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Analysis of Settlement Data From Sand-Drained Areas

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Rate of field consolidation is usually calculated from changes with time of piezometer readings. Presented here is a technique for analysis of field settlement observations to determine the field rate of consolidation and total settlement for sand-drained areas. This technique is developed from equal strain consolidation theory. The approach is demonstrated and verified using field data from three construction sites. For each site, the settlement data were analyzed for rate of consolidation and total settlement. The coefficient of consolidation values extracted from the settlement data are compared to those calculated from changes in pore pressures. Total settlement indicated by the analysis is compared to the maximum settlement observed at each platform. Piezometers are important for controlling construction. However, by use of this technique the complete analysis of field data can be achieved independent of piezometer readings.

Analysis of field data for rate and amount of settlement provides a check on design parameters and assumptions. A review of field data from previous projects in similar soil deposits can be a valuable guide to the most economic design. This is particularly important when considering vertical sand drains, since the expense of drain installation must be offset by faster consolidation.

Vertical sand drains have been used for nearly 50 years to shorten the time to achieve settlements in clay layers (1). There is, however, some question about sand drain effectiveness in sensitive clays when displacement methods are used to form the drains (2, 3, 4, 5, 6, 7). A judgment on the effectiveness of sand drains usually requires analysis of field data for rate and amount of settlement and the comparison of these field values to the parameters predicted from laboratory tests. The analysis of field data also provides information on soil disturbance due to the method of drain installation, since disturbance tends to decrease the rate of consolidation and increase the amount of settlement.

Field data often include information on rate of filling, piezometer readings, and settlement platform observations. Field values of rate of consolidation are normally computed from the change with time of the excess pore pressures as indicated by piezometers. The amount of ultimate consolidation settlement the fill will experience is usually computed from the change in piezometer readings and settlement platform observations. Steps in the analysis using piezometer readings have been outlined by Johnson (1) and Moran and others (8).

Settlement platforms are less expensive to install and easier to maintain than piezometers. The determination of rate of consolidation from settlement data alone has been considered difficult or impossible (1). Presented here, however, is a simple technique for using settlement data only to analyze for rate of consolidation and total settlement. The technique is based on equal strain consolidation theory for vertical sand drains and is as easily applied as the conventional analysis of pore pressures. The values of rate, as indicated by the coefficient of consolidation, analyzed by this settlement method are compared to values determined from piezometer readings. Computed total settlements are compared to the maximum observed settlement. This technique allows more extensive analysis of field data from sand-drained areas.

THEORETICAL BASIS

Consolidation in a sand-drained area can be envisaged as dissipation of excess pore pressures in the vertical and radial directions with reasonably well-defined boundary conditions. The average consolidation reflects the dissipation in both directions and can be written (9)

$$1 - U_c = (1 - U_R)(1 - U_v) \qquad (1)$$

where

U_c = average consolidation of the clay layer,
U_R = average consolidation if only radial flow to the sand drains occurs, and
U_v = average consolidation if only vertical drainage occurs.

The expression for average consolidation due to radial drainage, assuming equal strain, is (10)

$$U_R = 1 - \exp[-2T_R/F(n)] \qquad (2)$$

$$F(n) = \{[n^2/(n^2 - 1)]\ln(n) - (3n^2 - 1)/4n^2\} \qquad (3)$$

where

$n = r_e/r_w$,
r_e = effective radius of the sand drain,
r_w = radius of the sand well,
$T_R = C_R t/r_e^2$ (dimensionless time factor),
C_R = coefficient of consolidation in the radial direction, and
t = time.

The relation for vertical consolidation, which can be found in most textbooks on soil mechanics is (9, 11, 12)

$$U_v = 1 - \sum_{m=0}^{m=\infty} 2/M^2 \exp(-M^2 T_v) \tag{4}$$

$$M = (2m + 1)\pi/2 \tag{5}$$

where

T_v = time factor = $C_v t/H^2$,
C_v = coefficient of consolidation in the vertical direction, and
H = maximum drainage path.

