Smith at the California Division of Highways in their San Francisco office. The basic idea that these things could be done was a faint memory from a summer in the mid-1950s when I, equipped with one year of college mathematics but no knowledge of surveying or highway geometry, worked under the supervision of Eric Stokes. He was then using methods similar to those presented here, but I am not familiar with the details of how he applied them. The encouragement of these four engineers, the many things they taught me, and the example presented by their great concern for quality in design are all gratefully acknowledged.

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Superelevation and Curvature of Horizontal Curves

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This paper deals with the various parameters required for design of a horizontal curve, namely the relation between superelevation rate and curve radius. These parameters are derived from the characteristics of a person-vehicle-environment system. The lateral acceleration that constitutes the physical output from this system determines the critical case, for which the radius is minimum. The data used here are taken from studies dealing with human factors and traffic characteristics. The maximum superelevation rate is determined by limiting the negative lateral acceleration for slow vehicles and by assuring safe driving for fast vehicles. The development of the relation between superelevation rate and curve radius for any constant design speed is based on driver expectation that the pressure exerted on the steering wheel will decrease with increasing curve radius. The findings are compared with the U.S. and German guidelines. The criteria developed here are concluded to be reasonable and the findings useful for design purposes.

The design guidelines of different countries recommend various correlations between horizontal curves and superelevation rates but display significant differences both in the permissible minimum radii and in the values of superelevation rates. Since horizontal curves constitute critical points along the entire road system, the need exists for determining consensus criteria prescribing the parameters required for the design of a horizontal curve.

The purpose of this work is to establish these criteria and the corresponding guidelines for the relation between the radius of the curve and the superelevation. The basic approach adopted is to choose parameters that express the desired values for attaining unforced driving—referred to as "natural" driving in this paper—in a horizontal curve for an actual driving-speed distribution.

The data employed to determine these values are taken from various studies of the human factor and traffic research. Such studies lead to indices that determine the norm for natural driving in a curve on one hand and to indices that determine actual speed distribution on the other. For this reason, the lateral friction factor does not constitute a leading parameter in establishing the sought correlation; however, data on road accidents in curves are important input data for the analysis.

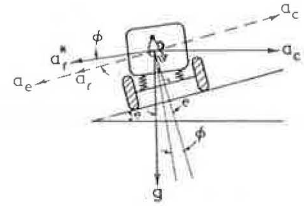
The approach to the study of the subject, as formulated above, forms the justification for the present work. Clearly, knowledge of the criteria for deriving the radii and superelevation rates enables any designer to evaluate the various implications of their determination.

SUPERELEVATION FORMULA

A car moving in a circular horizontal curve is in an equilibrium of accelerations, as given by the following formula (see Figure 1):

$$a_r = a_c - a_e \quad (1)$$

Figure 1. Scheme of accelerations during travel in a horizontal curve.



where a_r is the driver-perceived lateral acceleration, a_c is the centrifugal acceleration, and a_e is the superelevation acceleration. This formula is based on the fact that the value of the component of a_c acting in the direction of the superelevation slope is identical with the value of a_c itself, at a sufficient approximation. The expressions for the centrifugal acceleration and the superelevation acceleration are

$$a_c = V^2/R \tag{2}$$

$$a_e = ge \tag{3}$$

where V is the travel speed in the curve, R is the radius of the curve, e is the superelevation rate, and g is the gravity acceleration.

The assumptions inherent in the preceding equations are

1. The vehicle is a point mass,
2. The travel speed in the curve is constant,
3. The travel path of the vehicle has a constant radius identical with that of the curve itself, and
4. The longitudinal slope in the horizontal curve equals zero.

It is important to emphasize the difference between the real lateral acceleration, a_r, and the measured lateral acceleration, a_r^*. The latter is influenced by the angle of inclination of the car body (angle ϕ in Figure 1, denoted as body-roll angle). The real lateral acceleration is given by

$$a_r = a_r^* \cos\phi \tag{4}$$

where $\cos\phi$ is defined with the aid of the body-roll spring rate, α, as follows:

$$\cos\phi = 1/(1+\alpha) \tag{5}$$

or

$$a_r = a_r^*/(1+\alpha) \tag{6}$$

The accepted value for α is 0.2.

