Development of a Highway Construction Acceptance Plan

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Acceptance plans are being developed for highway construction inspection that require that the quality of a lot submitted by a contractor for acceptance be estimated by calculating the percentage that meets specification limits. This type of acceptance plan was initially developed in the early 1950s for use in Military Standard 414 for the inspection by variables of government procurement. The theory that underlies such acceptance plans is presented. Tables developed to facilitate the estimation of lot quality from small sample sizes are given. Four cases are considered: (a) Population mean $\bar{X}'$ and population standard deviation σ' are both known, (b) $\bar{X}'$ is known and σ' is unknown, (c) $\bar{X}'$ is unknown and σ' is known, and (d) $\bar{X}'$ and σ' are both unknown. In the fourth case, the one that most often applies in construction situations, two methods of estimation are possible: the range method and the standard deviation method. Although the range method has been used exclusively in highway construction, it is suggested that consideration be given to using the standard deviation method.

This paper presents the development of percentage within specification limits (PWL) tables for acceptance plans that require that an estimate be made of the percentage of submitted material that meets specification limits. One advantage of such acceptance plans is that the estimate of quality that is used is a more meaningful and concise index than either central tendency or central tendency and dispersion, two indexes that have been used almost exclusively in other acceptance plans. Central tendency, of course, tells nothing about the variability of the material and is therefore limited in its application in highway construction to those rare cases in which it can be assumed that the variability is known. Central tendency and dispersion, on the other hand, must be evaluated together in order to adequately describe the material in question; however, any comparison among several lots of different quality requires that these two measures be converted into a single percentage that meets specification limits.

Many highway agencies have been reluctant to adopt the PWL type of acceptance plan. A primary reason appears to be that specification writers do not have at their disposal a clearly defined path through the development of the underlying theory. The basic acceptance tables can be found in Military Standard 414 (1), but this standard presents the end product rather than the developmental rationale that is needed to fully understand the acceptance plans.

The purpose of this paper, therefore, is to summarize the theory that underlies the PWL type of acceptance plan to make it more convenient for those who might be interested in incorporating such a plan into their specifications. This paper should assist specification writers by filling the gaps that presently exist and thereby make it possible for adaptations of Military Standard 414 to highway construction to reach their maximum potential. A complete discussion of these acceptance plans can be found elsewhere (2, 3, 4).

ESTIMATION OF PWL

In estimating the quality of a lot of material, four cases can be considered. These cases are a function of the amount of information that is known or that can be assumed about the lot of material being submitted by a contractor or material supplier for acceptance. These cases may be listed as follows:

1. Population mean $\bar{X}'$ and population standard deviation σ' are known,
2. $\bar{X}'$ is known and σ' is unknown,
3. $\bar{X}'$ is unknown and σ' is known, and
4. $\bar{X}'$ and σ' are both unknown.

Military Standard 414 refers only to cases 3 and 4. Although case 4 is by far the most common in highway construction, the development of an estimate of quality is more complicated in this case because two parameters are unknown. Case 4 can best be understood if the theory is presented in steps starting with the simpler case 1.

Case 1

Case 1 presents no problem in highway construction because if $\bar{X}'$ and σ' are both known there is no need for an acceptance plan. In other words, if one knew the contractor's or material supplier's $\bar{X}'$ and σ', there would be no need to take a sample because the quality of the lot could easily be calculated. Assuming that the random variable (i.e., the quality characteristic) is normally distributed, the percentage meeting the specification limits is simply 100 percent minus the percentage of area under the normal distribution curve that is outside the lower specification limit L or outside the upper specification limit U or both. Thus, for double-limit specifications,

$$PWL' = 100 - 100\left[\int_{-\infty}^{(L-\bar{X}')/\sigma'} f(z)dz\right] - 100\left[\int_{(U-\bar{X}')/\sigma'}^{+\infty} f(z)dz\right] \quad (1)$$

where f(z) = standard normal density = $(1/2\sqrt{\pi})\ \exp(-z^2/2)$ or

$$(PWL'/100) = 1 - \left[\int_{-\infty}^{(L-\bar{X}')/\sigma'} f(z)dz + \int_{(U-\bar{X}')/\sigma'}^{+\infty} f(z)dz\right] \quad (2)$$

Note that in this case a prime appears above the PWL notation to denote a population parameter. In all other cases, the PWL notation without the prime is used.

