Theory of Roof Bolting

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There is currently an unfulfilled need in the field of rock mechanics for a rational, easily used system of rock bolt design. During the late 1950s and early 1960s, Panek and Lang performed independent studies on the nature of rock bolt behavior. Panek, working with bolt action in flat, laminated mine roof strata, attributed support to both suspension and friction and concluded that reinforcement by friction is a complex function of mine geometry, bolt spacing, and load. Lang, working with bolted gravel beams, developed essentially the same conclusions and, in effect, generalized Panek's work. By taking these two theories a step further, it is shown that rock support is a function of the rock bolt's power to enforce mechanical continuity on the rock. By using equations from two-hinged arch theory, it is possible to relate load directly to beam strength with the parameters of conventional structural analysis—load, strength, and beam geometry. Tables can thus be prepared that compare beam thicknesses and an offset dimension with span length. An example of such a table is included in the paper.

Most determinations of rock bolt length are made by some old timer in the field squinting at the exposed rock, spitting, and saying, "Twelve feet ought to do it" and "Put one there and there." This has given way a little, under pressure from modern rock mechanics, to the more sophisticated and expensive method of running a few tests on the depth of the field of influence around the opening (a rather nebulous term) combined with at least 48 h of central processor time on a finite element model before anybody spits and says 3.6 m (12 ft) ought to do it. However, rock mechanics, having progressed far enough to be able to convince people of the need for these expensive tests, has the responsibility now to produce a rational, easily used method by which these data can produce a specific length and spacing of rock bolt. The intention of this paper is to provide modifications to current theory that it is hoped will move rock bolt design toward this end.

CURRENT THEORIES OF ROOF BOLTING

Many important studies have been performed by the U.S. Bureau of Mines, particularly the work of Panek on the analysis of roof support (1, 2, 3, 4, 5). He considered bolt action in flat, laminated strata commonly found in the roofs of many mines. Panek attributes support to two mechanisms: suspension and friction.

The suspension effect "refers to the transfer of part of the weight of the weaker or thinner strata to one or more thick strata, which occurs when strata with differing tendencies to deflect are constrained to have equal deflection" (5). Suspension effects are heavily dependent on the geometry and mechanical properties of the bolted section.

The strengthening is then caused by the tying of the thicker, thereby stronger, beds to the thinner, weaker beds—the gain in strength being proportional to the cube of the bed thickness. During these analyses, no strengthening was allowed for any partial bonding between beds that would produce a composite beam effect. This was handled under the mechanism of friction.

The friction effect "refers to reduction of bending in a stratified roof due to clamping action of tensioned bolts, which compress the strata, thereby creating frictional resistance to displacement along planes of stratification" (5).

Panek concluded from essentially empirical data that reinforcement by friction is a complex function of mine geometry, bolt spacing, and bolt load. The latter is usually taken to be the maximum force the bolt can sustain over the design life of the bolt. Panek combined these effects into the concept of a reinforcement factor that is defined as follows:

$$RF = \text{maximum bending stress, unbolted roof} \div \text{maximum bending stress, bolted roof} \qquad (1)$$

To simplify the computations necessary to design a support system, Panek prepared the chart shown in Figure 1 (since the chart was prepared in U.S. customary units, no SI equivalents are given).

In practice, using the design method requires knowledge of the number of beds to be bolted, their thicknesses, and their moduli to arrive at any strength value better than an educated guess. For this reason, the design method was not well received by the industry.

Concurrently with Panek, Lang was also conducting several studies of bolt behavior (6, 7, 8). The emphasis was on heavily fractured ground rather than laminated mine roofs, and an extensive analysis of bolting patterns across various types of joints was presented. Particularly important, however, for the understanding of bolt behavior was a series of photoelastic studies that determined the effects of bolt spacing and length. It was found that bolts spaced closely enough produced a zone of uniform compression within the back and were much more effective at supporting the roof. The thickness of the beam of compressed rock was approximately the bolt length minus the spacing.