Design speed represents a predicted distribution of speeds, usually characterized by the fact that 85-95 percent of actual travel speeds are lower. The validity of this speed distribution is subject to an examination of the superelevation equation, in which a constant travel speed equaling the design speed is substituted. The two parameters specified below constitute the examination tools.

The first parameter, defined as the superelevation coefficient β^2, is the ratio of the superelevation acceleration to the centrifugal acceleration:

$$\beta^2 = a_e/a_c \tag{7}$$

This coefficient expresses the degree of safety of vehicles traveling at speeds differing from the design speed. For instance, with increasing β^2 for a constant superelevation rate, the value of the centrifugal acceleration decreases. Clearly, this decrease corresponds to a higher degree of travel safety.

The second parameter is defined as the comfort speed V_0, for which the lateral acceleration equals zero:

$$V_0 = \sqrt{Rge} \tag{8}$$

This speed is sometimes called the "hands-off" speed, as the car is carried in a circular path without any pressure having to be exerted on the steering wheel. In contrast, traveling at a speed lower than V_0 forces the driver to press on the wheel in a direction opposite to that of the turn of the wheel; this represents an unnatural driving operation in a curve. The comfort speed, therefore, must be as close as possible to the minimum travel speed expected in a curve in order to prevent most drivers from constrained driving. The relation between the above-mentioned two speed parameters and the superelevation equation may be shown as follows:

$$V_0 = \beta V_D \tag{9}$$

where V_D is the design speed. Also,

$$a_r = (V_0^2/\beta^2 R) - ge \tag{10}$$

Equation 10 is the superelevation equation, in which all basic parameters appear.

LATERAL ACCELERATION

Lateral acceleration comprises the physical output of the different variables for driving in a horizontal curve. Since the choice of travel speed is a result of the total person-vehicle-environment system, objective and subjective factors, such as qualifications, practical experience, and responses to stimuli, characterize the kinesthetic perception of the driver in conjunction with his or her visual perception. The output is, as mentioned above, the adopted lateral acceleration.

Some investigators—among them Ritchie, McCoy, and Welde (1); Leeming and Black (2); Moyer and Berry (3); and Kneebone (4)—have measured the value of the lateral acceleration of a vehicle in curves. The general trend of these measurements was that lateral acceleration value decreases with increasing travel speed (see Figure 2, which shows values of lateral acceleration corrected in accordance with Equation 6). This decrease results from drivers being overcautious when evaluating driving risk, which increases with increasing travel speed. The lateral acceleration values themselves were not uniform in the various measurements. However, the trend of these values when measured against travel speed is the same in almost all the measurements presented here.

The question that now arises is, from the complex of practical measurements, which correlation between design speed and lateral acceleration is recommended. The present work adopts the relation obtained by Ritchie, McCoy, and Welde, whose experiments involved 50 different drivers who drove an identical car along a fixed traveling course of 177 km with 227 curves (1).

Figure 3 shows the average lateral acceleration in these experiments for the entire driver population for the group of fastest drivers and for the group of slowest drivers. Analysis of these results shows that the correlation between lateral acceleration and curve speed for the entire population is closer to that for slow

Figure 2. Regression lines for the correlation between curve speed and the ratio a_r/g from different sources.