As an example of the use of Equation 2, it is assumed that a lot of bituminous concrete has a mean asphalt content $\bar{X}'$ = 6.0 percent with a standard deviation σ' = 0.25 percent. If asphalt contents between L = 5.6 percent and U = 6.4 percent meet the specification limits, then Equation 2 can be used to find the actual percentage of the lot that meets specification limits. Thus,

$$PWL'/100 = 1 - \left[\int_{-\infty}^{(5.6-6.0)/0.25} f(z)dz + \int_{(6.4-6.0)/0.25}^{+\infty} f(z)dz\right] \quad (3)$$

Thus, PWL'/100 = 1 - (0.0548 + 0.0548) = 0.8904, or PWL' = 89.04 percent.

Case 2

When $\bar{X}'$ is known but σ' is unknown, sampling inspec-

Table 1. Factors for making unbiased estimates of $\bar{\sigma}$ or $\bar{R}$.

Number of Observations in Subgroup n'	c_2 Factor	d_2 Factor	Number of Observations in Subgroup n'	c_2 Factor	d_2 Factor
2	0.5642	1.128	14	0.9353	3.407
3	0.7236	1.693	15	0.9490	3.472
4	0.7979	2.059	16	0.9523	3.532
5	0.8407	2.326	17	0.9551	3.588
6	0.8686	2.534	18	0.9576	3.640
7	0.8882	2.704	19	0.9599	3.689
8	0.9027	2.847	20	0.9619	3.735
9	0.9139	2.970	25	0.9696	3.931
10	0.9227	3.078	30	0.9748	4.086
11	0.9300	3.173	50	0.9849	4.498
12	0.9359	3.258	100	0.9925	5.015
13	0.9410	3.336			

Table 2. d_2^* factor for various numbers of subgroups of size n'.

Number of Subgroups of Size $n' = 5$	d_2^* Factor	Number of Subgroups of Size $n' = 5$	d_2^* Factor
1	2.474	8	2.346
2	2.405	10	2.342
3	2.379	12	2.339
5	2.358	20	2.334
6	2.353	35	2.331
7	2.349	∞	$2.326 = d_2$

tion is necessary to obtain an estimate of the quality of a lot. Since the only unknown term on the right side of Equation 2 is σ', the first inclination might be to estimate σ' from the sample data and substitute that estimate into Equation 2. It is not that easy, however, because the value of standard deviation or range that would be obtained from a small sample (i.e., $n < 30$) would provide a biased estimate of σ'. It has been shown (5, pp. 350-352) that the sample standard deviation $s = \sqrt{\Sigma (X - \bar{X})^2/(n-1)}$, the root-mean-square deviation $\sigma = \sqrt{\Sigma (X - \bar{X})^2/n}$, and the sample range R are all biased estimators of σ'.

To correct for this bias, one might use a table similar to Table 1 (5, p. 644). In Table 1, c_2 ($\sigma' = \bar{\sigma}/c_2$) is the unbiasing factor associated with the range. To use the table, one must understand that a sample (of size $n > 1$) can be thought of as consisting of one subgroup of size n or several subgroups of size n'. If m equals the number of subgroups, then $n = mn'$. The use of more than one subgroup is sometimes advantageous, especially if the sample range R is used to estimate σ'. The unbiased estimates can be based on calculating σ, s, or R from the entire sample when only one subgroup is available or on calculating $\bar{\sigma}$, $\bar{s}$, or $\bar{R}$ (i.e., the average σ, s, or R obtained from m individual subgroups). The unbiased estimates of σ' can therefore be σ/c_2, $s/c_2\sqrt{n/(n-1)}$, or R/d_2 whenever the sample consists of one subgroup of size n, or they can be $\bar{\sigma}/c_2$, $\bar{s}/c_2\sqrt{n'/(n'-1)}$, or $\bar{R}/d_2$ when m subgroups of size n' are used.

