Edge-of-Pavement Profiles

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When superelevation transitions occur within or adjacent to vertical curves, the profile of the edge of the road is sometimes distorted into shapes that are visually unpleasant or cause severe drainage problems, particularly on freeways or other wide roads. Such problems can be easily and quickly solved by simple algebraic methods that need no plotting of edge profiles to scale or resorting to curves more complicated than standard parabolic vertical curves if one takes advantage of the inherent simplicity of standard highway geometry. Methods for doing this are developed and demonstrated with a numerical example.

The horizontal and vertical alignment of highways has traditionally been thought of as a series of straight lines (tangents) connected together by curves. Probably as a carry-over from railway design—where horizontal curvature radically increases the force necessary to move the train—and for simplicity, most designers tried to keep the tangents as long as possible and the curves as short as possible. Roads built in hilly country 50 years ago were actually curvilinear (crooked) in spite of this design philosophy, but, as the introduction of motorized equipment lowered the cost of earthwork, the roads became straighter and straighter.

As Tunnard and Pushkarev (1) have documented so well with photographs, a frequent result was a type of long tangent, short curve alignment that not only seemed to exhibit a callous disregard for the integrity of the landscape, but tended to look silly in itself. To the extent that the general public reacted to this at all, the reaction was probably a sort of generalized objection to the building of roads and to the builders as well. Some professionals also reacted, however, and from this reaction a philosophy of curvilinear alignment developed wherein the designer fit long curves to the terrain and used short tangents as connecting elements.

In North America, this design philosophy suffered a severe setback during the rush of construction after World War II but eventually became commonly accepted, particularly during the period of emphasis on aesthetics of design in the United States in the mid-1960s. While many reference works on this type of design exist, I regard two (1, 2) as the most basic and comprehensive. Both reflect the ideas of progressive designers of the late 1930s (3).

Once one gets used to the idea of using curves rather than straight lines as the basic design elements, the design of curvilinear roads is not particularly difficult. Furthermore, the generous use of curvature allows a great deal of freedom in design, so one should be able to reduce construction costs by achieving a better fit to the terrain than is possible with a long-tangent, short-curve design.

The changeover is not completely without difficulties, however; a few new technical problems do arise. Perhaps the most obvious is that one must devote considerable attention to the provision of sufficient passing sight distance on two-lane roads. In addition, certain combinations of horizontal and vertical curvature can look very bad, so there has been considerable research into problems of the coordination of horizontal and vertical curvature and the general concept of the road as a three-dimensional object. Many books and papers (1, 2, 3) have devoted a good deal of attention to this subject, and some (4, 5) are entirely devoted to it.

A problem that has received much less attention, however, is the interaction of vertical curvature and superelevation transitions that occurs when vertical and horizontal curves of nearly equal length are superimposed to create a three-dimensional curve. The most obvious example of this interaction is the appearance of a little kink or dent in the edge of the roadway near the end of a superelevation transition, sometimes scarcely measurable, but very visible. Nearly every highway engineering manual and route surveying textbook contains a warning about this, usually coupled with the advice that a profile of the edge of the roadway should be plotted and examined for any irregularities. The advice is sound, but it should be obvious to any engineer who drives with a critical eye that the results often leave much to be desired.

An example of such a dent, along with the profile and superelevation diagram that produced it, is shown in Figure 1. The dent in the edge profile just to the left of km 10+000 is obvious in the figure and would be even more obvious on the ground, since the flat angle at which motorists view the roadway makes even very small undulations highly visible. The fact that the actual depth of the dent is tiny does not matter; the eye seems to be sensitive to irregularity of shape rather than to elevation.

To avoid such problems, many engineers abandon the superelevation diagram and design a smooth profile for each edge graphically with splines or French curves, a method recommended by the AASHTO Blue Book (8). This method is rather laborious, however, particularly since the resulting design can be included in contract plans only as a graphical curve or a series of grid point elevations. Splined curves are also positively guaranteed to produce indigestion if fed to a production-oriented computer program, so survey notes must be prepared by hand.

The basic premise of this paper is that an easier way is available. In many situations graphical methods are very desirable; in fact, I believe that engineers should use graphical methods more often than they do. In this particular case, however, an easier, quicker, and surer algebraic method is available. This method is hardly revolutionary; it really only amounts to understanding the basic geometry of the roadway and checking the values of certain critical parameters. There must be many engineers who have worked out similar methods. They are not, however, in common use in North America and appear in few, if any, of the standard texts and handbooks. A related but much less flexible approach has been used in the British computer program, JANUS (5).

