on the development of a frontage-road level-of service evaluation program that was conducted by the Texas Transportation Institute and sponsored by the Texas State Department of Highways and Public Transportation in cooperation with the U.S. Department of Transportation, Federal Highway Administration. We wish to thank Harold D. Cooner, Herman E. Haenel, and Elmer A. Koeppe of the Texas State Department of Highways and Public Transportation for their technical inputs and constructive suggestions throughout the duration of this project. The assistance provided by Murray A. Crutcher and the staffs in Houston and Dallas of the Texas Transportation Institute and of district 12 of the Texas State Department of Highways and Public Transportation is gratefully acknowledged. The contents of this paper reflect our views; we are responsible for the facts and accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Federal Highway Administration. This paper does not constitute a standard, specification, or regulation.

REFERENCES

1. A Policy on Geometric Design of Rural Highways. AASHO, 1965.
2. J. E. Leisch. Spacings of Interchanges on Freeways in Urban Areas. Trans., ASCE, Vol. 126, Pt. 4, 1961.
3. Highway Capacity Manual. HRB, Special Rept. 87, 1965.
4. C. J. Messer, D. B. Fambro, and D. A. Andersen. A Study of the Effects of Design and Operational Performance of Signal Systems. Texas Transportation Institute, Texas A&M Univ., College Station, Res. Rept. 203-2F, Aug. 1975.
5. P. M. Morse. Queues, Inventories, and Maintenance. Operations Research Society of America, Operations Research, No. 1, 1967.
6. K. A. Brewer, J. H. Buhr, D. R. Drew, and C. J. Messer. Ramp Capacity and Service Volume as Related to Freeway Control. HRB, Highway Research Record 279, 1969, pp. 70-86.
7. Highway Design Division Operations and Procedures Manual. Texas Highway Department, Pt. 4, 1970.
8. P. F. Everall. Urban Freeway Surveillance and Control: The State of the Art. Federal Highway Administration, 1972.

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Abridgment

Stimulus-Response Lane-Changing Model at Freeway Lane Drops

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Lane-changing behavior brought about by a freeway lane drop should differ considerably from the other types of lane-changing behavior considered in past studies because drivers in the dropped lane must merge into the adjacent through lane by or just downstream of the end of the lane-drop taper. Every lane-drop site has warning signs or pavement markings or both to alert drivers of the impending drop.

The stimulus-response lane-changing model was developed as a part of the freeway-lane-drop study (1) to relate driver lane-changing responses to various types of stimuli that forewarn of the lane drop. The model seeks to bridge the gap between macroscopic empirical observations of lane-changing behavior at freeway lane drops and microscopic human-behavior models that attempt to represent individual decision-making processes.

The lane-drop study collected approximately 2.5-3 h of traffic data at 18 mainline lane-drop sites by using a time-lapse photographic technique. Each site was divided into five 122-m (400-ft) subsections for data-reduction purposes; the boundary between the fourth and fifth subsections was always located at the end of the lane-drop taper. Traffic measures were reduced on a subsection or an entire-site basis. Lane-changing data reduced in this manner formed the basis for calibration of the model.

MODEL STRUCTURE

The basic behavioral assumption of the stimulus-response lane-changing model is that each sign or pavement marking or the view of the lane drop itself can be considered as a stimulus that will induce a certain proportion of drivers to change lanes. Thus, the model is most effective in representing the behavior of drivers who rely on warning signs and the visibility of the lane drop in their lane-changing decision-making processes. Drivers, such as commuters, who frequently traverse the lane-drop section probably respond more to their preexisting knowledge. The discussion below applies only to those drivers who are unfamiliar with the lane-drop site.

In the simplest form, suppose that each stimulus causes a certain proportion (p) of drivers to change lanes. Then, the probability (P_i) that a randomly chosen driver entering the lane-drop area within the dropped lane will change into the adjacent through lane in response to the ith stimulus obeys a geometric distribution with parameter p.

