2. R. D. Worrall and A. G. R. Bullen. Lane Changing on Multilane Highways. Department of Civil Engineering, Northwestern Univ., Evanston, IL, June 1969.

Publication of this paper sponsored by Committee on Freeway Operations.

**Mr. Stock was at JHK and Associates when this paper was prepared.*

Abridgment

Derivation of Freeway Speed Profiles From Point Surveillance Data

Philip J. Tarnoff and Wayne K. Kittelson, Alan M. Voorhees and Associates, McLean, Virginia

In the past, detailed analyses of the traffic flow on freeways have been hampered by the cost of acquiring large samples of vehicle trajectories along the freeway sections of interest. The analyst has been forced to spend hours laboriously studying aerial photographs or performing numerous floating-automobile runs to obtain the required level of detail.

The use of freeway surveillance systems has been restricted by the inability of real-time surveillance systems to measure traffic flow over a continuous segment of roadway. The use of point sensors (usually implemented as loop detectors) in these systems has prevented them from being effectively applied to surveillance applications such as incident detection.

The purpose of this paper is to describe a technique that permits the derivation of individual vehicle trajectories from sensor data. This technique was developed by using data that were collected on a freeway surveillance and control system in the Los Angeles area and supporting data that were collected on the John Lodge Freeway in Detroit.

THEORETICAL BACKGROUND

This research is based on the application of sampling theory that has long been used in the communications field. A mathematical derivation of this theory is given elsewhere (1, 2).

Spectral Characteristics

One of the basic theorems of engineering analysis states that any arbitrary function of time [f(t)] can be described in terms of an infinite summation of sinusoidal components. If f(t) is periodic, the frequency of its sinusoidal components will be required. The description of a waveform in terms of its frequency spectrum rather than of its variation in time is known as the Fourier transform. It is the Fourier transform analysis that forms the basis for the development presented below.

Sampling Theory

It is a basic theorem of communications theory that (1)

If a signal whose highest frequency is W cycles has been sampled at a rate of 2W samples/s, the samples are in the form of impulses whose area is proportional to the magnitude of that instant, the sampled signal may be reconstructed by passing the impulse train through an ideal low-pass filter whose cutoff frequency is W cycles.

The process described by this theorem is shown graphically in Figure 1 (1, p. 24). This figure shows an arbitrary function of time [f(t)] being sampled by a commutator switch and then passed through an ideal low-pass filter where x(t) is reconstructed and matches f(t). Thus, the low-pass filter is a device that will pass only those frequency components of a time-varying waveform that are less than W cycles/s.

Black (2) has further extended this theory by a theorem that states that, if derivatives of f(t) are available, the sampling rate (Δt) must be

$$\Delta t = (R + 1)/2fc \tag{1}$$

where R = number of derivatives available and fc = cutoff frequency (the highest component frequency) of input. Thus, the frequency with which a function must be sampled in time is inversely proportional to the number of derivatives used in the sampling process and directly proportional to its cutoff frequency.

Translation to Sampling in Space

To be useful for this application, it is necessary that the sampling theorem be modified to take into account the fact that the sampling is occurring in space rather than in time. Thus, by substituting space (s) for time (t) in the original theory, we obtain the overall version of the sampling equation as

$$f(s) = \sum_{k=-\infty}^{\infty} \{ f(k\Delta s) + (s - k\Delta s)f'(k\Delta s) + [(s - k\Delta s)^2/2!]\, f''(K\Delta s) + \ldots + [(s - k\Delta s)^R/R!]\, f^{(R)}(k\Delta s)\} \times \{\sin[(\pi/\Delta s)(s - k\Delta s)]/(\pi/\Delta s)(s - k\Delta s)\} \tag{2}$$

In equation 2, s is the location on the freeway at which the speed is being computed, $k\Delta s$ refers to the detector location (assuming equal detector spacing), and $(s - k\Delta s)$ is the distance between the location at which the speed $[f(k\Delta s)]$ is being estimated and the detector from which the data is being used at a given instant of time. These relations are shown in Figure 2. Equation 2 shows that the further a detector is from s, the lower its influence will be on the overall speed computation. Thus, although the summation indicates that an infinite number of detectors must be sampled to reconstruct the speed at all points along the freeway, it will be possible to use a more limited number of detectors in the vicinity of s.

The required detector spacing (Δs) can similarly be derived by translating Equation 1 from the time domain to the space domain to give

$$\Delta s = (R + 1)/2fc \quad (3)$$

where fc = cutoff frequency of the traffic-stream speed variations in space. The actual values of fc and Δs are discussed below.