It should be noted that the factor to be used in making estimates from the sample range (R or $\bar{R}$) must be chosen with caution. Although d_2 ($\sigma' = \bar{R}/d_2$) is the correct unbiasing factor, it has been found that for a small number of subgroups (i.e., $m < 20$) a slightly larger factor (d_2^*) will give better precision even though the estimate of σ' will be somewhat biased. Although Military Standard 414 uses the symbol c in place of d_2^*, the d_2^* designation is used by most statisticians and is used in this paper.

Unlike d_2, d_2^* varies with the number of subgroups. Data given in Table 2 (5, p. 93) show the effect of the number of subgroups on d_2^* for a subgroup of size $n' = 5$.

Note in Table 2 that d_2^* becomes essentially a constant (d_2) when the sample contains about 20 or more subgroups.

It should also be noted that, although the c_2 and d_2 factors given in Table 1 correct the bias in the estimate of σ', the mere substitution of an unbiased estimate of σ' in Equation 2 does not result in an unbiased estimate of PWL'. (This can be seen in Equation 1. If the true value of σ' in a certain situation is 3, for instance, the average PWL' obtained by using σ' estimates of 2, 3, and 4 is not equal to the PWL' obtained with $\sigma' = 3$.) Although it is biased, the estimate of PWL' obtained through the substitution for σ' is nonetheless a good estimate. The unbiased estimate of σ' that is preferred for the substitution into Equation 2 is σ/c_2 (or $\bar{\sigma}/c_2$) since σ is the maximum likelihood estimate of σ' (6, p. 257). In the case of one subgroup of size n that represents a particular lot of material, the estimated PWL can thus be obtained by using the following equation, which is analogous to Equation 2:

$$PWL/100 = 1 - \left[\int_{-\infty}^{c_2(L-\bar{X}')\sigma} f(z)dz + \int_{c_2(U-\bar{X}')/\sigma}^{+\infty} f(z)dz \right] \qquad (4)$$

As an example to demonstrate the use of Equation 4, it is assumed that an asphalt content sample of size $n = 5$ taken from a lot that has a known $\bar{X}'$ of 6.0 percent indicates a root-mean-square deviation $\sigma = 0.25$ percent. For a specification that has $L = 5.6$ percent and $U = 6.4$ percent, the estimated quality then becomes

$$PWL/100 = 1 - \left[\int_{-\infty}^{0.8407(5.6-6.0)/0.25} f(z)dz + \int_{0.8407(6.4-6.0)/0.25}^{+\infty} f(z)dz \right] \qquad (5)$$

Thus, $PWL/100 = 1 - (0.0885 + 0.0885) = 0.8230$, or PWL = 82.30 percent.

It should be noted that a sample statistic $c_2 (L - \bar{X}')/\sigma$ or $c_2(U - \bar{X}')/\sigma$ must be calculated to obtain the estimate. This sample statistic follows a normal distribution; however, as will be seen in case 4, not all sample statistics provide this convenience. Further, it should be noted that the Equation 4 estimate is a function of σ (since σ is the only unknown term and is calculated from sample data). As indicated, other estimates are possible—for example, those that are a function of s or R. No matter which estimate is used, however, the only information to be used from the sample data in case 2 is a measure of variability.

Case 3

As in case 2, numerous equations are possible for estimating PWL' when σ' is known. All of these estimates should be based on a sample statistic that is a function of central tendency. The statistic selected for use in Military Standard 414 is $\sqrt{n/(n-1)}\,(L - \bar{X})/\sigma'$ or $\sqrt{n/(n-1)}\,(U - \bar{X})/\sigma'$. Additional information regarding the development of this statistic is available elsewhere (7). As the statistic is developed, the best estimate of PWL' when $\bar{X}'$ is unknown and σ' is known can be expressed as

$$PWL/100 = 1 - \left[\int_{-\infty}^{\sqrt{n/(n-1)}\,(L-\bar{X})/\sigma'} f(z)dz + \int_{\sqrt{n/(n-1)}\,(U-\bar{X})/\sigma'}^{+\infty} f(z)dz \right] \qquad (6)$$

Figure 1. Symmetrical beta distributions ($\alpha = \beta$).

