  can vary from decision maker to decision maker. For example, a traveler without a driver's license or an available automobile would not generally be viewed as having the alternative of driving alone.

In the destination-mode choice component of the home-related shopping model, each feasible mode and shopping destination in the metropolitan area is an alternative available to the household. The utility of each alternative  $i$  and the probability of its being selected are determined by the attributes given in the table below. The coefficients given in the table are estimates of the  $\beta$ 's in the utility function and show how a unit of each attribute affects the utility of any alternative  $i$ . Therefore, for example, the utility of any destination-mode alternative is reduced by the travel time and cost that it entails and increased by the scale of retail activity as measured by total retail employment and retail employment density.

Variable in Logit Model	Estimated Coefficient	t-Ratio
Constant for automobile mode	0.797	—
Constant for CBD destinations	1.184	2.21
Dummy for CBD destinations in automobile utility function	-0.946	-1.74
Number of automobiles divided by household size	5.330	5.95
LN[total travel time (min) $\times$ income (\$)]	-0.130	-10.57
Out-of-pocket cost for automobile in cents per mile	-0.021	-10.08
Transit cost (cents) $\times$ household size	-0.022	-4.55
Density of retail and service employment per acre in destination zone	0.006	4.22
LN (retail and service employment in destination zone)	0.494	13.70

The estimates in the table have been derived from data on San Francisco shoppers and applied to predict shopping travel in Denver. The constant for the automobile mode, however, has been recalibrated for Denver by a nonstatistical procedure described below. Therefore, there is no t-ratio for that coefficient. Summary statistics for the model as estimated from the San Francisco data include:  $L^*(0)$  (the value of the log likelihood function when all parameters are zero, i.e., when every alternative has the same probability) = -2477;  $L^*(\hat{\beta})$  (the value of the log likelihood function at the maximum likelihood coefficient values) = -1610;  $\chi^2 = 1733$  (this statistic is equal to  $-2 [L^*(0) - L^*(\hat{\beta})]$ , asymptotically distributed as chi square with the number of degrees of freedom equal to the number of parameters estimated, and provides a test against the null hypothesis that all parameters are zero); NOBS (the number of households in the sample) = 572; NCASES (the number of available alternatives in excess of one per household used in the estimation) = 43 846; and the percentage of households for which the alternative with the highest nonstochastic component of utility was actually selected = 14.3.

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In addition to the destination-mode choice predictions, a second component of the home-related shopping sub-model predicts the expected frequency of shopping trips from home. The frequency model is a single nonlinear equation in which the daily total of household home-to-shop and shop-to-home vehicle trips ( $Q$ ) is a function of household size ( $X_1$ ), household income ( $X_2$ ), home-zone retail employment density ( $X_3$ ), and the expected utility of a shopping trip ( $X_4$ ). The functional form of the model, together with the coefficients estimated by nonlinear least squares for the SRGP model, is given by

$$Q = 0.609 / \{ 0.0737 + \exp[-0.342X_1 - 0.515X_2 + 0.115(\ln X_3) - 0.527X_4] \} \quad (13)$$

Most of the relations described in Equation 13 are self-explanatory. For example, the total number of daily shopping trips increases where household income is high but decreases where the residential zone of the household is characterized by a high density of retail employment. A high level of home-zone retail employment presumably leads households to substitute short shopping trips on foot for the journeys by transit and automobile that are predicted by the model.

The expected utility of a shopping trip ( $X_4$ ), which is positively related to the number of trips, requires fur-

ther definition. It measures the expected value of the utility produced when a shopper makes a trip to the mode-destination alternative that yields the highest possible satisfaction. Mathematically,

$$X_4 = E[\text{Max}_{i \in D_n}(U_{in})] \quad (14)$$

It can be shown (8) that, whenever the utility of each individual alternative is a random variable characterized by the Gumbel distribution, as is true in the logit model, the expected utility of a shopping trip can be expressed as

$$X_4 = E[\text{Max}_{i \in D_n}(U_{in})] = \ln \sum_{i \in D_n} e^{\beta Z_{in}} \quad (15)$$

All the information required to calculate  $X_4$  is, therefore, available from the destination-mode component of the home-related shopping submodel. The variable  $X_4$  is the critical link between the frequency component and the destination-mode component. Since it includes all of the level-of-service and socioeconomic variables given in the previous table, its inclusion in the frequency model makes the frequency of shopping travel responsive to transportation policy.