VERTICAL CURVES

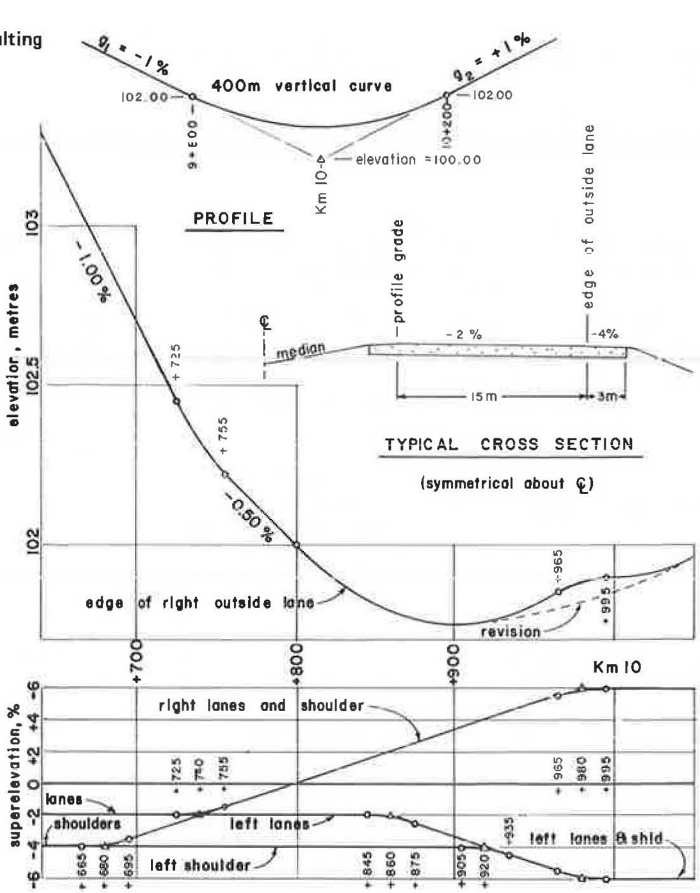
The first step in developing the method is to review the basic geometry of the parabolic vertical curves used in highway design. Two tangent grades, g_1 and g_2, are joined by a segment of a parabola defined by the quadratic equation

$$y(x) = y_0 + g_1 x + (r/2)x^2 \qquad (1)$$

where

$y(x)$ = elevation at a horizontal distance x from the beginning of the vertical curve (BVC),
y_0 = elevation at the beginning of the vertical curve,

Figure 1. Superelevation diagram with 30-m curves and resulting edge-of-pavement profile for right lanes.



$r = (g_2 - g_1)/L$, and
L = horizontal length of the vertical curve.

The slope or grade at any point on the curve can be obtained by differentiating Equation 1:

$$dy(x)/dx = g_1 + rx \quad (2)$$

Of more interest in the problem considered here is the rate of change of grade:

$$d^2y(x)/dx^2 = r \quad (3)$$

That the rate of change of grade is constant is, of course, a fundamental property of a parabolic vertical curve (6). Instead of differentiating Equation 1 to obtain Equations 2 and 3, we could as well have postulated that a constant rate of change of grade was a desirable property, and then obtained Equations 2 and 1 by integration. Clearly this must be what happened historically: someone noted that a constant rate of change of grade not only was the simplest way to join two grades but also produced the shortest possible curve satisfying a criterion of the form $d^2y(x)/dx^2 \leq$ some number.

The value of r, or its reciprocal, can be used as a measure of curvature. Use of the reciprocal is, in fact, common and has an advantage in that a parabolic highway or railway curve with rate of change of grade r is very nearly the same curve as a circular curve with radius 1/r. The design handbooks listed in the references all use the reciprocal. Two European references (2, 10) identify 1/r as a radius and denote it by the symbols H and R_v, respectively. North American references (8, 9) instead use K = 1/100r, the distance required to accomplish a 1 percent change in grade.

In what follows we shall use r as our measure of curvature rather than K because the equations would be more complicated with K. Most design calculations, however, could be made just as easily with K as with r.

A NOTE REGARDING UNITS

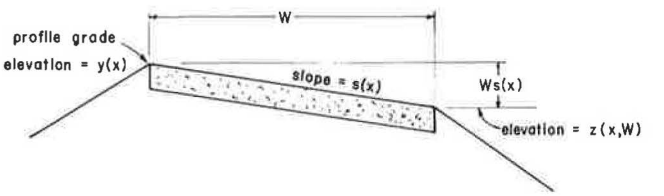
This paper has been written at a rather awkward time so far as units are concerned; both the traditional system of feet and 100-foot stations and the metric system are in current use in North American practice. In Canada, in fact, one might find an engineer working on two projects concurrently, one measured in feet and the other in meters.