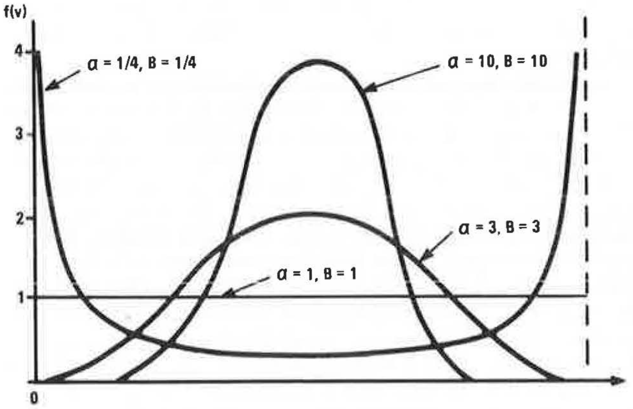
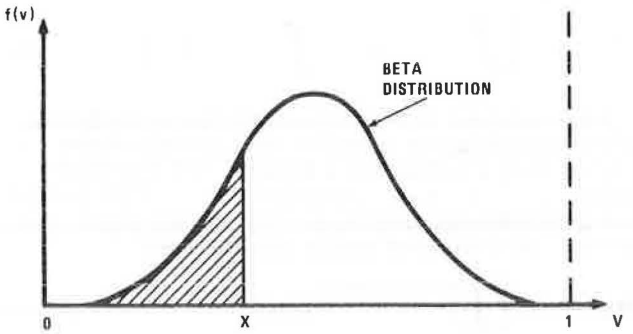


Figure 2. Incomplete beta function ratio.



$$I_x(\alpha, \beta) = \frac{\text{shaded area}}{\text{area under Beta distribution (equals 1)}}$$

Table 3. Incomplete beta function ratio I_x (α, β) for parameters of standard deviation method.