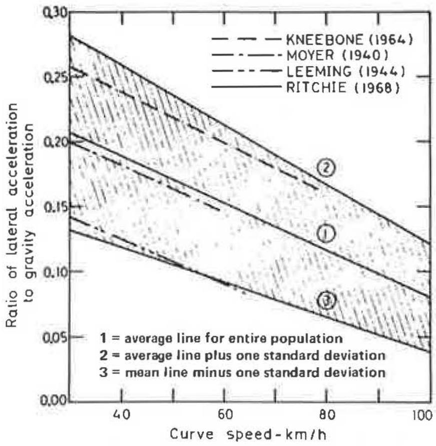
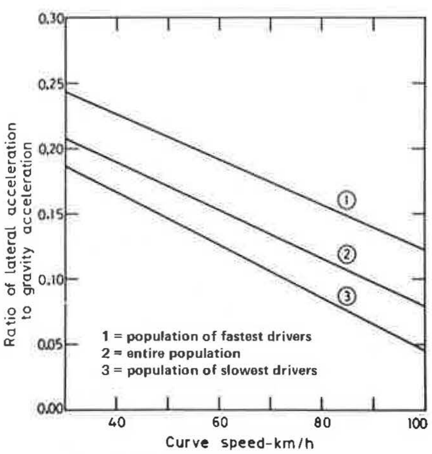


Figure 3. Regression lines for the correlation between curve speed and the ratio a_r/g from Ritchie, McCoy, and Welde's data.



drivers than that for fast drivers. Stated another way, the average population adopts values of lateral acceleration that are closer to those of slow drivers. Which correlation is the most relevant for replacing the curve speed with the design speed?

It would seem that one should choose the speed for the population of slow drivers, since it contains the lowest acceleration values. However, the critical values normally are not taken as practical values for design; instead, the values chosen are those that are certain for the majority of, say, 80 percent of events. Therefore, the correlation between lateral acceleration and curve speed for the entire population is the one adopted here as the design relation between the lateral acceleration and the design speed. This correlation, furthermore, expresses the physical output of the input data of driving behavior, including the choice of a driving path with a radius not necessarily identical to the curve radius.

Adoption of the correlation obtained by Ritchie, McCoy, and Welde for the design of superelevation rates in horizontal curves receives added justification from the fact that it lies in the middle range of the various measurement results (see Figure 2). The distribution of the correlation over the range of standard deviation in each direction (in about 67 percent of the cases) shows the approximate limits of the above range.

The mathematical expression for the adopted correlation is

$$a_r/g = (0.262 - 0.001\,82\,V_D) \quad (11)$$

where a_r/g is the ratio of lateral acceleration to gravity acceleration, and V_D is the design speed in kilometers per hour.

Since this correlation is valid up to a speed of 100 km/h, extrapolated values are suggested for higher speeds by reducing by 25 percent the slope of a_r as a function of V_D. This reduction is based on the work of the Fourth Committee on Highway Design, operated jointly by Germany, France, and Switzerland (5). The design values of a_r as a function of design speed are presented in Table 1.

SUPERELEVATION DESIGN VALUE

As stated previously, the value of the superelevation coefficient β^2 is connected with travel safety. High values of β^2 are desirable for high travel speeds accompanying the design speed. Conversely, for low travel speeds accompanying the same design speed, low values of β^2 are desirable, as derived from Equation 9. These contrasting trends can be bridged by relating the values of β^2 to the value of the design speed. At high design speeds safety is a more severe factor for the fast vehicle, and it is thus logical that the β^2 values should increase with increasing design speed. As a consequence, the need exists for a superelevation rate that is independent of the design speed on one hand and has a suitable maximum value on the other. To achieve this, Equation 7 may be substituted in Equation 1:

$$\beta^2 = 1/[(a_r/a_c) + 1] \quad (12)$$

According to Equation 12, increasing β^2 is accompanied by a reduction in the value of a_r/a_c. Because lateral acceleration decreases with design speed, the increase in β^2 with the design speed is conditional on assigning to a_c at least a fixed value. Among the accepted maximum values of superelevation, i.e., in the range of 0.06 -0.10, one has now to choose a value satisfying the criteria advanced in the beginning of this section.