In general, this paper will use SI (Systeme International d'Unités) units, following in most details the usage recommended (7). Vertical distances will always be expressed in meters, but location along the centerline, or chainage, will be expressed as a distance in kilometers from some arbitrary zero point plus a distance in meters: a point 9 km from the starting point will be identified as km 9 or km 9+000 and a point 500.36 m farther from the starting point as km 9+500.36, for example.

Equations 1, 2, and 3 are unit free; they are valid in any units. The horizontal and vertical distances can be measured in meters, feet, chains, furlongs, fathoms, or even inches; it does not matter what the unit of measurement is, so long as the same unit is used throughout the calculation. It should be noted that the grades g_1, g_2, and dy(x)/dx are dimensionless ratios, so a grade of 0.02 means exactly the same thing in any system of units; the rate of change of grade r is not dimensionless, though, and has units of reciprocal meters (1/m) or reciprocal feet (1/ft). Typical values are of the order of 10^{-4} in either system.

Since numbers less than 0.1 are awkward to remem-

Figure 2. Cross section of simple one-lane road at location x.



ber and use in mental calculations, it is common practice to express grades in percentages. The difficulty with too many zeros is obviously much more severe with the values of r normally encountered, so in this paper all values of r will be expressed as a multiple of 10^{-4}, for example 0.5×10^{-4}/m instead of 0.000 05/m. This can be thought of as 0.5 percent per hundred meters. In traditional units, 0.5×10^{-4}/ft can be thought of as 0.5 percent per station.

One additional warning about units is necessary for designers working with the AASHTO Blue Book (8) or RTAC metric standards (9). These handbooks express vertical curvature in terms of K, defined as the distance, in feet or meters respectively, to accomplish a 1 percent grade change. Thus the units of K are really ft/percent or m/percent, but table headings in the handbooks indicate that they are in feet and meters, respectively, because the percentage is included in the definition. The tabled values of K in these two reference books can, therefore, never be used directly in unit-free equations, but must always be converted to feet or meters by multiplying by 100.

SUPERELEVATION TRANSITIONS

The foregoing discussion of vertical curves used elementary calculus in order to clarify the concepts, but the mathematical methods used in ordinary practice are algebra and arithmetic. In treating superelevation, it is not even usual to use algebra; one simply draws a superelevation diagram such as the one shown in Figure 1 and reads the values graphically. As a matter of fact, this is a good approach. Since the diagram is easy to draw and provides adequate accuracy if an appropriate scale is used, why should one bother to write an equation?

In order to develop the relationship between vertical alignment and superelevation, however, we shall continue to use the language and notation of calculus because they provide an easy way to describe what is happening. For ordinary applications we shall revert to simple arithmetic.

To begin, let us suppose that our road has only one lane and no shoulders, with the profile along the left edge, as shown in Figure 2. This simple road could be a ramp or half of a two-lane road; the numerical example in Figure 1 is half of an eight-lane freeway. The width of the lane is W; the elevation of the left edge is, by definition, y(x); and we define a function s(x) = superelevation rate at location x to describe the superelevation. In the section on vertical curves, x was defined as the horizontal distance from the beginning of the vertical curve. In this section we shall not define it so precisely; x is simply a number that identifies our location, the horizontal distance along the centerline from some arbitrary zero point measured in the same units as all other distances. We shall never write an expression for s(x); it is simply a name for the function described by a superelevation diagram such as the numerical example in Figure 1. It is important to realize, however, that superelevation diagrams have the same mathematical form as profiles: straight lines joined together by parabolic curves.

To obtain the elevation z(x, W) of a point on the right edge of our one-lane road, a distance W right of a point on the centerline at location x, we simply read s(x) from the superelevation diagram, multiply by the width W, and add the product to the profile elevation g(x):

$$z(x, W) = y(x) + Ws(x) \tag{4}$$

Differentiating Equation 4 twice, we obtain

$$dz(x, W)/dx = dy(x)/dx + dWs(x)/dx \tag{5}$$

$$d^2z(x, W)/dx^2 = d^2y(x)/dx^2 + d^2Ws(x)/dx^2 \tag{6}$$

In this paper, it will always be assumed that the width of the road, W, is constant. For tapers, one must replace W with W(x) before differentiating.