By summarizing these two theories, it can be seen that Panek's theory is a special case of Lang's more general theme, and that support depends on the following factors: interlayer movements (continuity) and suspension, end fixity of the beam, end restraint, and end shear. The most important of these seems to be the continuity or interlayer movement. The enforcement of continuity, or prevention of interlayer movement, can be analyzed by looking at interlayer stresses. The most apparent stress is the direct stress applied by the bolt to the rock beam. This stress would produce an increase in normal force that would translate into a frictional force along the bedding. The stress F, mobilized to prevent interjoint movements in this situation, is then

$$F = P \tan \phi \qquad (2)$$

where P is the normal stress and ϕ is the friction angle. Though this approach is commonly used, the additional strength supplied in a real case is quite low. Since the layer movement is over the entire cross section of the beam, the bolt load should be transferred into a stress and frictional force should be a frictional stress. A 1.2- by 1.2-m (4- by 4-ft) pattern and 9100 kg (20 000 lb) of bolt load yield a normal load increase of approximately 89 kPa (13 lbf/in^2). This increase in normal force will add only 89 to 103 kPa (13 to 15 lbf/in^2) increased friction strength at best to enforce continuity. The actual amount, of course, depends on the value of the tangent of friction along the laminated surfaces and could be much less.

Analysis of a fixed-end beam 5.5 m (18 ft) long, 1.2 m (4 ft) wide, and 1.8 m (6 ft) high loaded uniformly at 689 kPa (100 lbf/in^2) yields a maximum horizontal shear stress near the rib of 1552 kPa (225 lbf/in^2) and an extreme fiber stress of 2965.5 kPa (430 lbf/in^2) at the midspan of the beam. Clearly, the resistance of 89 kPa (13 lbf/in^2) caused by the increase in friction attributable to roof bolting is of little value in overcoming these forces in maintaining continuity. Roof bolts, however, are a proven method of support. Another mechanism, then, must be invoked to explain their action.

It is possible that this mechanism could be attributed to the interlocking of beds along their contacts. Einstein, Bauhn, and Mirschfeld (9) performed theoretical and laboratory studies on friction in jointed rock masses and reported, "It is conceivable that ... on a microscopic scale and for rough surfaces, interlocking will be the dominant characteristic (governing friction)." Normal rock found in cavern roofs rarely presents the smooth planar laminations used in model studies. Even in horizontally laminated roofs, the material encountered in practice tends to separate along weak, nearly horizontal bedding planes until (a) a weak vertical flaw is encountered or (b) bedding plane A-A becomes stronger than bedding plane B-B, and the separation moves to the weaker bed. The result is a series of nearly parallel flat surfaces with short, steep connections. A typical lamination interface may look like that shown in Figure 2.

The strength of this laminated beam depends on the lack of relative motion between the laminations. In order for relative movement to take place between two rock units in contact along an interlocking surface,

1. The shear stress on the plane A-A or B-B (Figure 2), whichever is stronger, must be overcome;
2. The tensile strength in the rock on vertical plane C-C (Figure 2) between the two irregularities must be overcome; or
3. The projecting portions of each unit must ride up and over one another.

In the average rock, shear strength and tensile strength are of the same order of magnitude as the shear and fiber stresses calculated above, i.e., 1379 to 3448 kPa (200 to 500 lbf/in^2). However, the normal sagging associated with an opening provides assistance to the mechanism of unlocking. Opening of joints and bedding surfaces by gravity allows the laminations to move relative to one another and act as N independent beams with a resulting loss in strength by a factor of N. The rock bolts oppose this action and mechanically enforce continuity on the roof beam.