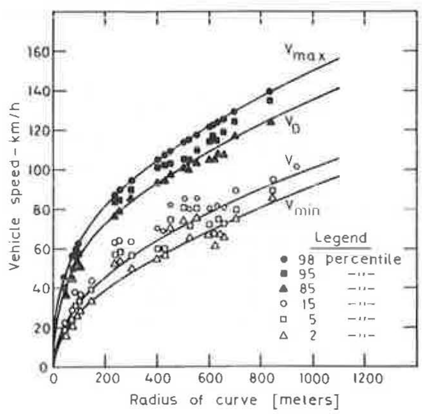
Since the choice of high values for the superelevation rate will result in a higher level of β^2 and, consequently, higher values of V_0 (see Equation 9), a value of 0.08 for the superelevation rate seems to ensure the maintenance of a reasonable limiting speed. In any case, a calculation check for the a_r values given in Table 1 and for e = 0.08 leads to the values of centrifugal acceleration, superelevation coefficient, and minimum radii, given in the same table. These values are checked by comparing the results of superelevation coefficient and centrifugal acceleration to commonly accepted values on one hand and to the reasonability of the derived speed distribution on the other.

As for the values of β^2 and a_c, it is important to mention the work of Spindler (6). Analysis of his observations proves that β^2 varies in the range from 0.3 to 0.5 when the values of centrifugal acceleration vary from 0.15 to 0.3 g. The horizontal curves that served for these experiments had radii varying from 30 to 500 m. Comparison of these findings with the results in Table 1, within the range of relevant radii, proves the reasonability of the values of both superelevation and lateral acceleration for this part of the check.

Table 1. Values of lateral acceleration for design, centrifugal acceleration, superelevation coefficient, and minimum radii as functions of design speed.

Design Speed (V_D) (km/h)	Ratio of Lateral to Gravity Acceleration (a_r/g) Value	Standard Deviation	Ratio of Centrifugal to Gravity Acceleration (a_c/g)	Superelevation Coefficient (β^2) (%)	Curve Radius (R) (m)
30	0.207	0.075	0.287	27.9	25
40	0.189	0.070	0.269	29.7	47
50	0.171	0.066	0.251	31.9	78
60	0.153	0.061	0.233	34.3	122
70	0.135	0.056	0.215	37.2	179
80	0.116	0.051	0.196	40.8	257
90	0.098	0.046	0.178	44.9	358
100	0.080	0.041	0.160	50.0	492
110	0.075	0.038	0.155	51.5	615
120	0.071	0.036	0.151	53.0	751
130	0.067	0.034	0.147	54.5	905
140	0.063	0.032	0.143	56.0	1080

Figure 4. Correlation between curve radius and vehicle speeds, on the basis of measured speed distributions.



As for the speed distribution, the following two assumptions are made: first, the "low critical speed" (V_{min}) is defined as that corresponding to a negative lateral acceleration of 0.02 g, which is equivalent to a lateral acceleration of 0.02 g, with an opposite direction to a_c for a car traveling on a straight line with a normal crown of 0.02 cross-fall; second, the "high critical speed" (V_{max}) is defined as that corresponding to the design lateral acceleration increased by one standard deviation (see Table 1). Calculation of these speeds based on the data in Table 1 and Equation 9 is given in Table 2.

The check of the suitability of the speed distribution shown in Table 2 is based on the following procedure. Different sources (7, 8, 9, 10, 11) cite actual distributions of travel speeds along straight sections. From these sources one can determine the values of the speeds corresponding to the following percentiles: 98, 95, 85, 15, 5, and 2. These are shown in Figure 4. Their dependence on the radius value is expressed by comparing the actual travel speed that corresponds to the 98th percentile with the high critical speed (V_{max}) given in Table 2. In other words, the calculated correlation between V_{max} and the curve radius comprises the position of the points representing the actual travel speed corresponding to the 98th percentile; the remaining points, representing the other speed percentiles, are indicated relative to that position. Figure 4 also shows the calculated correlations between the design speed (V_D), the comfort speed (V_o), the low critical speed (V_{min}), and the curve radius.

It is seen from this figure that the speed distribution derived from the superelevation calculations indeed approaches the actual speed distribution.