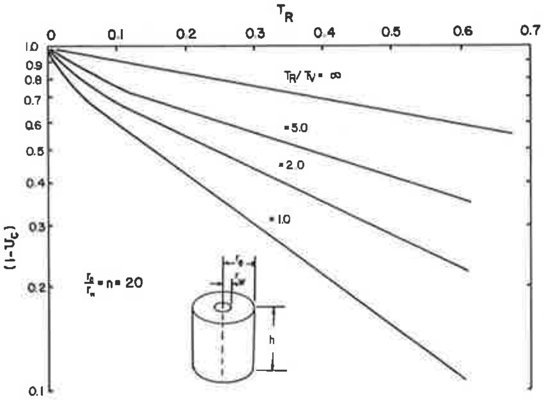
An example is presented to illustrate the expected field behavior. Using Equations 2 and 4, the progress of average consolidation under combined vertical and radial flow can be computed. A family of curves was computed using a value of $n = 20$ and a different value of the ratio T_R/T_v for each curve. The ratio of the time factors includes the ratios of the coefficients of consolidation as well as the squares of the respective drainage paths. The results of these computations are shown in Figure 1. The shape of the curves for combined consolidation is similar to the shape of the consolidation curve for radial drainage alone in that the logarithms of $(1-U_c)$ plots as a straight line at times greater than $T_R = 0.1$. The effect of vertical drainage is to steepen the slope. The curves shown in Figure 1 indicate that at higher values of the time factor, the slopes of a field settlement-time curve can be described with the aid of the equation:

$$1 - U_c = \exp[-2T_{Rc}/F(n)] \tag{6}$$

where $T_{Rc} = C_{Rc} t/r_e^2$, and C_{Rc} = combined coefficient of consolidation.

The combined coefficient of consolidation (C_{Rc}) is greater than the coefficient in the radial direction (C_R) because of the influence of vertical drainage.

Figure 1. $(1 - U_c)$ versus T_R for combined radial and vertical flow.



TECHNIQUE FOR FIELD DATA

When filling is complete and consolidation is proceeding, the observed settlements can be described by the equation (9, 13)

$$\rho = \rho_r + U_c \rho_{cf} \tag{7}$$

where

ρ = observed settlement at the time of interest,
ρ_r = rapid settlements accompanying fill placement, and
ρ_{cf} = final consolidation settlement.

Substituting the value of U_c from Equation 6 into Equation 7 yields

$$\rho = \rho_t - \exp[-2C_{Rc}t/F(n)r_e^2]\,\rho_{cf} \tag{8}$$

where $\rho_t = \rho_r + \rho_{cf}$.

Differentiating Equation 8 with respect to time

$$d\rho/dt = \exp[-2C_{Rc}t/F(n)r_e^2]\,[2C_{Rc}\rho_{cf}/F(n)r_e^2] \tag{9}$$

$$\log d\rho/dt = \log[2C_{Rc}\rho_{cf}/F(n)r_e^2] - [0.868C_{Rc}/F(n)r_e^2]\,t \tag{10}$$

Equation 10 indicates that if the slopes of the time settlement curve are determined at three or more times after filling and the logarithm of each slope is plotted against the time at which the slope is determined, the plot forms a straight line whose slope is proportional to the combined coefficient of consolidation (C_{Rc}). The final consolidation settlement does not affect the slope but appears in Equation 10 as an intercept.

In general, a reliable estimate of the total settlement ($\rho_t = \rho_r + \rho_{cf}$) can be obtained by an additional manipulation of Equation 8.

$$\ln(\rho_t - \rho)/\rho_{cf} = -2C_{Rc}t/F(n)r_e^2 \tag{11}$$

Applying Equation 11 at two different times and subtracting

$$\ln(\rho_t - \rho_2)/(\rho_t - \rho_1) = -[2C_{Rc}/F(n)r_e^2]\,(t_2 - t_1) \tag{12}$$

where ρ_1 = observed settlement at time t_1 and ρ_2 = observed settlement at time t_2.

Equation 12 contains only one unknown, ρ_t, since C_{Rc} is now known from Equation 10.

Verification

The applicability of this technique for field data was verified by comparing the coefficients of consolidation determined by Equation 10 to those determined by analyzing pore pressures. The total settlements computed from Equation 12 for data over a limited period were compared to the maximum observed settlements. The technique presented here can be applied to any sand-drained area where settlement data are available.

The data for each settlement platform were plotted against time and a smooth curve drawn through the points as shown in Figure 2. Although the tangent of the settlement-time curve at various points is required, the normal to the curve can be more accurately located by eye by use of a front-surface mirror. The mirror is placed on the curve at time of interest and rotated until the curve and its reflected image appear symmetrical in the vicinity of the mirror. A pencil line is then drawn along the mirror, which is now perpendicular to the curve. Using a mirror allows the person doing the anal-

Figure 2. Settlement versus time from field data.