However, the geometric distribution is only an approximation, because it requires an infinite number of stimuli to cause all drivers to leave the drop lane. In actuality, only a finite number of stimuli, say n, are present. An improved representation can be obtained by assuming that the beginning of the lane-drop taper constitutes a final (n + 1st) stimulus that will cause all

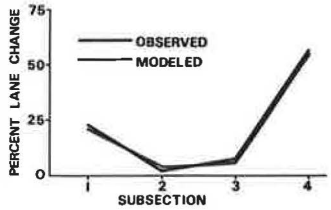

Figure 1. Observed and modeled lane-changing behavior at typical good-fit site.

drivers still in the drop lane to commence their lane changes. The complete response distribution with this modification is

$$P_i = \begin{cases} (1-p)^{i-1}p & (i \leq n) \\ (1-p)^n & (i = n+1) \\ 0 & \text{(otherwise)} \end{cases} \tag{1}$$

The actual lane-change-completion distribution over the length of the entire lane-drop area is of more interest than the response distribution. The drivers reacting to any one stimulus complete their lane changes over distance according to some probability density function [$g(x)$], where x represents distance relative to the location of that stimulus. This density function is referred to as the lane-change-completion density function. The density function of the gamma distribution, with parameters k and λ, was selected to represent lane-change completion in the model because this distribution has the flexibility to conform its shape to a wide range of possible lane-changing distributions. Furthermore, the gamma density function has a zero probability associated with x values less than zero (where it is anticipated that no lane changing in response to the stimulus should take place).

If the ith stimulus is located at coordinate d_i relative to a common origin, then the density function for lane changing in response to that stimulus relative to this common origin is $g(x - d_i)$. However, it is not reasonable to assume that the first lane-change completion arising from a stimulus will occur precisely at the location of that stimulus. Therefore, a location parameter (R) is included so that the density function for lane-change completion in response to stimulus i becomes $g(x - R - d_i)$.

Lane-change completion in response to a given stimulus is a random variable that is the sum of (a) distance traveled while perceiving the stimulus, (b) distance traveled while reacting to the stimulus, and (c) distance traveled while changing lanes in response to the stimulus. In this paper, the first two components are treated as constants, which allows handling their effects as part of R. It is realized that this treatment of the first two components is only approximate and so offers an area for further model improvement.

The overall lane-change-completion density function [$f(x)$] for the site is the weighted sum of the separate densities [$g(x - R - d_i)$]; the weights are the P_i given by Equation 1:

$$f(x) = \sum_{i=i}^{n} [(1-p)^{i-1} p \times g(x - R - d_i)] + [(1-p)^n \times g(x - R - d_{n+1})] \tag{2}$$

Finally, as presented here, the model has been generalized by assuming that signs and pavement markings are distinct types of stimuli and that each will affect drivers differently. Consequently, one impact value (P_s) will be used for signs and another impact value (p_m) will be used for pavement markings. An expression for the density function for such a generalization has been developed elsewhere (1).

PARAMETER ESTIMATION

The parameters k and λ were estimated from the mean and standard deviations of the lane-changing times given elsewhere (2). These estimated values are (1 m = 3.3 ft)

Side of Drop	k	$\lambda (m^{-1})$
Right	2.83	0.0305
Left	2.22	0.0174

The other parameters (R, p_s, and p_m) (at those sites that have pavement markings) were estimated individually from the observed data for each site by using a least-squares estimation procedure (1).

EVALUATION OF MODEL RESULTS

A total of 14 mainline lane-drop sites were selected for analysis with the model. The other four mainline sites were excluded from the analyses because lane-drop changing might be influenced by nearby same-side ramps not accounted for in the model.

Good model fits that had reasonable estimated parameter values were achieved at rural sites deemed to have few commuters, but poor fits occurred at urban sites that had heavy commuter traffic. This was not surprising; commuters can be expected to have limited reaction to signs and pavement markings because of their preexisting knowledge of the drop. These drivers will merge out of the drop lane farther upstream of the drop.

Observed and modeled behavior at a typical good-fit site are shown in Figure 1. The bimodal lane-changing pattern exhibited at this location can be explained very effectively by the model.

Based on the results of the seven sites for which the model fits the data well, the average values for the three parameters fitted are R = 67 m (220 ft); p_s = 0.28; and p_m = 0.14. For these seven sites, the impact of signing appears to be greater for left-side drop sites than for right-side drop sites. A second interesting finding is that pavement markings tend to have less impact than signs.

FUTURE WORK

A need exists for a more realistic portrayal of the perception and reaction distance components of the lane-change-completion distribution. Specifically, these two components should be treated as random variables. A critical need exists for better empirical data. Especially useful would be the locations of individual lane-change completion, which would allow the use of maximum-likelihood estimation techniques. Lastly, the model should be generalized to include the situation of driver familiarity with the lane drop. A possible model modification to handle this behavior is given elsewhere (1).