x^a	$\alpha = \beta = 0.5$ (n = 3)	$\alpha = \beta = 1.0$ (n = 4)	$\alpha = \beta = 1.5$ (n = 5)	$\alpha = \beta = 2.0$ (n = 6)	$\alpha = \beta = 2.5$ (n = 7)
0.01	0.063 768 6	0.010 000 0	0.001 692 6	0.000 298 0	0.000 053 7
0.02	0.090 334 5	0.020 000 0	0.004 772 8	0.001 184 0	0.000 300 7
0.03	0.110 824 7	0.030 000 0	0.008 741 4	0.002 646 0	0.000 819 8
0.04	0.128 188 4	0.040 000 0	0.013 417 1	0.004 672 0	0.001 664 5
0.05	0.143 566 3	0.050 000 0	0.018 693 0	0.007 250 0	0.002 875 8
0.06	0.157 542 4	0.060 000 0	0.024 496 3	0.010 368 0	0.004 486 1
0.07	0.170 463 4	0.070 000 0	0.030 772 2	0.014 014 0	0.006 521 8
0.08	0.182 554 9	0.080 000 0	0.037 478 0	0.018 176 0	0.009 004 2
0.09	0.193 973 4	0.090 000 0	0.044 578 4	0.022 842 0	0.011 950 6
0.10	0.204 832 8	0.100 000 0	0.052 044 0	0.028 000 0	0.015 374 7
0.11	0.215 219 0	0.110 000 0	0.059 849 4	0.033 638 0	0.019 287 6
0.12	0.225 198 9	0.120 000 0	0.067 972 4	0.039 744 0	0.023 697 5
0.13	0.234 825 5	0.130 000 0	0.076 393 4	0.046 306 0	0.028 610 3
0.14	0.244 141 8	0.140 000 0	0.085 094 6	0.053 312 0	0.034 029 9
0.15	0.253 183 3	0.150 000 0	0.094 060 2	0.060 750 0	0.039 958 3
0.16	0.261 979 8	0.160 000 0	0.103 275 5	0.068 608 0	0.046 395 9
0.17	0.270 556 3	0.170 000 0	0.112 727 0	0.076 874 0	0.053 341 1
0.18	0.278 934 3	0.180 000 0	0.122 402 3	0.085 536 0	0.060 791 3
0.19	0.287 132 6	0.190 000 0	0.132 289 7	0.094 582 0	0.068 742 2
0.20	0.295 167 2	0.200 000 0	0.142 378 5	0.104 000 0	0.077 188 6
0.21	0.303 052 5	0.210 000 0	0.152 658 3	0.113 778 0	0.086 123 8
0.22	0.310 801 1	0.220 000 0	0.163 119 4	0.123 904 0	0.095 540 2
0.23	0.318 424 2	0.230 000 0	0.173 752 7	0.134 366 0	0.105 429 1
0.24	0.325 931 9	0.240 000 0	0.184 549 4	0.145 152 0	0.115 780 9
0.25	0.333 333 3	0.250 000 0	0.195 501 1	0.156 250 0	0.126 585 0
0.26	0.340 636 7	0.260 000 0	0.206 599 9	0.167 648 0	0.137 830 1
0.27	0.347 849 4	0.270 000 0	0.217 838 1	0.179 334 0	0.149 504 1
0.28	0.354 978 4	0.280 000 0	0.229 208 1	0.191 296 0	0.161 594 0
0.29	0.362 030 1	0.290 000 0	0.240 703 0	0.203 522 0	0.174 086 4
0.30	0.369 010 1	0.300 000 0	0.252 315 8	0.216 000 0	0.186 967 0
0.31	0.375 924 0	0.310 000 0	0.264 039 7	0.228 718 0	0.200 220 9
0.32	0.382 776 7	0.320 000 0	0.275 868 2	0.241 664 0	0.213 832 8
0.33	0.389 572 9	0.330 000 0	0.287 795 0	0.254 826 0	0.227 786 8
0.34	0.396 317 1	0.340 000 0	0.299 813 9	0.268 192 0	0.242 066 4
0.35	0.403 013 3	0.350 000 0	0.311 918 8	0.281 750 0	0.256 654 8
0.36	0.409 665 5	0.360 000 0	0.324 103 8	0.295 488 0	0.271 534 7
0.37	0.416 277 4	0.370 000 0	0.336 363 1	0.309 394 0	0.286 688 4
0.38	0.422 852 6	0.380 000 0	0.348 691 0	0.323 456 0	0.302 097 7
0.39	0.429 394 3	0.390 000 0	0.361 081 8	0.337 662 0	0.317 744 4
0.40	0.435 905 8	0.400 000 0	0.373 530 0	0.352 000 0	0.333 609 6
0.41	0.442 390 2	0.410 000 0	0.386 030 3	0.366 458 0	0.349 674 4
0.42	0.448 850 6	0.420 000 0	0.398 577 1	0.381 024 0	0.365 919 5
0.43	0.455 289 7	0.430 000 0	0.411 165 2	0.395 686 0	0.382 325 5
0.44	0.461 710 5	0.440 000 0	0.423 789 4	0.410 432 0	0.398 872 6
0.45	0.468 115 7	0.450 000 0	0.436 444 3	0.425 250 0	0.415 541 1
0.46	0.474 508 0	0.460 000 0	0.449 124 8	0.440 128 0	0.432 311 0
0.47	0.480 889 9	0.470 000 0	0.461 825 7	0.455 054 0	0.449 162 0
0.48	0.487 264 2	0.480 000 0	0.474 542 0	0.470 016 0	0.466 074 1
0.49	0.493 633 4	0.490 000 0	0.487 268 5	0.485 002 0	0.483 026 9
0.50	0.500 000 0	0.500 000 0	0.500 000 0	0.500 000 0	0.500 000 0

[a]The value I_x (α,β) for x greater than 0.50 is the complement of that for 1 - x. For example, when $\alpha = \beta = 2.5$, the value of I_x (α,β) for 0.61 is obtained by subtracting the value 0.317 744 4 for 0.39 from 1; i.e., 1 - 0.317 744 4 = 0.682 255 6.

where PWL is an estimate that is a function of the sample mean $\bar{X}$.

Equation 6 is very similar to Equation 2, the most obvious difference being the $\sqrt{n/(n-1)}$ factor, which is introduced in Equation 6 because $\bar{X}'$ is not known but is estimated by $\bar{X}$. The larger the sample, the better is the estimate of $\bar{X}$. Hence, as n approaches ∞, $\sqrt{n/(n-1)}$ tends to become 1. It may also be noted that Equation 6 is also similar to Equation 4.