It should be pointed out that the design speed (V_D) is obtained as the speed corresponding to the 85th-95th percentile, that the comfort speed (V_o) is obtained as the speed corresponding to the 5th-15th percentile, but that the low critical speed (V_{min}) is obtained as the speed corresponding to the percentile below 5. The findings given here are highly significant and strengthen the recommendations in this section.

Still one more check is made to prove that a superelevation of 0.08 is preferable to superelevation rates of 0.10 and 0.06: First, if β^2 values are calculated for a superelevation of 0.06 up to a design speed of 80 km/h, they will be found to be lower than the recommended value of 0.3. This means that application of this superelvation rate does not ensure driving safety at the high travel speeds accompanying the design speeds. Second, it can be proved that V_o accompanying a superelevation of 0.10 equals V_{min} accompanying a superelevation of 0.08. Thus, application of a superelevation of 0.10 would result in negative radial accelerations for a greater proportion of vehicles and would not ensure driving safety at low travel speeds accompanying the design speeds. In conclusion, the application of a superelevation of 0.08 reflects the correct balance between the low superelevation of 0.06 and the high superelevation of 0.10.

CORRELATION BETWEEN RADIUS OF CURVE AND SUPERELEVATION RATE

The previous discussion and the numerical values given in Table 2 relate to the critical case only, where the curve radius assumes the absolute minimum value. The correlation between design speed and lateral acceleration (Equation 11) can serve only that case, since it is based on the average of the lateral acceleration values obtained in Ritchie, McCoy, and Welde's experiments. This average represents the maximum value of lateral acceleration that can still be considered tolerable during driving in a horizontal curve. It thus serves to establish the minimum curve radius. The radii given in Table 2 should, therefore, be considered as the absolute minimum radii for the different design speeds at a constant superelevation rate of 0.08.

Calculation of the superelevation for radii exceeding the absolute minimum is based on the criterion described below.

A road designed according to a design speed must satisfy the common expectation of drivers that the

Table 2. Distribution of speeds as a function of design speed.

Design Speed (V_D) (km/h)	Curve Radius (R) (m)	Low Critical Speed (V_{min}) (km/h)	Comfort Speed (V_0) (km/h)	High Critical Speed (V_{max}) (km/h)
30	25	13.8	15.8	33.9
40	47	18.9	21.5	45.0
50	78	24.4	28.3	56.0
60	122	30.5	35.2	67.5
70	179	36.9	42.7	78.5
80	257	44.3	51.5	89.8
90	358	52.5	60.3	101.0
100	492	61.2	70.7	112.0
110	615	69.3	78.9	122.8
120	751	78.0	87.4	133.6
130	905	87.3	96.0	144.3
140	1080	97.4	104.8	155.0

Table 3. Correlation between a number of factors and $(\beta_{max}/\beta_{min})^2$.

	Δ_0				
$(\beta_{max}/\beta_{min})^2$	V_D = 60 km/h	V_D = 120 km/h	R_{max}/R_{min}	a_{r1}/g	a_{r2}/g
1.0	0	0	4	0	-0.005
1.2	10	19	4.8	-0.003	-0.008
1.4	17	34	5.6	-0.006	-0.009
1.6	25	49	6.4	-0.008	-0.010
1.8	36	64	7.2	-0.010	-0.012

Figure 5. Relation between curve radius and superelevation rate, including absolute minimum radii.

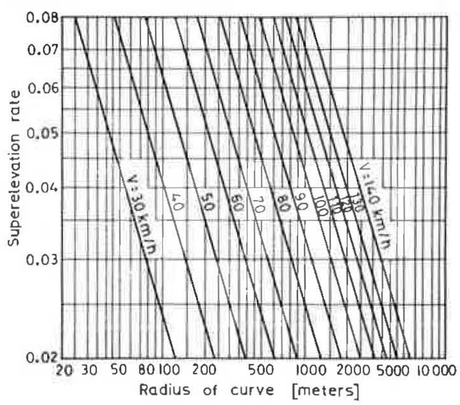
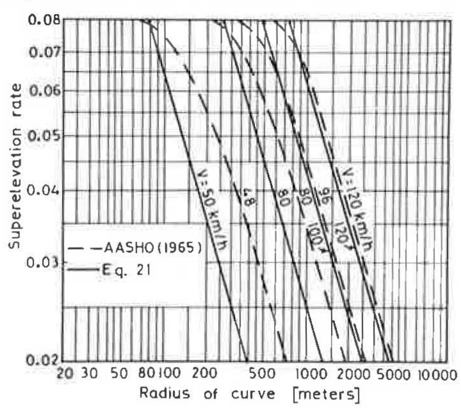