From Equations 4, 5, and 6 we see that the shape of the edge of the lane, described by its elevation z(x, W), slope dz(x, W)/dx, and rate of change of grade $d^2z(x, W)/dx^2$, is simply the sum of the profile and the effect of the superelevation transition. For a numerical example, consider the right lanes of the road described in Figure 1. For the moment, we shall ignore the shoulders. Within the linear portion of the superelevation diagram,

$$dWs(x)/dx = (15\text{ m}) \times [+0.06 - (-0.02)]/240\text{ m} = +0.005 \tag{7}$$

The grade along the outside edge of pavement is 0.005 more than the grade along the profile at every point between km 9+755 and km 9+965. To make this calculation, we can use either of two mathematically equivalent approaches. The most direct is to rewrite the last term of Equation 5 as Wds(x)/dx, W times the slope of the superelevation diagram. The superelevation changes linearly from -2 to +6 percent in 240 m (ignoring the curves in the superelevation diagram for the moment) and from 0 to +6 percent in 180 m, so

$$Wds(x)/dx = (15\text{ m}) \times (+0.06 - 0)/180\text{ m} = +0.005 \tag{8}$$

However, Equation 5 was written with dWs(x)/dx rather than Wds(x)/dx because I do not usually make the calculation in such a straightforward mathematical way, but according to the following more physical logic. Between km 9+800 and km 9+980 the superelevation changes 6 percent. Since the pavement is 15 m wide, this is a (0.06) × (15 m) = 0.90 m rise relative to the centerline. This rise is accomplished over a 180-m distance, so the grade necessary to accomplish it is

$$dWs(x)/dx = 0.90\text{ m}/180\text{ m} = +0.005 \tag{9}$$

To obtain the actual grade along the outside edge of pavement, we simply calculate the grade along the profile and add 0.005. For example, at km 9+820, we are 20 m into the profile vertical curve, so

$$\begin{aligned} dz(x)/dx &= dy(x)/dx + 0.005 = g_1 + 20r + 0.005 \\ &= -0.01 + [20 \times (0.5 \times 10^{-4})] + 0.005 = -0.004 \end{aligned} \tag{10}$$

Proceeding to the rate of change of grade, we can continue the same line of reasoning. Prior to km 9+965 the right edge of the road was rising at +0.005 relative to the centerline, but beyond km 9+995 it is parallel to the centerline. This -0.005 change is accomplished in a 30-m parabolic curve, so the rate of change of grade due to the superelevation transition must be -0.005/30 m = -1.667×10^{-4}/m. If we call this rate of change of grade due to the superelevation transition r_s, we can rewrite Equation 6 as

$$r_e = r + r_s \tag{11}$$

where $r_e = d^2z(x, W)/dx^2$ is the rate of change of grade along a line offset a horizontal distance W from the centerline. In Equation 11 it is clear that r_e is just the sum of the rate of change of grade due to the curves on the profile and on the superelevation diagram. In our example, r_e for the superelevation curve centered at km 9+980 is

$$r_e = r + r_s = +0.5 \times 10^{-4}/\text{m} - 1.667 \times 10^{-4}/\text{m} = -1.167 \times 10^{-4}/\text{m} \tag{12}$$

At this point, it should be very obvious why the profile of the outside edge of the right lanes in Figure 1 looks like a dented bowl. The dent is the portion from km 9+965 to km 9+995 where $r_e = -1.167 \times 10^{-4}$/m is negative, in contrast to the surrounding portion of the bowl where $r_e = r = +0.5 \times 10^{-4}$/m is positive. It is also quite obvious that we can eliminate the dent only by making r_e positive everywhere, which will happen if the absolute value of r_s is less than the absolute value of r. This can be accomplished either by lengthening the curve in the superelevation diagram or by shortening the curve in the profile. Usually, we prefer to do the former, although a situation occasionally arises where it is desirable or even necessary to change the profile.

In our example, we need to lengthen the curve in the superelevation diagram until $|r_s| < r = 0.5 \times 10^{-4}$/m, or $|-0.005/\ell| < 0.5 \times 10^{-4}$/m, where ℓ is the length of the curve in the superelevation diagram. Clearly we will achieve equality if $\ell = 100$ m and satisfy the inequality if $\ell > 100$ m. If we were to use $\ell = 100$ m, we would have $r_e = r + r_s = 0$ and the edge-of-pavement profile would be a straight line from km 9+930 to km 10+030.

