Tunnel-Boring Penetration Rate and Machine Design

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The experimental and theoretical findings of an ongoing research program on tunnel borability at the Colorado School of Mines are presented. A theoretical approach has been used to formulate predictor equations to calculate the forces involved in rock cutting with sharp and dull disc cutters. Concurrent with the theoretical analysis, extensive laboratory testing was undertaken with both sharp and artificially dulled disc cutters. These tests were carried out in three rock types and with full-size field cutters. Penetration, spacing, and edge angle, in that order, were found to affect cutting forces to a large extent whereas cutter diameter was found to be a variable of small significance. The effect of cutter wear on cutter forces was found to depend on the spacing of cuts and to decrease with increased spacing. It was concluded that wear is more detrimental to cutter performance at closer spacing of cuts. Predicted cutter forces were found to agree very well with those measured in laboratory cutting experiments. The theoretical behavior of dull cutter forces confirmed experimental observations in that the closer cuts were spaced the greater was the reduction in cutting efficiency of a disc cutter because of wear. Field boring data from a Jarva machine currently in operation in Chicago were procured and compared with predicted values, and agreement was very good. This represents an initial success in the application of the predictor equations to boring cases in the field.

The need for more efficient, faster underground excavation techniques has been well expounded in the literature in recent years. Mechanical tunnel boring offers a great potential for meeting the requirements of efficient underground excavation although its application to hard rock boring is yet to become widely accepted. Among other factors, inaccurate predictions of borability have been the major cause of many failures experienced in the application of mechanical tunnel boring to hard rocks.

This study is concerned with establishing a theoretical approach to predicting field boring performance. Because of its simple geometry and wide use in mechanical tunneling, the theoretical analysis has centered on the disc roller type of cutter. Based on the geometry of cutting and some fundamental assumptions, predictor equations have been developed to predict disc cutting forces.

This paper gives a brief account of the steps followed in deriving the predictor equations. A detailed description of these equations and an in-depth discussion of laboratory cutting tests performed concurrently with the theoretical analysis are given elsewhere.

THEORETICAL ANALYSIS OF DISC CUTTER-ROCK INTERACTION

Figure 1 shows an idealized representation of a system of disc cutter–rock interaction together with the parameters involved. Essentially, these parameters can be grouped in two main categories: those that describe the geometry of the disc cutter (edge angle and diameter) and those that pertain to the geometry of the cutting (cutter penetration and spacing).

Several predictor equations for disc-cutter performance are offered in the literature, but these analyses relate to a single, independent cut and therefore cannot be used to predict field borability because the effects of spacing are not known.

To formulate a predictor equation that includes the effect of spacing and other effects, a comprehensive understanding of cutter-force behavior with varying levels of spacing must be established. Miller (1) and Ozdemir (2), by using two different scales of laboratory rock-cutting devices, obtained extensive laboratory cutting data that enabled them to observe the effect of spacing on disc cutting forces. Figure 2, which shows some of their findings, shows the influence of an increase in the spacing of cuts on the vertical force required to obtain a fixed penetration. Regardless of cutter geometry and rock type, the vertical force on the cutter required to achieve a given penetration shows a continual increase as spacing increases because of the larger volume of rock that needs to be broken by each cut.

What is important, however, is the particular trend of the force curve follows. As shown in Figure 3, the overall force behavior with increasing cut spacing can be divided into three distinct zones. Zone 1 covers the low spacing values that result in extensive crushing of the rock, zone 2 corresponds to those spacing values over which the force increase is nearly linear, and zone 3 includes the spacing values beyond optimum, a point at which the increase in force departs from linearity.

When one examines this observed relation between force and spacing, it is not difficult to realize that the relation, especially for zones 1 and 2, can be expressed by means of a linear equation of a certain slope and intercept as shown in Figure 3. The intercept of the predictor behavior is taken as the load on the cutter that results from a series of cuts positioned on a rock surface in a situation such as that shown in Figure 4 so that complete crushing of the rock occurs. This situation exists at a spacing value somewhere within zone 1 and, depending on the exact location of this spacing, the predictor curve will be above or below or will exactly coincide with the observed behavior. Nevertheless, the arrangement of cutters shown in Figure 4 will result in individual cutter forces of

\[ F_1 = C \times AC \]  

(1)

where

- \( F_1 \) = vertical force on the cutter (i.e., intercept of the predictor curve),
- \( C \) = rock uniaxial compressive strength, and
- \( AC \) = area of contact of the cutter with the rock measured at the level rock surface.