To predict home-related shopping travel in Denver, the home-related shopping submodel has been run by using 253 households sampled from the 1971 Denver home interview survey. Household residential locations and alternative shopping destinations are described by a total of 274 separate zones, and forecasting results are summarized at the level of 10 superzones, one of which is the CBD. Sample results have been appropriately factored to represent the entire population.

Two modeling issues must be addressed here. First is the issue of transferability. Can models estimated for San Francisco legitimately be used to predict shopping travel in Denver? Since these models are based on the decisions of individual households, it should be possible to estimate a model in one city and use at least some of the estimated parameters in another as long as the cities have populations with similar tastes. Atherton (9), Atherton and Ben-Akiva (10), and Pecknold and Suhrbier (11) all present evidence that most parameters are transferable among U.S. cities as diverse as Boston, Milwaukee, New Bedford, Philadelphia, San Francisco, and Washington. The only exceptions are constant terms that determine mode shares and total daily shopping trips. Therefore, the automobile constant in the preceding table and the two constant terms in the frequency model have been adjusted so that the aggregate mode shares and the total daily shopping trips predicted for the Denver region match reported Denver values derived from the 1971 home interview survey. For the logit model, this procedure chooses a value for the constant that is the maximum likelihood estimate for the Denver sample, conditional on the values of the transferred parameters.

A second issue arises because shopping destinations specified in the destination-mode model are not the individual shopping centers available to households but groups of shopping centers that represent the sum of shopping opportunities in each destination zone. Estimation of a disaggregate behavioral model is possible when alternatives are grouped as long as the model is structured to guarantee the following property: When any two destinations with identical characteristics are combined, the resulting probability of choosing the combined zone is equal to the sum of the two probabilities for the destinations treated separately. Lerman (12) has shown that this property, termed homogeneity, can

be guaranteed in a logit model if the model includes a variable that represents the natural log of group size and if its coefficient is constrained to unity. In the original SRGP destination-mode model, the natural log of retail employment was chosen to measure group size. This choice, however, is unsatisfactory for the home-related shopping model. In the home-related shopping model, retail employment in any zone is determined endogenously by a simultaneous process in which shoppers are attracted to places with high retail employment and varied shopping opportunities and retail employment grows or declines in step with consumer demand. Constraining the retail employment coefficient to unity in the destination-mode artificially inflates the number of shoppers who choose destinations where retail employment is high and tends to preclude equilibrium levels of zonal retail employment at values other than zero or infinity. Therefore, for use in the home-related shopping model, the original SRGP destination-mode model has been reestimated without the constraint. It is the set of coefficients from the reestimated model that is shown in the preceding table. Had it been possible to completely respecify the destination-mode model, homogeneity could have been preserved by choosing another measure of group size, such as acreage of the destination zone. Budget limitations precluded this option.

#### Work-Related Shopping Trip Submodel

The second source of shopping trips represented in the disaggregate model is noon-hour shopping by workers. To forecast noon-hour trips, a model of work-related shopping trips originally developed for a study of the Bunker Hill area of Los Angeles has been adapted (13). Because the noon-hour model was originally estimated only for workers in the CBD, the work-based shopping model has been applied only to Denver's downtown zones. Noon-hour shopping is generally considered a substantial fraction of total sales only in the CBD.

Like the destination-mode component of the home-related submodel, the work-related model is a joint multinomial logit model. The probability that a worker will select any one of several mode-destination alternatives depends on the utility yielded by the attributes of each. Relevant attributes and coefficients that measure their contribution to utility are given below [the coefficients are taken from the Los Angeles travel demand model (13)]:

<u>Variable in Logit Model</u>	<u>Estimated Coefficient</u>
Automobile mode constant	-0.592
Walk mode constant	0.115
Minibus mode constant (free fare)	-2.376
Regional bus mode constant (\$0.25 fare)	-2.434
Total travel time in minutes	-0.052
Out-of-pocket travel cost for automobile in cents per mile	-0.008
Trip attraction density per acre at destination	0.032
Employment per acre at origin zone (zero-frequency alternative only)	0.008
Zero-frequency constant	8.578
LN (area of destination zone in acres)	1.000

Unlike the home-related shopping analysis, the destination, mode, and frequency of work-related shopping are predicted by a single choice model. During any noon-hour period, each worker may make a single trip, described by a particular mode-destination combination, or may choose not to travel at all. The alternative of no travel (zero frequency) is explicitly included in the set of alternatives.