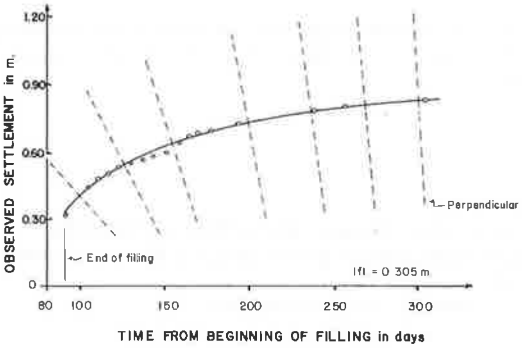
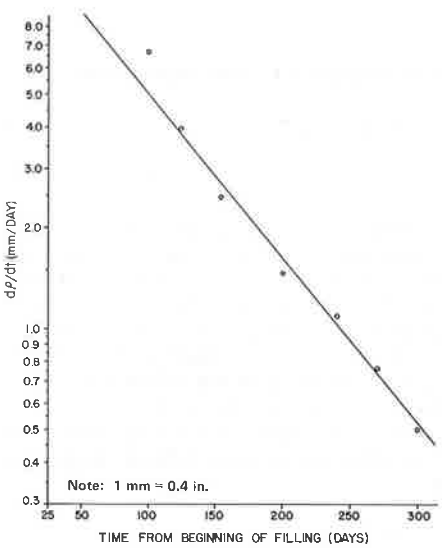


Figure 3. Log $d\rho/dt$ against time.



ysis to concentrate on the curve in the vicinity of each selected time. A front-surface mirror eliminates refraction. Figure 2 shows the perpendiculars determined with a mirror. The slopes of the perpendiculars were then converted to the tangents with

$$(\Delta y/\Delta x)_{tangent} = -1/[(\Delta y/\Delta x)_{normal}\,(x\ \text{scale}/y\ \text{scale})^2] \tag{13}$$

A typical semilog plot of the tangents ($d\rho/dt$) against time is shown in Figure 3. As can be seen from Figure 3, the points fall close to a straight line for times between 125 and 300 d. The slope of this straight line was used in Equations 10 and 12 to determine C_{R_0} and ρ_t.

Also note from Figure 3 that the plotted slope of the settlement-time curve at 110 d is above the line formed by the slopes at greater times. This behavior was observed in several plots. The predicted effect of vertical drainage is to steepen the consolidation-time curve as shown in Figure 1. This effect is greater at times less than $T_R = 0.1$.

This technique was applied to field data from three highway construction sites.

Site Descriptions

Site 1: Southern Tier Expressway

This site is located in Jamestown, New York. Details of the site were recorded by Leathers (14). The profile

Figure 4. Plan and center line profile, Southern Tier Expressway, ramp KJ, site 1.

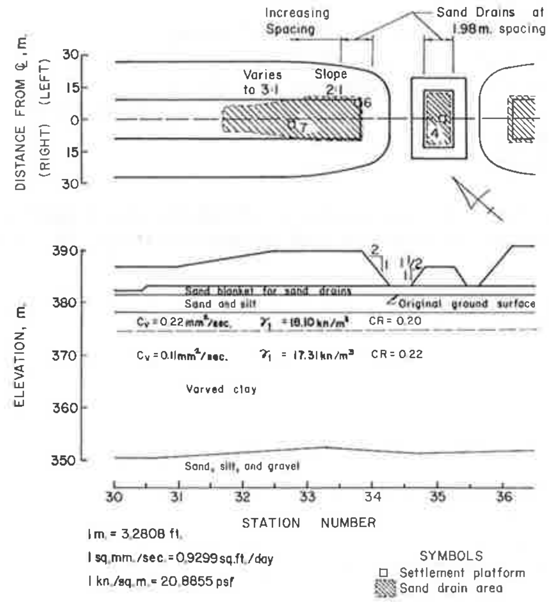
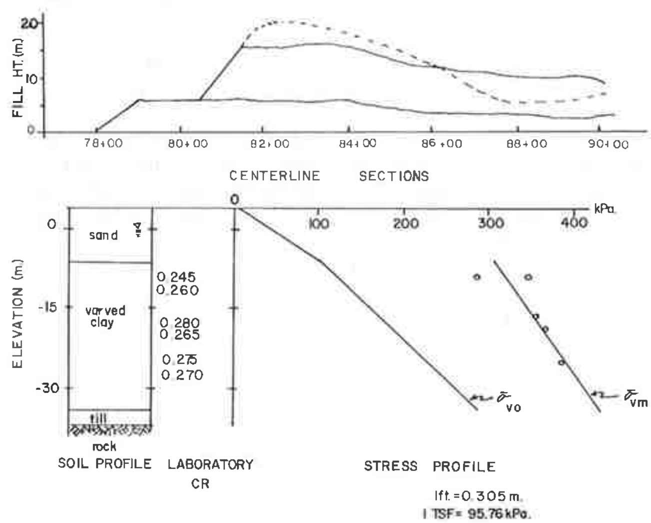