Computing Derivatives

Examination of the terms in Equation 1 for the reconstruction of speeds based on speeds measured by individual sensors shows that a potential problem is created because it is necessary to compute derivatives of the speed [$f'(k\Delta s)$] at each of the sensors. This problem arises because the derivative is a function of space rather than of time; i.e.,

$$f'(k\Delta s) = df(k\Delta s)/ds \quad (4a)$$

and

$$f''(k\Delta s) = d^2f(k\Delta s)/ds^2 \quad (4b)$$

Obviously, a point sensor such as a loop detector is not capable of measuring speed changes in space because, by definition, this equation is capable only of measuring data at a particular point along the roadway. Yet the analysis of cutoff frequency discussed below indicates that, to arrive at feasible sampling rates, it is necessary to compute at least the first and second derivatives of the speed change in space.

This problem is solved elsewhere (3); the relationship between the derivatives in time (t) and space (s) is

$$\partial u/\partial s = (-1/u_w)(\partial u/\partial t) \quad (5a)$$

and

$$\partial^2 u/\partial s^2 = (1/u_w)^2(\partial^2 u/\partial t^2) \quad (5b)$$

where u = speed and u_w = speed-propagation rate.

u_w is related to the slope of the freeway flow (q) versus density (k) curve and can be expressed as

$$u_w = dq/dk \quad (6)$$

Thus, for a particular section of freeway, it will be necessary to calibrate a flow-density curve as a part of the implementation process. Then, a value of u_w can be estimated by using the vehicle flows measured by the detectors. By knowing u_w and computing $\partial u/\partial t$ directly from the vehicle speed data, we can calculate the derivatives of speed with respect to space.

The use of this approximation permits the substitution of the derivative of speed with respect to time for the derivative of speed with respect to space. This derivative with respect to time is readily computed from the detector data.

FEASIBILITY OF APPLYING PROCESS

From the above discussion, it is obvious that the sampling theorem can be applied to the case of freeway-traffic-speed estimation only if the cutoff frequency of the speed profiles is low enough to permit a reasonable detector spacing as computed by Equation 2. To determine the minimum acceptable detector spacing, it was necessary to determine the frequency spectrum for actual freeway speeds as recorded during floating-automobile runs. Two sources of data were used for this determination—some collected on the John Lodge Freeway in Detroit at or near the Calvert, Chicago, Hamilton, and Gladstone overpasses and some were collected for this project on the 67-km (42-mile) loop in Los Angeles.

By using the worst-case frequency obtained from these sources, we can then determine the detector spacing (assuming that the first and second derivatives are computed) necessary to perfectly reconstruct vehicle speed in the following way: s = (R + 1)/fc = 3/0.002 =

Figure 1. Sampler and low-pass filter for reconstruction of signal (delay of low-pass filter neglected).

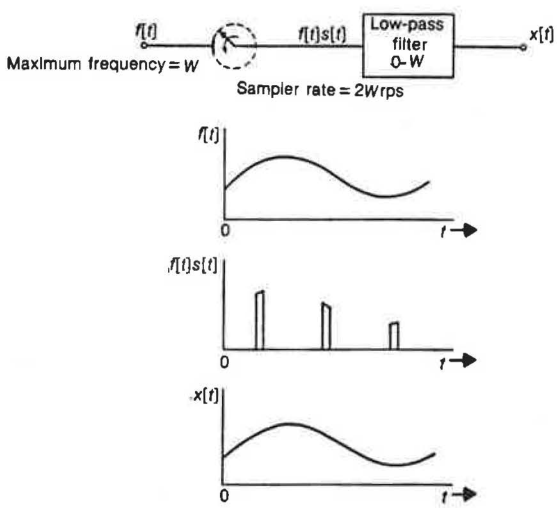
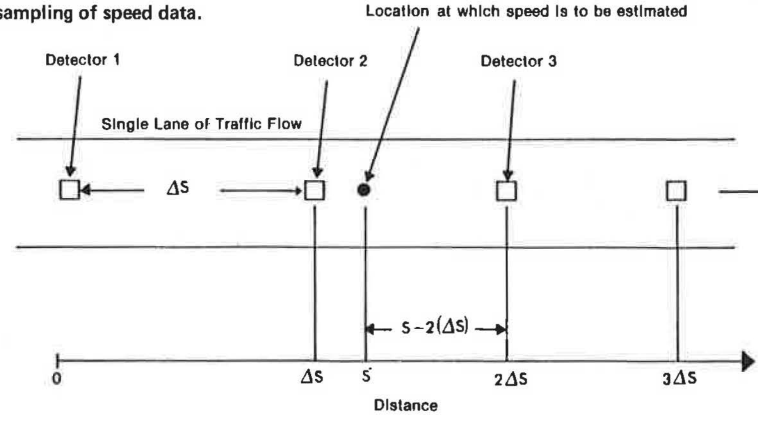


Figure 2. Relationships used in sampling of speed data.