The work-related model has been applied to each of five zones in the Denver CBD. These zones provide the geographic detail that describes both the location of workplaces and the alternative shopping destinations. Note in the table that this model makes no use of the socioeconomic characteristics of workers; such data are not likely to be readily available for forecasting. For this reason, the model predicts travel behavior for a representative worker at each workplace zone rather than operating on a sample of travelers as in the home-related model. For each workplace zone, it predicts the share of workers who choose each mode-destination alternative and the share who choose to make no trip at all. Total daily shopping travel is obtained by multiplying these shares times the total number of workers at each workplace zone.

Two adjustments have been made in applying the Los Angeles model to Denver. In the Los Angeles model, the attractiveness of each shopping destination is determined by its trip attraction density, which predicts, for example, that the number of shopping trips to any zone depends on the amount of retail floorspace per zone acre. For Denver zones, we have employment figures but no data for floorspace. For use in the model, employment figures have been converted to floorspace by using typical ratios of CBD floorspace per employee estimated from Boston data.

Second, the coefficients of the Los Angeles model imply an unrealistically low value of time for noon-hour shoppers: \$0.15/h. This low value resulted from Los Angeles data in which money costs were constant for all trips by each mode except automobile. As a result of the high correlation between cost and mode, coefficients of the cost variable and of the constants representing different modes are poorly specified, although their combined effects are correctly estimated. Evidence on work trips for several cities indicates that workers often value travel time in the range of \$3.00 to \$5.00/h (11). We have therefore readjusted the coefficients of cost and of each modal constant so that the combined effect of the constant plus cost remains identical to that estimated by the Los Angeles model but the cost coefficient is consistent with a value of time of \$3.85/h. This adjustment is embodied in the results given in the table above.

#### Tests of Policy and Equilibration Procedure

Before the impacts of new transportation policies can be estimated, both the home-related and work-related shopping models must be run to predict a base number of trips to each destination zone given existing policy. Because both models predict the number of trips rather than the number of shopping visits per trip tour, some average number of shopping visits per trip must be assumed. For work-related shopping, evidence from Los Angeles suggests an average of 1.25 shopping trip ends per work-based shopping tour. To estimate the number of trip ends per home-based trip, we relied on estimates by downtown Denver merchants that the number of shoppers visiting each store is evenly divided between home-based shoppers and workers on noon-hour trips. The disaggregate model system produces this result for CBD destinations if an assumption of two shopping trip ends per home-based trip is made. This assumption has been adopted for all home-related trips. For both kinds of trips, the extra shopping visits that make up the tour are assumed to take place in the same superzone as the original home-based or work-based link of the trip.

To test any policy, the first step of the equilibration procedure is to modify level-of-service variables to reflect the effect of the policy on the Denver transporta-

tion system. Then both the home-related and work-related models are run to forecast total shopping trip ends for each zone. These results are aggregated to the 10-superzone level.

The change in shopping trip ends in each superzone is then computed as a percentage of base-year values. The percentage change in zonal retail employment is assumed to be equal to the percentage change in shopping trip ends. This procedure generates new levels of both retail and total employment in each zone. Both the home- and work-related submodels are run again with the revised employment levels as inputs, and the resulting trip-end forecasts are used to further revise retail and total employment. The process is repeated until the changes in trip ends and retail employment in each zone are relatively small.