Figure 5. Soil and fill profile for the high fill at east approach to Putnam Bridge, site 2.



shown in Figure 4 gives the basic soils and properties at the site. The major deposit is a layer of varved clay about 27.4 m (90 ft) thick, which is covered by sand and silt and underlain by sand, silt, and gravel. Sand drains were installed by jetting. Minimum sand drain spacing was 2.0 m (6.5 ft), increasing toward the northwest under the higher embankment as shown in Figure 4. The drain spacing at settlement platforms 4 and 6 was assumed as 2.0 m. The drain spacing at settlement platform 7 was about 5.5 m (18 ft). The diameter of the sand drains was 0.3 m (12 in).

Site 2: East Approach to the Putnam Bridge

The Putnam Bridge spans the Connecticut River a few kilometers south of Hartford, between Wethersfield and Glastonbury, Connecticut. The east approach required a high fill that was stabilized by berms. Sand drains 0.45 m (18 in) in diameter were installed with a continuous flight auger (6). Spacing of the sand drains near the functioning piezometers was 3.05 m (10 ft). The fill and soil profiles are shown in Figure 5. The varved clay is about 27.4 m (90 ft) thick. A layer of sand covers the varved clay and a stratum of till separates the clay from bedrock.

Site 3: I-95, Portsmouth, New Hampshire

This site is located in New Hampshire at the intersection of the New Hampshire and Spaulding turnpikes. Details of the soils and construction have been published elsewhere (15, 16). A typical profile shows a shallow layer of organic materials, a 1.5-m (5-ft) clay crust, and then 6- to 11-m (20- to 35-ft) soft, somewhat sensitive, marine clay underlain by a sandy glacial till over bedrock. The marine clay had a liquidity index between 1.5 to 2.0. The sand drain spacing varied between 2.7 and 4.9 m (9 and 16 ft), depending on location along the highway. Most of the sand drains are beneath the high part of the fill. The drain diameter is 0.30 m (12 in). The Dutch jet-bailer method of jetting was used to install the sand drains (16).

RESULTS

Southern Tier Expressway and Putnam Bridge

The results from these two projects are reported in Table 1. The method outlined by Johnson (1) was used in compute C_{Ro} from the piezometer readings. Values of ρ_t were computed from Equation 12.

The values of C_{Ro} for the Southern Tier Expressway from settlement data are about the same as those computed from piezometer readings for platforms 6 and 7 and are slightly low for platform 4. At both of these sites the drain spacing is small compared to the vertical thickness of the clay deposit. The influence of vertical drainage is small and C_{Ro} will be close to C_R.

The values of ρ_t from the settlement data are approximately equal to the maximum observed settlements. The maximum observed settlements may contain some secondary compression settlements.

Only two piezometer groups at the east approach to the Putnam Bridge functioned long enough to determine a value of C_{Ro}. However, as can be seen from Table 1, the values of C_{Ro} as analyzed from the settlement data alone are about the same as values determined from the piezometer readings. The values of ρ_t are slightly larger than the maximum observed settlements, indicating that the consolidation was not complete, which was supported by the piezometer readings.

I-95, Portsmouth, New Hampshire

The consolidation at this site was influenced most by vertical drainage. The thickness of the clay layer was about 12.2 m (40 ft). Since laboratory data were available for C_v, the values of C_R were separated from C_{Ro}.

The separation of C_R from C_{Ro} requires that either the value of C_v or the ratio C_R/C_v be known. Since the combined behavior is similar to the radial behavior, Equation 1 can be written

$$\exp\{-[2/F(n)]\,(T_{Rc} - T_R)\} = 1 - U_v \qquad (14)$$

where the subscripts indicate the coefficient of consolidation used to compute the time factors. To use Equation 14, substitute the approximate expressions for U_v (11) and take the natural logarithms of both sides, which yields

$$\text{if } T_v \leq 0.28 \; \{-[2/F(n)]\,(T_{Rc} - T_R)\} = \ln[1 - (4T_v/\pi)^{1/2}] \qquad (15a)$$

$$\text{if } T_v \geq 0.28 \; [-2/F(n)]\,(T_{Rc} - T_R) = \ln(8/\pi^2) - \pi^2 T_v/4 \qquad (15b)$$

When computing the time factors for use in Equation 15, an adjustment must be made for construction time. This is normally done, in accordance with the method of Taylor, by halving the construction time (11).