The area of contact is represented in Figure 5 by the shaded portion of the boat-shaped area. Note that this area is half the total contact area because the cutter is rolling over the cutter surface and the rock behind the vertical axis of the cutter is already broken and cannot exert any forces on the cutter. The desired area of contact, which can be determined from geometrical considerations, is \( R^2(\phi - \sin\phi \cos\phi)\tan\alpha/2 \). Thus, the intercept of the predictor curve becomes

\[ F_1 = CR^2(\phi - \sin\phi \cos\phi)\tan\alpha/2 \]  

(2)

To determine the slope of the predictor curve, the exact mechanism of rock failure between two adjacent cuts must be known.

There are two possible means by which a disc cutter can cause rock failure between adjacent cuts. Rock can...
fail either by tension or by shear. A combination of these two mechanisms, one succeeding the other, is also probable. In this study, a detailed analysis of rock failure was performed for the purpose of determining which failure mode prevails. To determine the conditions in which a shear failure would be most likely to occur, several types of qualitative observations were made:

1. Analysis of fracture patterns beneath craters produced by disc cutters,
2. Examination of high-speed films of disc cutting action,
3. Microscopic examination of rock failure surfaces, and
4. Extensive observation of rock surface profiles found in laboratory studies that used disc cutters.

How a shear failure of the interlying rock material between two adjacent cuts would occur is shown in Figure 6. The figure shows the geometry of adjacent cuts and the approximate loading of the interlying material by the pressure bulb that surrounds the penetrating edge of the cutter. Finely crushed rock contained in this bulb, which behaves in a near-plastic manner, is in a state of triaxial compression and produces a hydrostatic loading on the surrounding intact rock in a fashion shown by the arrows. This loading configuration then causes a pure shear failure of the rock material that lies between two adjacent cuts. Note that it is said to be a pure shear failure since no normal loading is exerted on the failure plane. It is also believed that the radial tensile cracks formed around the pressure bulb aid in shearing action by creating an initial path for the shear plane to follow.

Figure 1. Rock cutting with disc cutter.

Figure 2. Vertical force versus cut spacing at fixed cutter penetration of 0.25 cm.

Figure 3. Vertical force versus cut spacing: observed behavior and behavior to be predicted.
Once the occurrence of a shear failure of rock material between two adjacent cuts is determined, the second part of the predictor equation—the part that defines the slope—can easily be obtained by determining the force required to cause this shear failure. Consider the idealized multiple-cut situation shown in Figure 7. The crushed zone was not superimposed in this figure to preserve graphical clarity. To generate a shear failure of rock toward the existing cut or cuts, the side force $f_s$ should be large enough to overcome shearing resistance. That is,

$$f_s = \tau \times SA$$

(3)
where 

\[ \tau = \text{rock unconfined shear strength (cohesion)} \] and 

\[ SA = \text{area of failure surface}. \]

At the instant of chip formation, the failure surface will have a nearly rectangular configuration with a length that approaches the chord length (Figure 5) and a width approximately equal to the spacing of the cuts minus the width of the crushed zone. Thus, the area of the failure surface is 

\[ SA = R\phi(s - 2p \tan \alpha/2) \] 

which results in the following expression for side force:

\[ f_2 = \tau R\phi(s - 2p \tan \alpha/2) \]  

(4)

Assuming there is no cutter-rock friction, the resultant is perpendicular to the side of the cutter edge. Therefore, we can write \( f_2 = f_2 \times \tan \alpha/2 \) or \( f_2 = \tau R\phi(s - 2p \tan \alpha/2) \tan \alpha/2 \) since \( f_2 = 2f_1 \); then

\[ F_1 = 2R\phi(s - 2p \tan \alpha/2) \tan \alpha/2 \]  