Trial runs of several iterations of the model have indicated that, for many different policy options, the change in trip ends at each iteration is a constant fraction of the change at the previous iteration. Given this pattern of convergence and an initial change of  $\Delta J_i$  in the number of trip ends in zone  $i$ , the total change can be approximated by

$$\Delta J_i + \rho \Delta J_i + \rho^2 \Delta J_i + \rho^3 \Delta J_i + \dots = \Delta J_i \sum_{j=0}^{\infty} \rho^j = \Delta J_i / (1 - \rho) \quad (16)$$

where  $\rho$  is the fraction that relates changes in trips at successive iterations. In accordance with a previously specified assumption, retail employment in any zone is expected to change ultimately by a percentage equal to the total change in shopping trip ends. From Equation 16, this percentage is given by

$$[\Delta J_i / (1 - \rho)] / J_i \quad (17)$$

where  $J_i$  is the number of trip ends before the policy change.

This approximation greatly reduces the number of iterations required to operate the entire model system because  $\Delta J_i$  is computed at the first iteration and a reasonable estimate of  $\rho$  can be obtained in two or three iterations. This approximation has been used for all policy analyses in this paper.

It must be noted that the exact mathematical properties of the iterative equilibration procedure are still unknown and that the limited computational experience obtained in this study is insufficient to reach any definitive, empirically based conclusion about the stability of the model. Trial runs that led to the approximation adopted above indicated some instability after the first several iterations, and changes in the definition of zones and superzones affected the pattern of convergence. The conceptual appeal of the disaggregate system argues strongly for further research into its convergence properties and further refinement of its specification.

#### TRANSPORTATION CONTROL POLICIES AND ESTIMATED IMPACTS

Both the aggregate and disaggregate models have been used to estimate the impact of several hypothetical transportation control measures on retail sales in the Denver CBD. These measures include

1. Implementation of an automobile-free zone or elimination of convenient parking spaces so that a shopper arriving by automobile must walk an extra 2.5 min after parking,
2. A similar policy that leads to an extra 5 min of walking after parking, and
3. A 5-min reduction in waiting time for a transit

Table 1: Tests of policy impacts: aggregate and disaggregate models.

Policy	Total Ultimate Percentage Change in Sales (aggregate model)	Ultimate Percentage Change in Trip Ends and Retail Employment (disaggregate model)		
		Response to Change in Level of Service	Response to Change in Retail Center Size	Total
Increase in one-way walk time for parkers				
2.5 min	-17.5	-5.5	-24.3	-29.8
5.0 min	-32.7	-9.9	-32.7	-42.6
Reduction of 5.0 min in one-way wait time for transit riders	+0.0	+0.8	+4.3	+5.1

trip to or from the CBD (for noon-hour bus trips within the CBD, for which average wait times are currently estimated at 5 min, a reduction of 2.5 min has been assumed).

The results, which are given in Table 1, raise several issues worthy of brief discussion.

Both models forecast a substantial sensitivity of retail sales to reductions in access for shoppers who travel by automobile and negligible impacts for improvements in transit service. When a seemingly insignificant 2.5 min is added to automobile trips to the CBD, the forecasted drop in retail activity ranges from 17.5 to nearly 30 percent. Policies that restrict automobile access and seek to preserve demand for downtown shopping by improving transit service will likely fail and precipitate instead a decline in the downtown retail center. Transit improvements that are politically and financially possible may be more effective offsets to reductions in automobile access in cities like Boston and New York, where transit service is already extensive enough to attract large fractions of riders to downtown destinations. The vast majority of American cities, however, resemble Denver more closely than they resemble Boston or New York.

The availability of two estimates of policy impact obtained by using completely dissimilar techniques illustrates an important benefit of the narrowly focused modeling strategy described in this paper. The ability to model the same policy simultaneously with alternative models is a result of the reduced cost inherent in less comprehensive models. The alternative results can be compared for consistency, and their quantitative forecasts can be used as bounds for policy impacts. If the forecasts produced by different methodologies are at least qualitatively consistent with one another, as in this case, confidence in their accuracy is enhanced.

The structure of the disaggregate model makes it possible to identify two separate sources for the decline in retail activity that accompanies a reduction in automobile access: an initial response to the change in level of service and subsequent adjustments that reinforce the first response as shoppers react to changes in the size of retail centers and the shopping opportunities they offer. The immediate impact of transportation policy may represent only a fraction of the ultimate change in activity. According to the disaggregate model, a 2.5-min increase in the time of an automobile trip leads directly to a 5.5 percent decline in retail activity. However, the chain of further adjustments by shoppers who find the smaller retail center less attractive induces a further decline of 24.3 percent.