As an example to show the use of Equation 6, it is assumed that an asphalt content sample of size n = 5, taken from a lot that has a known σ' of 0.25 percent, shows a sample mean $\bar{X}$ = 6.0 percent. For a specification that has L = 5.6 percent and U = 6.4 percent, the estimated quality then becomes

$$PWL/100 = 1 - \left[\int_{-\infty}^{1.118(5.6-6.0)/0.25} f(z)dz + \int_{1.118(6.4-6.0)/0.25}^{+\infty} f(z)dz\right] \quad (7)$$

Thus, PWL/100 = 1 - (0.0367 + 0.0367) = 0.9266 or PWL = 92.66 percent.

Case 4

The above discussion has set the pattern for case 4. As in cases 2 and 3, to obtain an estimate of PWL′ a sample of size n > 1 must be taken on which measurements of a quality characteristic are made. A statistic that is known to follow a certain distribution is then computed. The estimate of PWL′ can then be obtained by finding the appropriate area under the distribution being considered.

In accordance with the procedure outlined in Military Standard 414, two methods of estimating quality are presented for case 4: (a) the standard deviation method and (b) the range method. It is important to realize before these two methods are discussed that the normal distribution cannot be used in case 4 since matters have become more complicated now that $\bar{X}'$ and σ' are both unknown. As developed elsewhere (7), the sample statistics that are used to provide the best estimate of PWL′ in this case follow a symmetrical beta distribution. An explanation of the reason for using the beta distribution can be found in a paper by Lieberman and Resnikoff (8). The discussion that follows will provide a brief introduction to the beta distribution and will also provide a table of this distribution, which is often difficult to obtain.

A random variable v is said to be distributed as the beta distribution if the density function is given by

$$f(v) = [\Gamma(\alpha+\beta)/\Gamma(\alpha)\Gamma(\beta)]\, v^{\alpha-1}(1-v)^{\beta-1} \qquad 0 \leq v \leq 1 \quad (8)$$

with parameters α and β, both of which are positive constants. When α is equal to β, the distribution is symmetric as shown in Figure 1.

Figure 3. Representation of the estimate of PWL′ using double specification limits when PWL is a function of $\bar{X}$ and s ($\bar{X}'$ and σ' unknown).

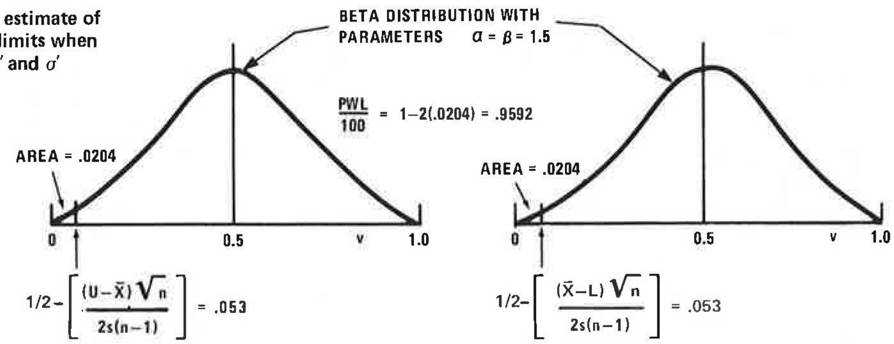


Table 4. Values of d_2^* and ν for use in range method estimate of case 4.

Subgroups (m)	Factor	Size of Subgroups (n′) 2	3	4	5	6	7	8	9	10	15
1	d_2^*	1.41	1.91	2.24	2.48	2.67	2.83	2.96	3.08	3.18	3.55
	ν	1.00	1.98	2.93	3.83	4.68	5.48	6.25	6.98	7.68	10.8
2	d_2^*	1.28	1.81	2.15	2.40	2.60	2.77	2.91	3.02	3.13	3.51
	ν	1.92	3.83	5.69	7.47	9.16	10.8	12.3	13.8	15.1	21.3
3	d_2^*	1.23	1.77	2.12	2.38	2.58	2.75	2.89	3.01	3.11	3.50
	ν	2.82	5.66	8.44	11.1	13.6	16.0	18.3	20.5	22.6	31.9
4	d_2^*	1.21	1.75	2.11	2.37	2.57	2.74	2.88	3.00	3.10	3.49
	ν	3.71	7.49	11.2	14.7	18.1	21.3	24.4	27.3	30.1	42.4
5	d_2^*	1.19	1.74	2.10	2.36	2.56	2.73	2.87	2.99	3.10	3.49
	ν	4.59	9.31	13.9	18.4	22.6	26.6	30.4	34.0	37.5	52.9