The reader can see how this would look by laying a straight edge tangent to the edge-of-pavement profile at km 9+930 and km 10+030 on Figure 1. It eliminates the dent, but leaves a flat spot. Since flat spots on round bowls are usually considered undesirable, we shall usually insist on strict inequality. The dashed curve on Figure 1 has $\ell = 140$ m. This length was chosen rather arbitrarily to illustrate the effect of using $\ell > 100$ m; whether or not it was a good choice will be discussed in a later section. In any case, it is clear that the problem can be solved only by using a curve much longer than is standard practice. It should also be noted that any solution worked out graphically with a spline will involve a revision in the superelevation over a distance of at least 100 m.

MULTIPLE PLANES

Most modern roads have paved shoulders, so they cannot be considered a single plane. In addition, many highways are built with different cross slopes for each lane. The generalization of the preceding theory to cover multiple planes is straightforward.

Suppose that the road has several planes numbered 1, 2, 3, ... If plane number i has width w_i and superelevation $s_i(x)$ and we are interested in the shape of a line offset a distance $W = w_1 + w_2 + \ldots + w_n$ from the profile grade, then

$$z(x, W) = y(x) + \sum_{i=1}^{n} w_i s_i(x) \tag{13}$$

Taking derivatives as before,

$$dz(x, W)/dx = dy(x)/dx + \sum_{i=1}^{n} dw_i s_i(x)/dx \tag{14}$$

$$d^2z(x, W)/dx^2 = d^2y(x)/dx^2 + \sum_{i=1}^{n} d^2w_i s_i(x)/dx^2 \tag{15}$$

Thus, we only need to add together the effects of what happens in each plane to obtain the total effect.

DESIGN EXAMPLE

To illustrate the analysis of multiple planes, we return to Figure 1, but we shall consider what happens to the edge-of-shoulder profile rather than the profile of the edge of the outside lane. This is the line we are really interested in, the actual physical edge of the roadway. It is not only the most clearly visible line, but also the one that determines the shape of the bridge rail if there is one.

Since we already know that there is a problem on the right side at km 9+980, we shall start there. The width W is now 18 m, but at this location we still have a single plane, since the shoulder and lanes all have the same superelevation. Thus the grade change caused by the superelevation transition is $dWs(x)/dx = (18 \text{ m}) \times (+0.06 - 0)/180 \text{ m} = +0.006$ and the rate of change of grade due to superelevation is $r_s = -0.006/30 \text{ m} = -2 \times 10^{-4}$/m. Since the profile grade is curving at $r = +0.5 \times 10^{-4}$/m, there will clearly be a dent in the edge of the shoulder unless we increase the superelevation curve length until $|r_s| \leq 0.5 \times 10^{-4}$ or $\ell > 0.006/(0.5 \times 10^{-4}/\text{m}) = 120$ m.

Usually we would choose to use a length greater than the minimum to avoid a flat spot, but this time we shall use $\ell = 120$ m. This will give us a 120-m straight line in the right edge of the shoulder profile on a grade of +0.2 percent, as shown in Figure 3. A short tangent might look bad, but 120 m seems long enough to stand on its own as a geometric element. Since we are on the high side of superelevation, the very flat grade will be acceptable, though some very careful gutter design might be necessary if the road were in excavation.

The next question to be asked is, Where else might there be dents in the edge of the shoulder profile? After a little reflection, it should be obvious that there are only two possibilities, both on the left roadway, at km 9+860 and at km 9+920. Notice that no calculations were necessary to reach this conclusion; these are the only other superelevation diagram curves that could cause dents, because all the other curves are either outside of the profile vertical curve or curve in the same direction as the profile. In the latter case r_s has the same sign as r, so the effect of the superelevation curve is to make the curvature sharper, not to change its direction.

To analyze the two possible trouble spots, it is not necessary to repeat all the calculations we made for the curve at km 9+980 on the right lanes. Instead, we observe that the curves at km 9+860 and km 9+920 look exactly the same on the superelevation diagram as the curve at km 9+980 on the right lanes. Therefore $ds(x)/dx$ and $d^2s(x)/dx$ are exactly the same as before; the only difference in the three curves is the width of the plane, W or w_i.

At km 9+860 only the lanes are curving, so W = 15 m. We found at km 9+980 that $|r_s|$ for a plane 18 m wide was 2.0×10^{-4}/m, so for W = 15 m, $|r_s| = (2.0 \times 10^{-4}/\text{m}) \times (15 \text{ m}/18 \text{ m}) = 1.667 \times 10^{-4}$/m and at km 9+920 where only the shoulder is curving due to superelevation $|r_s| = (2.0 \times 10^{-4}/\text{m}) \times (3 \text{ m}/18 \text{ m}) = 0.333 \times 10^{-4}$/m. Comparing these numbers with $r = 0.5 \times 10^{-4}$, we see that there will be a dent at km 9+860, but not at km 9+920.