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REFERENCES

1. J. J. Wang, W. A. Stock, B. C. Johnson, and I. J. Fullerton. Improving the Safety and Operation of Freeway Lane Drops. JHK and Associates, San Francisco, Sept. 1977.

2. R. D. Worrall and A. G. R. Bullen. Lane Changing on Multilane Highways. Department of Civil Engineering, Northwestern Univ., Evanston, IL, June 1969.

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Abridgment

Derivation of Freeway Speed Profiles From Point Surveillance Data

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In the past, detailed analyses of the traffic flow on freeways have been hampered by the cost of acquiring large samples of vehicle trajectories along the freeway sections of interest. The analyst has been forced to spend hours laboriously studying aerial photographs or performing numerous floating-automobile runs to obtain the required level of detail.

The use of freeway surveillance systems has been restricted by the inability of real-time surveillance systems to measure traffic flow over a continuous segment of roadway. The use of point sensors (usually implemented as loop detectors) in these systems has prevented them from being effectively applied to surveillance applications such as incident detection.

The purpose of this paper is to describe a technique that permits the derivation of individual vehicle trajectories from sensor data. This technique was developed by using data that were collected on a freeway surveillance and control system in the Los Angeles area and supporting data that were collected on the John Lodge Freeway in Detroit.

THEORETICAL BACKGROUND

This research is based on the application of sampling theory that has long been used in the communications field. A mathematical derivation of this theory is given elsewhere (1, 2).

Spectral Characteristics

One of the basic theorems of engineering analysis states that any arbitrary function of time [f(t)] can be described in terms of an infinite summation of sinusoidal components. If f(t) is periodic, the frequency of its sinusoidal components will be required. The description of a waveform in terms of its frequency spectrum rather than of its variation in time is known as the Fourier transform. It is the Fourier transform analysis that forms the basis for the development presented below.

Sampling Theory

It is a basic theorem of communications theory that (1)

If a signal whose highest frequency is W cycles has been sampled at a rate of 2W samples/s, the samples are in the form of impulses whose area is proportional to the magnitude of that instant, the sampled signal may be reconstructed by passing the impulse train through an ideal low-pass filter whose cutoff frequency is W cycles.

The process described by this theorem is shown graphically in Figure 1 (1, p. 24). This figure shows an arbitrary function of time [f(t)] being sampled by a commutator switch and then passed through an ideal low-pass filter where x(t) is reconstructed and matches f(t). Thus, the low-pass filter is a device that will pass only those frequency components of a time-varying waveform that are less than W cycles/s.

Black (2) has further extended this theory by a theorem that states that, if derivatives of f(t) are available, the sampling rate (Δt) must be

$$\Delta t = (R + 1)/2fc \tag{1}$$

where R = number of derivatives available and fc = cutoff frequency (the highest component frequency) of input. Thus, the frequency with which a function must be sampled in time is inversely proportional to the number of derivatives used in the sampling process and directly proportional to its cutoff frequency.

Translation to Sampling in Space

To be useful for this application, it is necessary that the sampling theorem be modified to take into account the fact that the sampling is occurring in space rather than in time. Thus, by substituting space (s) for time (t) in the original theory, we obtain the overall version of the sampling equation as

$$f(s) = \sum_{k=-\infty}^{\infty} \{f(k\Delta s) + (s - k\Delta s)f'(k\Delta s) + [(s - k\Delta s)^2/2!]\, f''(K\Delta s) + \ldots + [(s - k\Delta s)^R/R!]\, f^{(R)}(k\Delta s)\} \times \{\sin[(\pi/\Delta s)(s - k\Delta s)]/(\pi/\Delta s)(s - k\Delta s)\} \tag{2}$$

In equation 2, s is the location on the freeway at which the speed is being computed, kΔs refers to the detector location (assuming equal detector spacing), and (s - kΔs) is the distance between the location at which the speed [f(kΔs)] is being estimated and the detector from which the data is being used at a given instant of time. These relations are shown in Figure 2. Equation 2 shows that the further a detector is from s, the lower its influence will be on the overall speed computation. Thus, although the summation indicates that an infinite number of detectors must be sampled to reconstruct the speed at all points along the freeway, it will be possible to use a more limited number of detectors in the vicinity of s.