(5)

where \( F_2 \) is the vertical force required to shear the rock on both sides of the cutter. If there is a previous cut on only one side of the cutter, the preceding analysis and results still hold true because of a required balance of forces on the cutter edge. Having determined the two parts of the predictor equation, one then only needs to combine these two effects (Equations 4 and 5) to arrive at an equation for the total vertical force on the cutter. Thus, \( VF = F_1 + F_2 \) or

\[ VF = CR^2(\phi - \sin \phi \cos \phi) + 2R\phi(s - 2p \tan \alpha/2) \tan \alpha/2 \]  

(6)

As presented, Equation 6 will provide a theoretical estimate of cutter thrust for given levels of penetration, spacing, cutter geometry, and required rock properties. However, it may be desirable, as it would be in field boring practice, to calculate the cutter penetration for a given value of cutter thrust. This can be accomplished by rearranging the equation so that the penetration is the unknown parameter, but such a process is difficult because angle \( \phi \) is a function of penetration. It is feasible, however, to simplify the equation by inserting reasonable approximations for the cutter-rock contact area and angle \( \phi \), which gives

\[ VF = D^2p^{3/2}[4/3C + 2(s/p - 2 \tan \alpha/2)] \tan \alpha/2 \]  

(7)

The results provided by Equation 7 were found to be in excellent agreement with those from the exact solution (Equation 6) with a maximum deviation of around 2 percent for the most likely levels of pertinent variables.

Rearranging Equation 7 gives

\[ (4/3C - 4\tan \alpha/2)p^{3/2} + 2s/p - 2 \tan \alpha/2) = 0 \]  

(8)

which can then be solved for penetration by using various methods given in the literature.

The rolling (tractive) force on the cutter can now be obtained by multiplying the developed vertical force equation (Equation 7) by a constant that reflects the geometrical relation of these two force components. This constant (3) is commonly referred to as the cutting coefficient (CC) and is found to be

\[ CC = (1 - \cos \phi)^2/(\phi - \sin \phi \cos \phi) \]  

(9)

where \( \phi = \cos^{-1}(R - p)/R \)

Since rolling force (RF) is defined as \( RF = VF \times CC \), inserting the predetermined values of \( VF \) and \( CC \) provides the following analytical expression for the rolling force:

\[ RF = [(Cp^2 + 4\phi(s - 2p \tan \alpha/2))/D(\phi - \sin \phi \cos \phi)] \tan \alpha/2 \]  

(10)

To summarize the results of the foregoing theoretical analysis, the following equations were derived for predicting the vertical and rolling forces on a disc roller cutter (because the equations were formulated in U.S. customary units of measurement, no SI equivalents are given):

\[ VF = D^2p^{3/2}[4/3C + 2(s/p - 2 \tan \alpha/2)] \tan \alpha/2 \]  

(11)

and

\[ RF = [(Cp^2 + 4\phi(s - 2p \tan \alpha/2))/D(\phi - \sin \phi \cos \phi)] \tan \alpha/2 \]  

(12)

where

\[ VF = \text{vertical force on the cutter (lbf)}; \]
\[ RF = \text{rolling force on the cutter (lbf)}; \]
\[ C = \text{rock uniaxial compressive strength (lbf/in}^2); \]
\[ \tau = \text{rock unconfined shear strength (cohesion, lbf/in}^2); \]
\[ D = \text{cutter diameter (in)}; \]
\[ \alpha = \text{cutter included edge angle (deg)}; \]
\[ s = \text{spacing of cuts (in)}; \]
\[ p = \text{cutter penetration (in)}; \]
\[ \phi = \cos^{-1}(R - p)/R \qquad \text{where } R \text{ is the cutter radius (in)}. \]

The foregoing analysis and subsequently developed predictor equations expressed the force-spacing behavior with a linear equation of a certain intercept and slope. As discussed earlier, laboratory cutting results with disc roller cutters showed a linear increase in forces with increasing spacings up to an optimum value, and thereafter the increase in force followed more of a curvilinear trend. Thus, from a theoretical viewpoint, the developed equations characterized the force behavior over the spacing values below the optimum. Although the rock failure mechanism for spacings beyond the optimum is generally understood and confirmed by laboratory observations, no attempt was made to modify the developed equations to describe the force behavior in this region because (a) the problem appears to be complicated and probably needs a separate, in-depth study and (b) force behavior in this region does not depart appreciably from linear trend, especially up to maximum spacing values [about 7.5 cm (3 in)] commonly used on hard-rock-boring machines. Thus, the developed prediction equations are assumed to apply to spacings beyond the optimum as well.