For any reduction in automobile access, the aggregate forecast of overall decline in retail activity is substantially smaller than the disaggregate forecast. This disparity may well result from an excessive disaggre-

gate estimate of consumer response to changes in the size of retail centers. Recall that the source of this estimate is the equilibration procedure described earlier in this paper and that both this procedure and its results are subject to the uncertainties mentioned there. Further insight into this issue must await further investigation into the convergence properties of the disaggregate system.

Finally, the impacts described in this paper are those that result solely from transportation policies. Transportation control plans may also include compensatory nontransportation measures that enhance the attractiveness of downtown amenities or the uniqueness of downtown retail opportunities. If reductions in automobile access to downtown retail areas are required to meet environmental standards, careful consideration should be given to such measures so that efforts to improve air quality do not unintentionally further erode the urban core. For example, total automobile restriction in certain areas accompanied by improved pedestrian amenities has, at least in Europe, succeeded in offsetting the negative influence of reduced accessibility. This is only common sense. If shopping in the CBD offers a unique or especially pleasant experience, because of the amenities or the products available there, shoppers should be more willing to bear marginal reductions in convenience to shop downtown.

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# Optimizing Urban Mass Transit Systems: A General Model

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This paper describes a model for determining the general dimensions of an optimal mass transit system for an idealized urban area. The model is based on a circular city with a definite center and with density declining uniformly from the center in all directions according to the negative exponential function. The transit system consists of radial routes that emanate from the center and contain discrete stops. Only trips to or from the center are considered, and travel is assumed to occur only in radial and circumferential directions. The model represents total community costs of the system, defined to include travel time, operating costs, equipment, and construction. A recursive procedure was devised to find a simultaneous minimum with respect to the spacing of routes, number and spacing of stops on each route, and average headway. Numerical analyses were conducted for six hypothetical cities by using varying values for the parameters of the density function. In each case, three types of transit systems were compared: conventional bus service, buses on exclusive lanes, and rail rapid transit. The optimal system in the largest city examined was exclusive bus lanes; in the other five cases, the optimal system was conventional bus service. Other interesting relations that appeared in the results are summarized.

The United States has entered a new era of massive investment in urban mass transit, prompted by the willingness of Congress to authorize billions of dollars in federal aid for local transit systems. However, there is yet no systematic procedure for allocating these resources and determining whether a transit proposal is worthwhile. Each proposal is evaluated on an ad hoc basis, and considerable weight is given to the zeal of the proponents, political pressures, and the current availability of funds. Choice of technology has become a major issue in many areas, and the question of whether medium-sized cities should proceed with huge investments in fixed-guideway transit systems is particularly controversial.

This paper summarizes a dissertation aimed at determining the dimensions of an optimal transit system for an idealized urban area (1). The approach was to

hypothesize a circular city with a definite center and with density declining uniformly from the center in all directions. The transit system consists of routes that emanate from the center and contain discrete stops. By use of integral calculus, a model was derived that represented the total community costs of building and using such a system. By use of differential calculus, a procedure was developed to optimize the principal design variables in the system: the number of radial routes, their length, and the number and spacing of stops on each route. Numerical analyses compared three common forms of conventional transit: buses on city streets, buses on exclusive lanes, and rail rapid transit.

Such an abstract model cannot be mechanically applied to the complex, irregular pattern of a real city. Abstraction is an unavoidable compromise if a model is to be made mathematically tractable. Similar approaches have been followed in many previous studies of transit optimization. A few of these will be cited here; a fuller review can be found elsewhere (1).

Most previous studies can be divided into two geometrical approaches. One of these assumes a gridiron network of transit routes laid on a homogeneous infinite city, usually with a uniform density of trip ends. What may have been the first study of this type was done by Creighton and others (2) and involved both highway and transit grids; the object was to find the optimal combination of investment in the two modes. Holroyd (3) assumed a single grid of bus routes and derived a solution for the optimal spacing of routes and frequency of service. Two dissertations, one by Mattzie at Carnegie-Mellon (4) and the other by Woodhull at Rensselaer Polytechnic (5), also dealt with grid systems of transit routes.

The second approach is to examine a single transit line. Often one terminal is assumed to be in the central