When we change the length ℓ of the curve at km 9+860 to eliminate the dent, r_e will change in proportion to $1/\ell$, so we need to increase ℓ until $(1.667 \times 10^{-4}/m) \times (30\ m/\ell) \le 0.5 \times 10^{-4}/m$ or $\ell \ge 100$ m. If we were to use 100 m, however, we would get the result shown schematically in Figure 4.

The negative value of r_e between km 9+905 and km 9+910 occurs because the lane curve at km 9+860 and the shoulder curve at km 9+920 now overlap. To eliminate this section of downward curvature by lengthening the curves, we would have to make the lane curve considerably longer than 100 m, but if it were longer than 120 m, it would extend outside of the profile vertical curve and again give us a short section where r_e was negative. This would not be too bad since it would be between a section with $r_e = 0$ and one with $r_e > 0$ rather than between two sections with $r_e > 0$, but it still seems worth avoiding. Sometimes a good solution to such a situation can be obtained only by changing the profile.

Figure 3. Redesigned superelevation diagram and resulting edge-of-shoulder profiles.

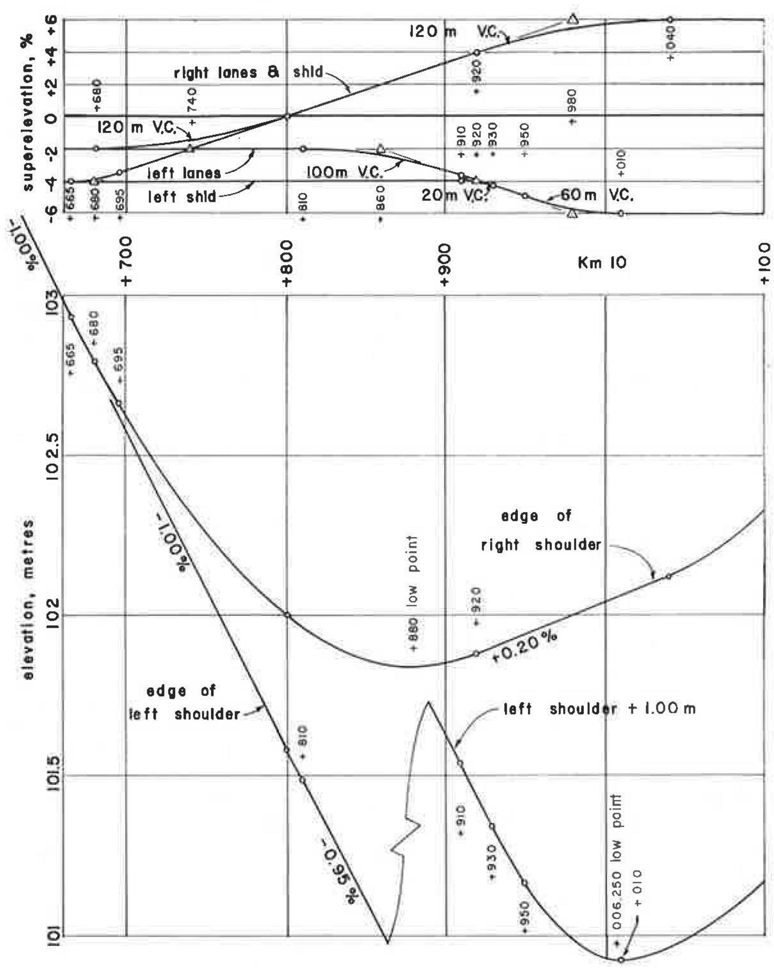
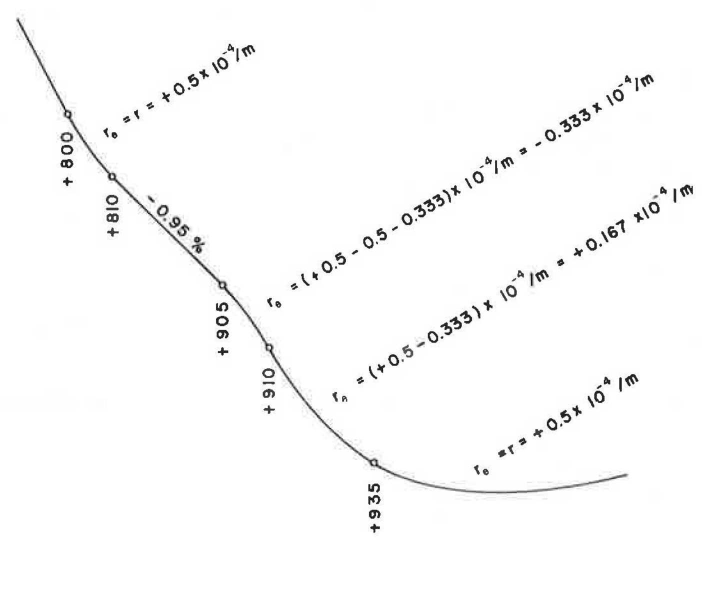