Figure 6. Comparison of curve radius and superelevation rate with AASHO values.



pressing effort on the steering wheel during driving in a curve will decrease with the increasing radius of the curve. The superelevation factor β^2 constitutes a measure of the extent of pressing effort on the wheel: the higher its value, the smaller the effort. An analysis of β^2 values in Table 1 proves that the pressing effort still considered tolerable by the driver population decreases with increasing travel speed in the critical cases, when curve radii are minimum.

Clearly, for a minimum radius corresponding to a given design speed, β^2 is as given in Table 1, but its value has to increase with increasing curve radius, and its maximum value is reached with the maximum radius (R_{max}), defined here as the radius for which the superelevation rate is 0.02. It is obvious that β should not be allowed to increase beyond a given value for the reason that this would lead to an increase in the comfort speed (V_0) and, consequently, in the frequency of occurrence of negative lateral acceleration (pressing the steering wheel in a direction opposite to that of the turn) in vehicles traveling at speeds slower than the design speed.

The implications of the variation of β with the curve radius for a given design speed (denoted as β_r) were examined with respect to (a) the increase in the number of vehicles developing negative lateral acceleration and (b) the value of negative lateral acceleration for the case where the curve radius equals R_{max}. For the purpose of this examination, it is assumed that the cumulative distribution of travel speeds is linear between the values (according to Table 3) of V_0, corresponding to the 15th percentile, and V_D, corresponding to the 85th percentile.

If, then, Δ_0 denotes the increase in the percentage of vehicles that develop negative lateral acceleration, one obtains

$$\Delta_c = [(85-15)/(V_D - V_0)]\ (\beta_r V_D - V_0) \tag{13}$$

After appropriate substitution with $V_0 = \beta_{min} V_D$, the value of Δ_0 is

$$\Delta_c = 70\beta_{min}/(1 - \beta_{min})\ [(\beta_{max}/\beta_{min}) - 1] \tag{14}$$

where β_{min} is the value corresponding to the critical case (Table 1) and β_{max} is the value corresponding to the case where the curve radius equals R_{max} with a superelevation rate of 0.02.

The value of lateral acceleration (a_{r1}) that develops at a travel speed of V_0 in a curve where the radius equals R_{max} with a superelevation of 0.02 is given as

$$a_{r1} = (V_0^2/R_{max}) - 0.02\,g \tag{15}$$

and, by appropriate substitutions, one gets

$$a_{r1} = \{[0.08/4(\beta_{max}/\beta_{min})^2] - 0.02\}\,g \tag{16}$$

The negative lateral acceleration developed for the proportion of vehicles Δ_0 varies from zero to the value of a_{r1} given in Equation 16. It is obvious that, for the proportion of cars traveling at a speed lower than V_0 (about 15 percent), the value of negative lateral acceleration will be larger, reaching the value of a_{r2} at a speed of V_{min}, as defined in Table 3, and as given in the following equation:

$$a_{r2} = \{[0.06/4(\beta_{max}/\beta_{min})^2] - 0.02\}\,g \tag{17}$$

The values of a_{r1}, a_{r2}, Δ_0, and R_{max}/R_{min} are summarized in Table 3 to show their dependence on $(\beta_{max}/\beta_{min})^2$.