The amount of force required to open the bedding surface depends on the geometry of the interlocking projects. Einstein, Bauhn, and Mirschfeld found that "In ... gypsum models with a single joint inclined at 30° to the major principal stress, failure occurs by sliding along the joint for all applied confining stresses between 0 and 1500 psi. In the models with a joint inclination of 60°, failure occurs only by fracture through intact material" (9). In the above condition, the joint inclination is almost entirely above 60°; most are nearly vertical. Therefore, it would appear that, if dilation can be prevented, the full shear strength of the rock can be mobilized along the nonsheared joint surfaces.

As shown in Figure 3, the tangential stress f along the joint caused by the lateral stress τ, which tends to cause overriding, is

$$f = \tau \sin \theta \qquad (3)$$

The tangential strength F along the joint caused by the bolting required to prevent overriding is

$$F = \sigma \sin \theta \qquad (4)$$

Under normal bolting practices, where σ = 103 kPa (15 lbf/in^2) and τ = 3103 kPa (450 lbf/in^2), to obtain a balance between these two stresses would require that the controlling joints be nearly vertical or one or two degrees from vertical. It should also be noted that this is an inverse chain effect in that the strongest link must be overcome before general failure occurs.

Figure 1. Roof-bolt design chart for friction effect.

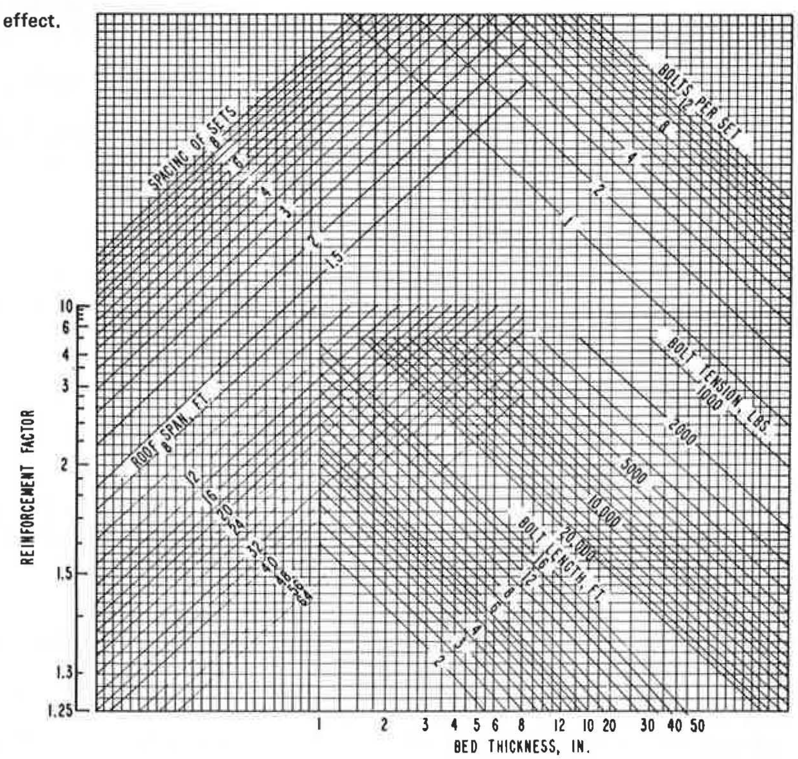


Figure 2. Typical laminated rock formation showing probable bedding point.

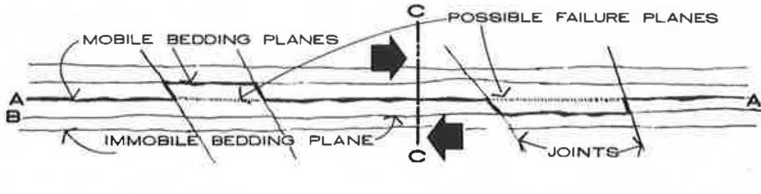


Figure 3. Stresses along joint asperity.