Equation 15 is convenient to use when $\overline{C_v}$ is known. When there is more confidence in the ratio C_R/C_v, Equation 15 can be rewritten

$$t = r_e^2 T_R/C_R = H^2 T_v/C_v \qquad (16a)$$

Therefore,

$$T_v = (C_v/C_R)(r_e^2/H^2)T_R \qquad (16b)$$

Equations 15a and 15b then become:

$$-[2/F(n)]\,(T_{Rc} - T_R) = \ln\Big[1 - \{[(4/\pi)(C_v/C_R)(r_e^2/H^2)]\,T_R\}^{1/2}\Big] \qquad (17a)$$

$$-[2/F(n)]\,(T_{Rc} - T_R) = +\ln(8/\pi^2) - (\pi^2/4) \times \{[(C_v/C_R)(r_e^2/H^2)]\,T_R\} \qquad (17b)$$

Equation 17 must be solved by trial and error. A few trials are usually sufficient to obtain the proper value of T_R. The best estimate of C_R by either Equation 15 or 17 is obtained by first computing T_R for a series of times, then determining C_R on an increment of time basis from

$$\Delta T_R = (C_R \Delta t)/r_e^2 \qquad (18)$$

The results of analysis from I-95 are shown in Table 2. Values of C_{Ro} and ρ_t were found as explained for Table 1. The value of C_R was estimated at 0.13 mm^2/s (0.12 ft^2/d) from laboratory data. Having appropriate values of C_R, H, C_{Ro}, and r_e, values of the time factors T_v and T_{Rc} can be calculated for any time. These time factors were then used in Equations 15 and 18 to determine C_R. As can be seen from Table 2, the values of C_R by both methods are comparable and the values of ρ_t are slightly larger in most cases than the maximum observed settlements.

COMMENTS AND CONCLUSIONS

Information on pore pressure behavior during construction is invaluable. Piezometers tend to have a limited functional life in areas experiencing large settlements. On routine projects piezometers are most needed to monitor pore pressures generated during filling. After pore pressures have peaked, malfunctioning piezometers are seldom replaced. Settlement platforms perform their functions for longer periods of time, making settlement data more readily available. This technique fills a need to be able to analyze settlement data when no piezometer readings are available.

The technique is relatively simple to apply. In addition to standard engineering office supplies, the technique requires a mirror, preferably a front-surface mirror. The analysis of data from each settlement platform requires about 1 h. Field settlement data alone can be analyzed to yield valid rate of consolidation and total settlement values.

Table 1. Summary of C_{Rc} results, sites 1 and 2.

Project Name	Settlement Platform No.	Sand Drain Spacing (m)	C_{Rc}[a] (piezometer) (mm^2/s)	C_{Rc} (settlement data) (mm^2/s)	ρ (m)	Maximum Observed Settlement (m)
Southern	4	2.0	0.27-0.71	0.13	0.21	0.22
tier	6	2.0	0.12-0.26	0.15	0.42	0.45
expressway	7	5.5	0.70-1.69	1.02	0.48	0.51
East approach	1	3.0		0.12	0.41	0.39
Putnam Bridge	2	3.0	0.15	0.18	0.88	0.84
	3	3.0		0.14	1.48	1.28
	4	3.0		0.29	1.19	1.08
	7	3.0		0.09	0.47	0.43
	11	3.0	0.16	-	-	1.55

Note: 1 mm²/s = 0.93 ft²/d; 1 m = 3.3 ft.

[a] As reported by Leathers (14) and Long and Healy (17).

Table 2. Summary of C_R results, site 3.

Project Name	Settlement Platform No.	Sand Drain Spacing (m)	C_{Rc}[a] (piezometer) (mm^2/s)	C_{Rc} (settlement data) (mm^2/s)	ρ_t (m)	Maximum Observed Settlement (m)
I-95	C-1	2.7	0.27-0.48	0.38	1.37	1.34
Interchange	C-2	2.7	0.22-0.35	0.37	1.01	1.01
	C-3	3.8	0.09-0.48	0.15	1.28	1.01
	C-4	2.7	0.15-0.37	0.37	1.34	1.31
	C-5	3.8	0.16-0.59	0.10	0.98	0.61
	C-7	2.7	0.10-0.27	0.27	1.25	1.19
	C-8	4.9	0.32-1.08	0.92	0.73	0.82

Notes: 1 mm²/s = 0.93 ft²/d; 1 m = 3.3 ft.

[a] As reported by Gifford (15) and Ladd, Rixner, and Gifford (16).

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