Γ in Equation 8 is the symbol for a gamma function. The gamma function $\Gamma(A)$ is defined by

$$\Gamma(A) = \int_0^\infty x^{A-1} e^{-x} dx \qquad A > 0 \tag{9}$$

It can be shown—as, for example, by Miller (9)—that if A is a positive integer, $\Gamma(A) = (A-1)!$. If B is a positive half-integer greater than 1 (i.e., 1.5, 2.5, 3.5, and so on), one may write B = m + 0.5 where m is an integer, and it can be shown that $\Gamma(B) = (B-1), (B-2), (B-3), \ldots, (B-m)\ \Gamma(0.5)$ where $\Gamma(0.5) = \pi$.

In working with the beta distribution, the incomplete beta function ratio is normally used. The incomplete beta function ratio $I_x(\alpha, \beta)$ is defined by

$$I_x(\alpha,\beta) = [\Gamma(\alpha+\beta)/\Gamma(\alpha)\Gamma(\beta)] \int_0^x v^{\alpha-1}(1-v)^{\beta-1} dv \tag{10}$$

As Figure 2 shows, the incomplete beta function ratio gives an area under the beta distribution from v = 0 to v = x.

$I_x(\alpha, \beta)$ has been tabulated by Pearson (10) for α and β values of integers and half-integers less than or equal to 50. Although Pearson's tables are extensive, only values of $\alpha = \beta$ are required to solve the equations that apply for case 4. It will be shown below that the parameters α and β of the beta distributions developed for the standard deviation method are (n/2) - 1; therefore, α and β are always half-integers for that method.

Table 3 (10) is a table of the incomplete beta function ratio for the standard deviation method parameters. Table 3 was obtained from Pearson's tables. Only those beta distributions that are required for n = 3 through n = 7 are tabulated.

Standard Deviation Method

The equation for estimating PWL′ by using the standard deviation method of case 4 is

$$PWL/100 = 1 - \left(\int_0^{\max\{0,(1/2) - [(U-\bar{X})\sqrt{n}/2s(n-1)]\}} d\beta[(n/2)-1] + \int_0^{\max\{0,(1/2) - [(\bar{X}-L)\sqrt{n}/2s(n-1)]\}} d\beta[(n/2)-1] \right) \tag{11}$$

where PWL is an estimate that is a function of both $\bar{X}$ and s and $d\beta\,[(n/2)-1]$ is a symmetrical beta density function with parameters α and β both equal to $[(n/2)-1]$.

A symmetrical beta distribution that has parameters α and β greater than 1 (i.e., n > 4) is similar in appearance to the normal distribution (Figure 1); however, whereas the normal random variable z is continuous over an infinite range, the beta random variable v is continuous over a range from 0 to 1. Figure 3 shows the estimate of PWL′ obtained by means of Equation 11 for the case of an asphalt content sample of size n = 5 that yields a sample mean $\bar{X}$ = 6.0 percent and a sample standard deviation s = 0.25 percent for a specification that has L = 5.6 percent and U = 6.4 percent. This estimate is

$$PWL/100 = 1 - \left[\int_0^{\max(0,0.053)} d\beta(1.5) + \int_0^{\max(0,0.053)} d\beta(1.5) \right] \tag{12}$$

Thus, PWL/100 = 1 - (0.0204 + 0.0204) = 0.9592, or PWL = 95.92 percent.

Range Method

The equation for estimating PWL′ by using the range method of case 4 is

$$PWL/100 = 1 - \left(\int_0^{\max\{0,(1/2)-[d_2^*(U-\bar{X})\sqrt{\nu+1}/2\bar{R}\nu]\}} d\beta\{[(\nu+1)/2]-1\} + \int_0^{\max\{0,(1/2)-[d_2^*(\bar{X}-L)\sqrt{\nu+1}/2\bar{R}\nu]\}} \times d\beta\{[(\nu+1)/2]-1\} \right) \tag{13}$$

where PWL is an estimate that is a function of both $\overline{X}$ and $\overline{R}$, $d\beta\ \{[(\nu + 1)/2] - 1\}$ is a beta density function with parameters of α and β both equal to $[(\nu + 1)/2] - 1$, and

d_2^* = factor from Table 2 or Table 4;
ν = degrees of freedom [see Table 4, modified from Nelson (11)]; and
$\overline{R}$ = average range of subgroups ($\overline{R}$ = R where only one subgroup is used).