1500 ft (457 m). This spacing would be required to provide accurate estimates of vehicle speeds. It should be used when there is a desire for reliable incident detection or other types of speed estimation. Thus, the spectral analysis represented can be used as a tool for the estimation of required detector spacing.

Raw detector data obtained from the California Department of Transportation were used in an attempt to demonstrate the feasibility of processing detector data to develop estimates of the equivalent of floating-automobile speeds. Unfortunately, the detector spacing exceeded that computed above and, as a result, a reliable validation of the process discussed was not possible.

CONCLUSIONS

The discussion above has described an approach to the derivation of vehicle speed profiles from point surveillance data. Unfortunately, the limitation of the available data precluded a comprehensive evaluation of the procedure.

All indications are that this process can be successfully used for freeway applications. It is recommended, therefore, that further validation of the process be pursued. This validation is especially important because of the possible errors introduced by variations in vehicle lengths, difficulties in estimating speed propagation rates, and lane-changing maneuvers between the detectors. Currently, variations in vehicle length are compensated for by smoothing the input data to reduce the impact on speed processing, but this also introduces errors in the computation process.

REFERENCES

1. J. C. Hancock. An Introduction to the Principles of Communications Theory. McGraw-Hill, New York, 1961.
2. H. S. Black. Modulation Theory. Van Nostrand, Princeton, NJ, 1953, chap. 4.
3. P. Brach, W. Kittelson, J. Mehra, and P. Tarnoff. Vehicle Operation, Fuel Consumption, and Emissions as Related to Highway Design and Operation. Alan M. Voorhees and Associates, McLean, VA, 2nd Interim Progress Rept., Aug. 1977; Federal Highway Administration, Aug. 1977.

Publication of this paper sponsored by Committee on Freeway Operations.

Development of Efficient Procedure for Recreating and Analyzing Traffic Flow Patterns on Urban Freeways

Wayne K. Kittelson and Philip J. Tarnoff, Alan M. Voorhees and Associates, McLean, Virginia

The purpose of this paper is to describe an analytical procedure for the synthesis and analysis of the speed profiles of typical vehicles traveling on urban freeways. Much past research has been conducted in this area in an attempt to define and describe traffic operating characteristics on urban freeways; this paper represents an extension of the past research and as such develops new analytical tools and relies on an additional data-collection effort. The paper includes a detailed discussion of the floating-automobile experiments and the data-collection procedures that were used and a description of the results that were obtained. This description includes a discussion of the variables found to be important in classifying freeway segments and a summary of the results of the data-collection effort. The paper concludes with a description of the methodology developed for using the results of the analysis to generate (in either a manual or an automatic mode) typical speed profiles for vehicles traveling on any section of urban freeway.

The urban freeway is a major component of the total urban transportation network. From a service point of view, urban freeways can offer relatively high average travel speeds to a very large volume of traffic; today's urban freeways carry an estimated 13 percent of the annual urban travel although they represent only 2 percent of the total urban street length available. Because of the important part that freeways play in the overall transportation network, a great deal of effort has been expended in an attempt to understand and describe freeway traffic flow patterns.

The purpose of this paper is to describe an analytical procedure for the synthesis of the speed profiles of typical vehicles traveling on an urban freeway. Much research has been conducted in this area in an attempt to define and describe traffic operating characteristics on urban freeways (1); this paper represents an extension of this past research and as such develops new analytical tools and relies on an additional data-collection effort. The paper begins with a rather detailed discussion of the floating-automobile experiments and data-collection procedures and continues with a description of the results that were obtained. This description includes a discussion of the variables found to be important in classifying freeway segments and a summary of the results of the data-collection effort. The paper concludes with a description of the methodology that was developed to use the results of the analysis to generate (in either a manual or an automatic mode) typical speed profiles of vehicles traveling on an urban freeway.

FLOATING-AUTOMOBILE EXPERIMENTS AND DATA-COLLECTION PROCEDURES

The purpose of this section is to provide a detailed description of the floating-automobile experiments used in collecting the data for the freeway analysis. Currently, only a rather limited set of urban freeway data