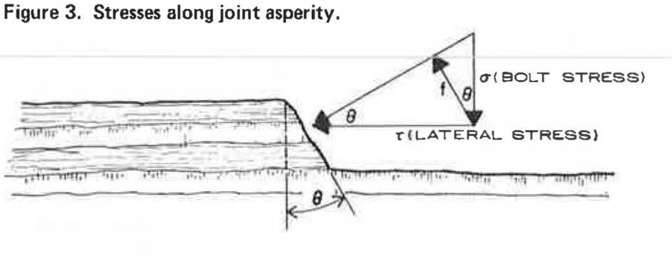


Figure 4. Stress beam produced by pattern bolting.

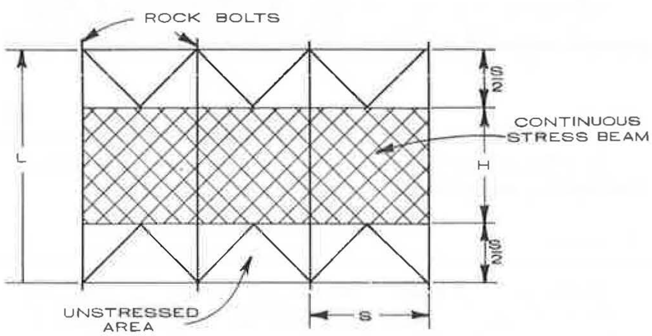
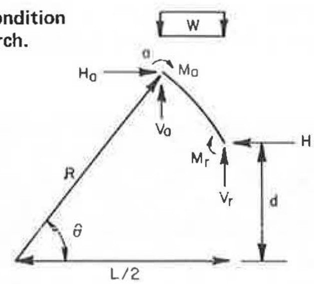


Figure 5. General stress condition in fixed-end, two-hinged arch.



The existence of these joints is based on direct observations; that they act in this manner is only theorized. This hypothesis could explain, however, that the obvious mechanical advantages gained by roof bolting are attributed to restriction of movement along joints or by providing continuity through mechanical means. Not much is known about this mechanism—in particular, how much tension is required to produce continuity—but the study of rock bolts made by Lang for other purposes was of sufficient detail to shed some light on how bolt tension may affect beam strength.

The data were obtained by loading a rock-bolted 1.2-m (4-ft) cube of crushed rock. The cube was supported laterally and instrumented to measure deflection and lateral pressure. This experiment showed that competent structures could be formed from completely incoherent, suitably bolted masses of gravel and that the mass exhibited elastic and plastic (strain-hardening) properties similar to those of intact rock masses. By using the data gathered in these tests, it can be further shown that an increase in bolt tension, which increases bolt-induced continuity, decreases beam deflection (Figure 4).

Lang also concluded elsewhere in his report that beam strength remains relatively constant until a threshold level is reached, and then failure occurs rapidly.

These two findings support the hypothesis that, regardless of the orientation of joints, cracks, or bedding planes, as long as the rock is laterally confined the rock bolts support by enforcing mechanical continuity and allow the rock to support itself with its own inherent properties. The beam produced by the interference of the individual bolt stress patterns is shown in Figure 4.

If this be the case, neglecting all the rock except for the cross-hatched area, a simple, conservative, conventional analysis can be performed on the remaining rectangular section by assuming that the material is held continuous by the rock bolts throughout this zone. Of the many conventional methods of analysis available, the one that seems to be most adaptable to the actual situation is the two-hinged analysis. If this beam is assumed to be a two-hinged arch separate from the surrounding rock material and is assumed to be loaded with a uniform vertical loading, the general stress condition in this arch stress beam would be as shown in Figure 5. Real loading conditions other than this assumption generally produce errors on the side of safety.

$$H_R = K_1 WL = \{[\int(Mmds/EI) + \int(Vvds/AG) + \int(Nn/AE)ds] \div [\int(M^2 ds/EI) + \int(v^2 ds/AG) + \int(n^2 ds/AE)]\} \quad (5)$$

where

$M = (W/2)[(L^2/4) - R^2 \cos^2 \theta]$,
$m = 1\,(R \sin \theta - d)$,
$ds = rd\theta$,
$V = (wl/2) \cos^2 \theta$,
$v = \cos \theta$,
$N = (wl/2) \cos \theta \sin \theta$,
$n = -\sin \theta$, and
$G = \frac{2}{5} E$.