Table 3 permits a consideration of the design value for $(\beta_{max}/\beta_{min})^2$. The maximum value of this expression is governed both by the extent of negative acceleration and by the magnitude of the additional proportion (Δ_0) of vehicles expected to acquire negative acceleration. As a reasonable maximum value of negative lateral acceleration (a_{r1}), one can adopt the value of $-0.005\,g$. Thus, it was established that $(\beta_{max}/\beta_{min})^2$ equals 1.25. In addition, it should be pointed out that the sensitivity to the change in superelevation rate with radius for values exceeding 1.25 but less than 1.4 is insignificant. This strengthens the choice of the design value.

The expected additional proportion of vehicles (Δ_0) for which negative lateral acceleration will vary from 0 to $-0.004\,g$ is only about 20 percent.

The correlation between β_r and any radius can be expressed by the following equation:

$$(\beta_r/\beta_{min})^2 = (R/R_{min})^{0.14} \tag{18}$$

For R_{max}/R_{min}, $(\beta_{max}/\beta_{min})^2$ equals 1.25. Under this condition it can be proved that R_{max}/R_{min} equals 5. The correlation can now be found between the superelevation rate and the curve radius for any given design speed.

It can be proved that

$$a_r = \beta_r^2\,V_D^2/R \tag{19}$$

By making the various substitutions, the following results:

$$a_e = 0.08\,g\,(R_{min}/R)^{0.86} \tag{20}$$

The values of β^2_{max} and R_{min} are given in Table 1 as a function of the design speed.

The above correlation between curve radius and the superelevation rate constitutes a straight line on a logarithmic plot; thus

$$\log e = \log 0.08 + 0.86 \log R_{min} - 0.86 \log R \tag{21}$$

This correlation is displayed graphically in Figure 5.

COMPARISON WITH OTHER SOURCES

In order to evaluate the quality and reasonability of the findings derived here, a comparison was made with design guidelines existing in the United States (12) and Germany (13)(RAL-73).

Comparison With AASHO Guidelines

It may be seen from Figure 6 that the superelevation values according to the AASHO guidelines exceed those proposed in the present work up to a design speed of 100 km/h. Above this speed, the values are essentially equal. This fact results in higher negative values of lateral acceleration in slow vehicles traveling on curves designed according to AASHO. Figure 7 presents the correlation between negative lateral acceleration and curve radius for the following design speeds: 48, 80, and 120 km/h, and a traveling speed V_0 (comfort speed) corresponding to each case. The figure indicates two important phenomena: (a) the correlation reaches a peak, and the increase from the minimum radius to the radius corresponding to this peak does not comply with the driver's expectation (i.e., with a desired decrease in pressure on the steering wheel with increasing curve radius) and (b) the values of negative acceleration largely exceed the limiting value of 0.05 m/s² proposed in this work, and they even reach a value of 0.28 m/s² at a design speed of 48 km/h and a value of 0.19 m/s² at a design speed of 120 km/h. This requires high pressure on the wheel, with the pressure itself exerted in the unnatural direction. Thus, the values of negative lateral acceleration calculated from AASHO guidelines may be subject to question.

It is also important to point out that Δ_0 varies from 50 percent for a design speed of 80 km/h to 35 percent for a design speed of 120 km/h. These values of Δ_0 were determined by using Equation 14 and the values of β_{min} and β_{max} calculated from the AASHO data. Comparison of these Δ_0 values with those appearing in Table 3 for $(\beta_{max}/\beta_{min})^2 = 1.25$ shows that the former are greater and that the difference increases with decreasing design speed. This is in agreement with the principle that the lateral acceleration is zero or a higher value for a travel speed equal to the average running speed that corresponds approximately to the 70th percentile, on which the AASHO guidelines are based.

Another point is that the minimum radii of AASHO are significantly smaller than those given here for a design speed of 80 km/h or higher.

Considering the facts presented in this section, it seems reasonable to apply the data developed here in actual practice.