Figure 4. Sketch of left shoulder profile with 100-m curve at km 9+860 in superelevation diagram.



In this case, however, we have another option.

If we simply shorten the shoulder curve at km 9+920 to 20 m, as shown in Figure 3, it will not overlap a 100-m curve at km 9+860. This will give us $|r_s| = 0.5 \times 10^{-4}/m$ at km 9+860 and $|r_s| = (0.333 \times 10^{-4}/m) \times (30\ m/20\ m) = 0.5 \times 10^{-4}/m$ at km 9+920.

Interestingly enough (and quite by accident), both of these curves just exactly counteract the profile vertical curve, so the straight line on a -0.95 percent slope shown in Figure 4 will now extend all the way from km 9+810 to km 9+930. There will be no dents, but there is a 10-m curve in the edge of the lanes and shoulder centered at 9+805. Since the grade only changes 0.05 percent in this curve, it is a reasonably safe assumption that it will not show up at all; if it does, it will look like an angle point rather than a curve.

We have now eliminated all the dents but should still check the other superelevation curves to see if the design can be improved. On a two-lane road we would make the curve on the left side at km 9+980 the same length as the one on the right. On a freeway it is usually convenient to do so but not essential, and 120 m really does seem very long to use without a definite reason. On the other hand 30 m, though it does provide adequate sight distance for freeway speeds, is very short for such a wide road. Therefore, a 60-m curve will be used at km 9+980 as shown in Figure 3. This is probably about the shortest superelevation curve we should ever use for freeway lanes.

The subject of sight distance in superelevation transitions is complicated. In our example there is adequate sight distance because the profile vertical curve is very flat. If the profile had been designed to just satisfy sight distance standards, however, the 30-m curve would not have provided adequate sight distance in the outside lane. There can also be severe sight distance problems at crest curves in the superelevation diagram. On the high side of superelevation these problems are reduced because drivers can look diagonally across the lower lanes, but on the low side of superelevation in excavation the shoulder width may not be sufficient for this to be much help.

Wirasinghe of the University of Calgary has done some research on analytic methods for calculating the sight distance in superelevation transitions (11), but the complexity of his analysis of even very simple cases is discouraging. For practical purposes, it would seem that sight distance problems must be individually analyzed by graphical methods. Of course, a good understanding of the geometry of the roadway will be very helpful in identifying those cases where the likelihood of sight distance problems is sufficiently great to justify a graphical analysis.

The two curves that remain are at km 9+680 and km 9+740 on the right roadway, outside of the profile vertical curve. Nothing really needs to be done about these two curves, except possibly to lengthen the one at km 9+740 simply because 30 m is very short for a freeway curve. If we were to lengthen it to 60 m, however, we would be left with a 30-m straight line in the edge-of-shoulder profile between km 9+770 and km 9+800. To avoid this, I have used a 120-m curve so that the right edge-of-shoulder curves continuously from km 9+665 to km 9+920, as shown in Figure 3. This last change is admittedly a nicety rather than a necessity, but certainly a very inexpensive nicety.

DRAINAGE

So far, very little has been said about drainage, but it is often a major consideration in the design of superelevation diagrams. Figure 1 illustrates some of the problems that can arise. The figure shows the high edge of the road, but exactly the same things can happen on the low side where the shoulder is expected to function as a drainage channel. The most obvious potential drainage problem in Figure 1 is that the right edge profile has two points with zero grade, at km 9+900 and at km 10+000, with a very small hump between them. The second sag point was eliminated from the final design along with the dent centered at km 9+980.

Less interesting, perhaps, than the double sag points but very likely more serious is the danger of inadvertently creating grades that are too near zero for proper drainage. We have already seen that the 120-m tangent in the right shoulder profile in Figure 3 is on a grade of only 0.2 percent. To see what could have happened to us, suppose that g_1 and g_2 in Figure 1 had been -1.2 percent and 0.8 percent instead of -1.0 percent and 1.0 percent, respectively. Then the 120-m tangent would have been absolutely horizontal.