This produces

$$K_1 = (WL/2A)\{[L^2/4t^2(3NJ^2 - 2)(\pi/2\tan^{-1}N) - 3I^2 + 4] + 6/J^2\} \div \{[L^2/4t^2(2N^2 + J^2)(\pi/2 - \tan^{-1}N) - 3N] + 7/2(\pi/2 - \tan^{-1}N) - 3/2N\} \quad (6)$$

where

$N = 2d/L$ and
$J = 2t/L$.

These values will allow expressions to be written for beam stress by using the familiar equation

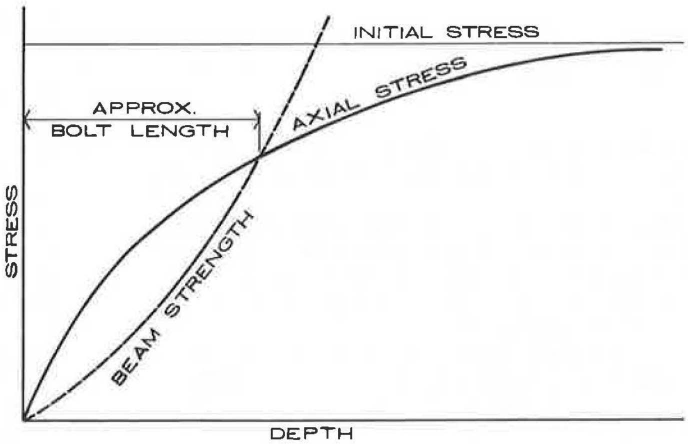
$$a = (M_a C/I) \pm (N_a/A) \quad (7)$$

The sum of the moments around a produces

$$M_a = V_r(L/2 - R\cos\theta) - W/2(L/2 - R\cos\theta)^2 - H_r(R\sin\theta - d) - M_r \quad (8)$$

Substituting $V_r = WL/2$ and reducing lead to

Figure 6. Relation between beam strength and in situ load.



$$M_a = W/2(L^2/4 - R^2 \cos^2\theta) - H_r(R\sin\theta - d) - M_r \quad (9)$$

If we let $H_r = K_1WL$ and $M_r = K_2WL^2$, then the equation reduces to

$$M_a = W/2(L^2/4 - R^2\cos^2\theta) - 2K_1WL(R\sin\theta - d) - 2K_2WL^2 \quad (10)$$

The normal and shear stresses at point a can be written

$$N_a = H_a \sin\theta - V\cos\theta \quad (11)$$

Substituting $H_a = H_r = K_1WL$ and $V_a = V_r - W_2L)(1 - \cos\theta)$ produces

$$N_a = K_1WL\sin\theta - W(L/2 - R\cos\theta)\cos\theta + WL/2\cos\theta \quad (12)$$

If, for the purpose of illustration, it is assumed that end rotation is allowed (hinged arch), K_2 is equal to zero and K_1 can be found by assuming the controlling condition to be at midspan where θ equals 90°:

$$\sigma_{90} = 3(WL^2/4t^2)K_3K_1(WL/t) \quad (13)$$

where

$$K_2 = 1 - 4K_1(1 + N^2)^{1/2} - N \text{ and}$$
$$\sigma_{90} = 3(K_3W/J^2) \pm 2K_1(W/J)$$
$$= W(3K_3 \pm 2K_1J)/J^2$$
$$= K_4W.$$

This equation then relates load directly to beam strength not only in a simple manner but in a manner commonly used by structural engineers. The upper boundary on the beam load would be a vertically applied uniform load equal in magnitude to the in situ stress level. However, this is, in many instances, ultraconservative because the in situ load is really the radial load, which varies from zero at the surface to the in situ level, a depth according to the following relation:

$$\sigma_{90} = \tfrac{1}{2}S_v[1 - (a^2/r^2)] + \tfrac{1}{2}S_v[1 - 4(a^2/r^2) + 3(a^4/r^4)] \quad (14)$$

where
S_v = vertical applied stress,
$a = L/2$, and
r = distance from hole center.