Comparison With RAL-73

Figure 8 shows that the superelevation values according to RAL-73 are essentially similar to those proposed in this work. The differences in the superelevation rates reach a limiting value of 0.005; the RAL-73 superelevation rates for small radii are lower and those for large radii are higher than the values presented here. The advantage of the data in the present work lies in the fact that the radii corresponding to a superelevation rate of 0.08 exceed those obtained from RAL-73. (The RAL-73 values for a superelevation rate of 0.08 are obtained by extending the curves in Figure 8 beyond 0.07, which is the maximum value in the German guidelines.) This fact is mentioned because a maximum superelevation of 0.08 is definitely appropriate for many conditions.

SUMMARY AND CONCLUSIONS

The present work adopts clear criteria for establishing the correlation between superelevation rates and curve radius. These criteria are derived from those factors that contribute to natural and unconstrained driving in a horizontal curve and to the distribution of travel speeds

Figure 7. Comparison of negative lateral acceleration and curve radius with AASHO values at a travel speed of V_o.

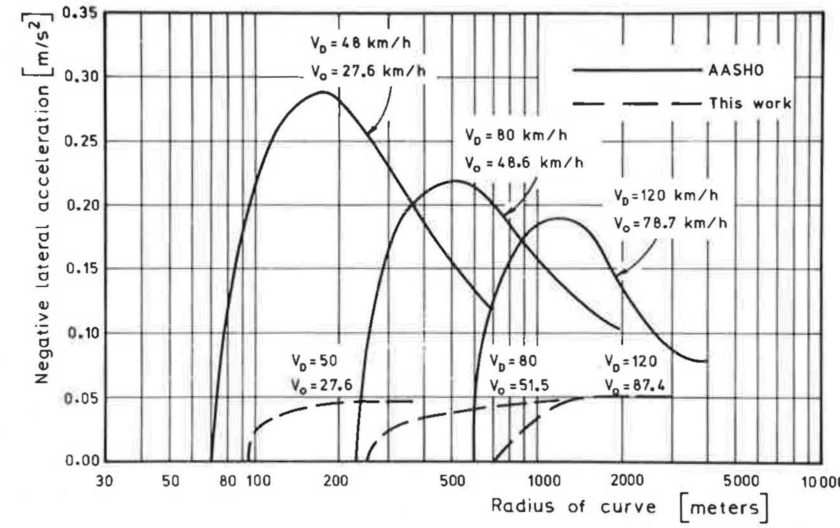
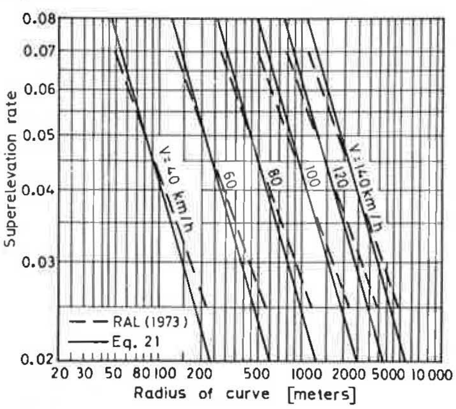


Figure 8. Comparison of curve radius and superelevation rate with RAL-73 values.



accompanying each given design speed.

The critical case, in which the radius is minimum, is defined by the maximum permissible lateral acceleration, which constitutes the physical output of the different input variables of driving parameters connected with the person-vehicle-environment system. The maximum superelevation rate derived from this case must permit natural driving for any actual speed distribution. Thus, the requirement for a slow vehicle is that the negative lateral acceleration, causing pressure on the steering wheel in a direction opposite to that of the turn, shall not exceed 0.2 m/s^2 for about 95 percent of all vehicles.

The correlation between superelevation rates and radius for a constant design speed is defined by the decrease in pressure exerted on the steering wheel with increasing curve radius. This pressure is expressed with the aid of an index, β^2, whose value increases with increasing curve radius. The function for this increase in β^2 also ensures a reasonable negative lateral acceleration in slow vehicles: 0.05 m/s^2 or less for 85 percent of all vehicles.

The results of the present work have been compared with AASHO and RAL-73 guidelines, and it appears that the criteria developed here are reasonable and that the findings are useful for design purposes.

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