For an even more troublesome situation, suppose that g_1 and g_2 had been -0.6 percent and 1.4 percent. Then the 30-m straight line we considered using between km 9+770 and km 9+800 in the right edge of shoulder would have been horizontal. Since the superelevation at km 9+800 is also zero, a very hazardous pond would have formed on the roadway. To avoid this sort of ponding, it is necessary to design so that the resultant of the grade and the cross-fall is sufficiently large to keep the water moving. For example, Deitrich, Graff, and Rotach (10) recommend keeping $[g^2(x) + s^2(x)]^{1/2} \geq 0.005$, where $g(x)$ is the flattest longitudinal grade at any point on the cross section, i.e., the minimum value of $dz(x, W)/dx$ encountered for any W.

The analysis necessary to determine whether any drainage problems have been caused by a superelevation transition can be somewhat more complex than the analysis for the edge-of-shoulder dents but is never really difficult. It is not necessary to draw edge profiles to scale, but schematic drawings similar to Figure 4 are recommended for all situations where it is not immediately obvious that no problem exists. Solving the problems that already exist is likely to be more difficult. A solution can sometimes be obtained by changing the axis of rotation, but it is often necessary to change the profile. Only in unusually fortunate circumstances will changes in the length of the superelevation transition or its curves be sufficient to remedy serious drainage problems.

SUMMARY

As demonstrated in the numerical example, the simplicity of the mathematics of vertical curves and superelevation transitions can be exploited to very easily check whether a design will have undesirable features such as edge-of-shoulder dents, roller-coaster edge profiles, and a number of possible drainage and appearance problems. The arithmetic required to check the shapes is very simple and requires far less time than would be required to make the same checks graphically by plotting edge-of-shoulder profiles. In addition, the algebraic approach yields new designs that conform to the normal format of profile and superelevation diagrams and, hence, are compatible with standard computer programs.

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Smith at the California Division of Highways in their San Francisco office. The basic idea that these things could be done was a faint memory from a summer in the mid-1950s when I, equipped with one year of college mathematics but no knowledge of surveying or highway geometry, worked under the supervision of Eric Stokes. He was then using methods similar to those presented here, but I am not familiar with the details of how he applied them. The encouragement of these four engineers, the many things they taught me, and the example presented by their great concern for quality in design are all gratefully acknowledged.

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Superelevation and Curvature of Horizontal Curves

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This paper deals with the various parameters required for design of a horizontal curve, namely the relation between superelevation rate and curve radius. These parameters are derived from the characteristics of a person-vehicle-environment system. The lateral acceleration that constitutes the physical output from this system determines the critical case, for which the radius is minimum. The data used here are taken from studies dealing with human factors and traffic characteristics. The maximum superelevation rate is determined by limiting the negative lateral acceleration for slow vehicles and by assuring safe driving for fast vehicles. The development of the relation between superelevation rate and curve radius for any constant design speed is based on driver expectation that the pressure exerted on the steering wheel will decrease with increasing curve radius. The findings are compared with the U.S. and German guidelines. The criteria developed here are concluded to be reasonable and the findings useful for design purposes.

The design guidelines of different countries recommend various correlations between horizontal curves and superelevation rates but display significant differences both in the permissible minimum radii and in the values of superelevation rates. Since horizontal curves constitute critical points along the entire road system, the need exists for determining consensus criteria prescribing the parameters required for the design of a horizontal curve.

The purpose of this work is to establish these criteria and the corresponding guidelines for the relation between the radius of the curve and the superelevation. The basic approach adopted is to choose parameters that express the desired values for attaining unforced driving—referred to as "natural" driving in this paper—in a horizontal curve for an actual driving-speed distribution.

The data employed to determine these values are taken from various studies of the human factor and traffic research. Such studies lead to indices that determine the norm for natural driving in a curve on one hand and to indices that determine actual speed distribution on the other. For this reason, the lateral friction factor does not constitute a leading parameter in establishing the sought correlation; however, data on road accidents in curves are important input data for the analysis.

The approach to the study of the subject, as formulated above, forms the justification for the present work. Clearly, knowledge of the criteria for deriving the radii and superelevation rates enables any designer to evaluate the various implications of their determination.

SUPERELEVATION FORMULA

A car moving in a circular horizontal curve is in an equilibrium of accelerations, as given by the following formula (see Figure 1):

$$a_r = a_c - a_e \quad (1)$$