If this load were plotted with depth, as shown in Figure 6, it would commence at zero and increase concavely downward; if strength plus depth were plotted on the same axis, it would commence at zero and increase concavely upward. The point at which the curves intersect is the design depth. These concepts may be used to modify the load portion of the K_4 value. Tables can then be prepared based on the two dimensionless ratios of beam thickness to span length (t/L) and offset dimension d shown in Figure 6 over the span length (d/L). The table below was prepared in this way:

	d/L				
t/L	0.00	0.30	0.60	0.90	1.20
0.10	0.10	2.15	2.34	2.44	2.71
0.11	2.10	2.21	2.28	2.38	2.67
0.12	2.02	2.12	2.18	2.29	2.59
0.13	1.92	2.01	2.07	2.17	2.49
0.14	1.81	1.90	1.95	2.05	2.37
0.15	1.70	1.78	1.82	1.92	2.24
0.16	1.59	1.66	1.70	1.79	2.12
0.17	1.49	1.35	1.59	1.67	1.99
0.18	1.39	1.44	1.47	1.55	1.87
0.19	1.29	1.34	1.37	1.44	1.75
0.20	1.20	1.24	1.27	1.34	1.64
0.21	1.11	1.15	1.17	1.24	1.53
0.22	1.03	1.07	1.09	1.15	1.43
0.23	0.95	0.99	1.01	1.07	1.33
0.24	0.88	0.91	0.93	0.99	1.24
0.25	0.82	0.84	0.86	0.91	1.16
0.26	0.75	0.78	0.79	0.84	1.07
0.27	0.69	0.71	0.73	0.78	1.00
0.28	0.64	0.66	0.67	0.72	0.92
0.29	0.58	0.60	0.61	0.66	0.85
0.30	0.53	0.55	0.56	0.60	0.78
0.31	0.48	0.50	0.51	0.55	0.72
0.32	0.44	0.45	0.46	0.50	0.65

This form of table is simple and easy to use. If the geometry of the openings is known, then the table is entered in the column with the appropriate d/L ratio, searched for the correct beam strength to load ratio, and exited with the proper beam thickness to span ratio. For example, a beam with a d/L ratio of zero, a beam strength of 4138 kPa (600 lbf/in²), and an undisturbed stress level of 3448 kPa (500 lbf/in²) would produce a beam thickness ratio of approximately 0.20. If the cavern span were 15.2 m (50 ft), it would require a beam thickness of 3 m (10 ft), or a 4.3-m (14-ft) bolt on a 1.2-m (4-ft) spacing would provide adequate protection.

The value of this method lives or dies on what the rock beam strength really is under bending stresses. I believe that, if research were done in this area, significant advancements could be made in placing rock-bolt design on a firm analytical footing. The 4138-kPa (600-lbf/in²) bending strength seems to be a conservative number. If research did indeed show this to be the case, current rock bolt practice is specifying bolt lengths that are grossly overdesigned. This would explain the lack of failures in properly placed and grouted, tensioned rock bolt systems.

CONCLUSIONS

There appear to be four major conclusions in this research:

1. Roof support by rock bolting is a function of the power of the rock bolts to enforce mechanical continuity on the rock.
2. A study should be made to determine the effect of bolt tension on beam stability and the threshold tension required to produce continuity and to identify any other important variable in predicting levels of mechanically induced continuity.
3. A study should be made to determine realistic values of rock beam strength to enable the use of K_4 tables and reduce the apparently high rock-bolt safety factors.
4. Laboratory or field tests should be developed to predict rock beam strength.

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