The developed predictor equations are meant to be used in estimating forces on sharp disc cutters. Since most tunnel-boring machines operate with cutters in some stage of dullness, the developed equations should be modified to consider the effect of wear on cutter performance. Before any modification, the wear surface to be analyzed should be defined. In this study, a toroidal wear surface was chosen to describe the geometry of worn disc cutters based on a survey of patterns of cutter wear in the field and discussions with manufacturers of cutters. Assuming a toroidal wear surface, the equations developed for sharp disc cutters were then modified to incorporate the presence of a wear surface along the cutter edge, providing a new set of equations for estimating the forces on dull disc cutters. Because of the complexity and length of the resulting equations, those equations are not presented in this paper. The equations and a detailed description of the steps
followed in their derivation are given elsewhere (3).

LABORATORY EQUIPMENT, INSTRUMENTATION, AND TESTING PROCEDURE

The laboratory testing for this investigation was carried out by using a large, stiff, linear rock-cutting machine. This equipment has been described in full elsewhere (1, 2, 3). The instrumentation included a triaxial load cell capable of resolving the load on the cutter into its three mutually perpendicular components and time-based digital integrators for recording forces.

The average values of the three force components (vertical, rolling, and side) were measured, and peak readings were taken. Occasionally, XYY’ plotters were introduced into the bridge circuits to provide analog copies of cutter forces and to allow visual observation of their behavior and interdependence.

The rock types tested were samples of St. Cloud gray granodiorite [trade name Charcoal Gray Granite (CG)], Holston limestone [trade name Tennessee Marble (TM)], Colorado Red Granite.

Figure 8. Predicted vertical forces versus vertical forces measured with sharp disc cutters in Colorado Red Granite.

Figure 9. Predicted vertical forces versus vertical forces measured with sharp disc cutters in Charcoal Gray Granite.
and orthoclase-bladed porphyritic granite of the Colorado red granite type [trade name Colorado Red Granite (RG)].

The testing procedure involved setting the disc at the required level of penetration and then traversing it over the rock surface and measuring the forces that act on the cutter. Each test consisted of making several passes over the rock surface; each pass contained several cuts taken at a fixed spacing. The test started with a smooth rock surface. Data for the first two or three passes were discarded, and these passes were used to condition the rock to create a rock surface similar to that found on a tunnel boring face.

The entire laboratory testing was carried out at a cutting velocity of 25 cm/s (10 in/s). This choice was based on the experimental results reported by Ozdemir (2) since cutting velocity was found not to affect cutter forces in the range above 12.5 to 25 cm/s (5 to 10 in/s).

LABORATORY CUTTING RESULTS

A very ambitious laboratory testing program was carried out with both sharp and artificially dulled disc cutters. All of these tests were performed according to designed experimental plans to permit the application of various statistical procedures to cutting results. For tests with sharp disc cutters, Latin square experimental

![Figure 10. Predicted vertical forces versus vertical forces measured with dull disc cutters in Colorado Red Granite.](image)

![Figure 11. Predicted vertical forces versus vertical forces measured with dull disc cutters in Charcoal Gray Granite.](image)
plans were constructed and used for cutting each type of rock; each plan included the following variables and their respective levels (1 cm = 0.39 in and 1 mm = 0.039 in):

<table>
<thead>
<tr>
<th>Variable</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutter edge angle (α)</td>
<td>60, 75, 90, 105, 120</td>
</tr>
<tr>
<td>Cutter diameter (D), cm</td>
<td>20, 25, 30, 35, 40</td>
</tr>
<tr>
<td>Cutter penetration (p), mm</td>
<td>1.25, 2.5, 3.75, 5.0, 6.25</td>
</tr>
<tr>
<td>Spacing of cuts (s), cm</td>
<td>2.5, 3.75, 5.0, 6.25, 7.5</td>
</tr>
</tbody>
</table>

The cutting results obtained from testing many planned combinations of the above levels of the variables were analyzed to determine the relative significance of each variable in affecting cutter forces and also to establish empirical relations between each variable and the cutter forces. Penetration, spacing, and edge angle, in that order, were found to affect cutter forces significantly whereas cutter diameter was shown to be a variable of minor significance.

In tests that used artificially dulled disc cutters, the following variables and their levels were investigated:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutter edge angle, °</td>
<td>60, 75, 90</td>
</tr>
<tr>
<td>Cutter wear (measured as tip loss w), mm</td>
<td>1.56, 3.80, 6.25</td>
</tr>
<tr>
<td>Cutter penetration, mm</td>
<td>1.25, 2.5, 3.75</td>
</tr>
<tr>
<td>Spacing of cuts, cm</td>
<td>2.5, 5.0, 7.5</td>
</tr>
<tr>
<td>Rock type</td>
<td>RG, CG, TM</td>
</tr>
</tbody>
</table>

In these tests, a partial factorial design that required a total of 81 laboratory tests was constructed and used. The results showed the pertinent variables to affect the dull cutter forces in much the same way that they affect sharp-disc cutter forces. That is, the presence of wear surface along the cutter edge did not seem to affect the characteristic behavior of disc cutter forces at increasing levels of the variables investigated. It was also found that the degree to which cutter wear affects cutter forces depends on the spacing of cuts and becomes smaller with increasing spacing. It was concluded that cutter wear would have a small effect on boring performance for the spacing values most commonly used on tunnel-boring machines equipped with disc roller cutters (about 7.5 cm (3 in)). Cutter wear, however, was found to reduce cutting efficiency considerably when cuts were spaced close to each other.

**COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS**

By using the developed predictor equations (Equations 7 and 10), theoretical force values were calculated and compared with those measured in laboratory tests performed with sharp and artificially dulled disc cutters in three rock types. The goal of successfully predicting the vertical force on the cutter was given the most attention because it is the vertical force on the cutter that determines how much the cutter can penetrate a given type of rock at a given cut spacing.

To observe visually the degree of correlation, the theoretical vertical force values are plotted against the measured ones in Figures 8 through 12. At first glance, these figures indicate good agreement between the measured and predicted vertical forces. To assign a form of statistical confidence to the degree of correlation, two separate regression analyses—standard and forced—were performed. The former is a well-known regression technique that uses the least-squares analysis method. The latter constitutes the regression line, which has an imposed slope of one. That is, the slope was taken as one, and thereafter the intercepts and the correlation coefficients were determined for each set of data by regression analysis. As seen in these figures, the correlation coefficients for all rock types tested are high enough to confirm the validity of the developed equa-
tions for both sharp and dull cutters.

For rolling forces, again the predicted and the measured values agreed very well, but the correlation coefficients were lower than they were for the vertical forces. It was suggested that ignorance of the cutter bearing friction in the development of the rolling force equation and the enormous sensitivity of rolling force to cutter penetration were responsible for the observed deviation of the predicted and measured values.

PREDICTION OF FIELD BORING PERFORMANCE

The field boring data supplied by Jarva, Inc., from one of their machines currently in operation in Chicago provided an excellent opportunity to verify the developed predictor equations for their applicability to predictions of field boring performance. Since the Chicago machine is boring through a relatively homogeneous and competent dolomitic limestone formation with very little jointing, the field data were considered ideal for comparison with the predicted values.

Theoretical advance rates were calculated by using a computer program that incorporated the developed predictor equations and was specifically written for use in predictions of field boring performance. The calculated advance rates and actual rates measured in the field are given in Table 1. The predicted rate of penetrations is clearly very close to those measured. This initial success in predicting field boring performance does not, of course, mean that the equations can be universally applied to cases of tunnel boring. To arrive at such a conclusion, more field data from different machines operating in different rock formations must be collected and compared with predicted data.

SUMMARY AND CONCLUSIONS

The developed predictor equations successfully predicted the forces acting on disc roller cutters in laboratory studies of borability with sharp and artificially dulled disc cutters. Moreover, they closely predicted the field rate of penetration of a Jarva tunnel-boring machine currently operating in dolomitic limestone in Chicago.

It is obvious that more field boring data must be collected and compared with the predicted values before the equations can be considered universal. In addition to parameters of machine design and operation, these data should also include any existing geological features in order to understand their effect on the predicted values.

Future work on this project will concentrate on collecting and procuring more distinct field boring data. The theoretical analysis performed for disc roller cutters will also be extended to include other commonly used rolling cutters such as disc-button and multikerf cutters.

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REFERENCES


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