Table 5. Incomplete beta function ratio I_x (α, β) for parameters of range method.

x^a	$\alpha = \beta = 0.467$ (n = 3)	$\alpha = \beta = 0.998$ (n = 4)	$\alpha = \beta = 1.414$ (n = 5)	$\alpha = \beta = 1.84$ (n = 6)	$\alpha = \beta = 2.25$ (n = 7)
0.01	0.072 396 8	0.010 072 5	0.002 289 5	0.000 517 9	0.000 126 3
0.02	0.100 241 0	0.020 117 2	0.006 086 0	0.001 843 9	0.000 595 3
0.03	0.121 347 6	0.030 151 7	0.010 771 1	0.003 866 8	0.001 469 4
0.04	0.139 038 2	0.040 179 6	0.016 137 0	0.006 520 6	0.002 782 2
0.05	0.154 580 0	0.050 202 5	0.022 069 3	0.009 788 1	0.004 555 8
0.06	0.168 616 2	0.060 221 7	0.028 487 8	0.013 612 5	0.006 804 9
0.07	0.181 525 6	0.070 237 6	0.035 336 2	0.017 974 1	0.009 539 4
0.08	0.193 553 5	0.080 251 1	0.042 571 0	0.022 849 0	0.012 765 7
0.09	0.204 868 9	0.090 262 2	0.050 156 5	0.028 215 2	0.016 487 4
0.10	0.215 594 7	0.100 271 2	0.058 063 7	0.034 052 9	0.020 705 7
0.11	0.225 823 0	0.110 278 5	0.066 267 7	0.040 343 7	0.025 420 3
0.12	0.235 625 3	0.120 284 2	0.074 747 3	0.047 070 4	0.030 628 9
0.13	0.245 058 1	0.120 388 4	0.083 483 6	0.054 216 6	0.036 328 1
0.14	0.254 107 2	0.140 291 3	0.092 460 0	0.061 766 9	0.042 513 3
0.15	0.202 990 1	0.150 293 1	0.101 661 6	0.069 706 4	0.049 178 7
0.16	0.271 558 4	0.160 293 8	0.111 074 9	0.078 020 9	0.056 318 0
0.17	0.279 898 4	0.170 293 4	0.120 687 5	0.086 696 6	0.063 923 4
0.18	0.288 032 9	0.180 292 1	0.130 488 3	0.095 720 1	0.071 986 9
0.19	0.295 981 3	0.190 289 9	0.140 466 6	0.105 078 4	0.080 499 8
0.20	0.303 761 0	0.200 286 9	0.150 612 9	0.114 759 0	0.089 452 3
0.21	0.311 386 4	0.210 283 2	0.160 918 0	0.124 749 5	0.098 834 7
0.22	0.318 871 0	0.220 278 7	0.171 373 4	0.135 037 8	0.108 636 3
0.23	0.326 226 5	0.230 273 7	0.181 971 1	0.145 612 1	0.118 846 1
0.24	0.333 463 3	0.240 268 0	0.192 703 5	0.156 460 9	0.129 452 7
0.25	0.340 591 0	0.250 261 7	0.203 563 3	0.167 572 7	0.140 444 0
0.26	0.347 618 0	0.260 254 9	0.214 543 6	0.178 936 2	0.151 808 1
0.27	0.354 552 2	0.270 247 6	0.225 638 1	0.190 540 5	0.163 532 1
0.28	0.361 400 5	0.280 239 8	0.236 840 3	0.202 374 5	0.175 603 3
0.29	0.368 169 7	0.290 231 7	0.248 144 2	0.214 427 6	0.188 008 4
0.30	0.374 865 6	0.300 223 0	0.259 544 0	0.226 689 0	0.200 734 0
0.31	0.381 493 9	0.310 214 0	0.271 033 9	0.239 148 2	0.213 766 2
0.32	0.388 059 7	0.320 204 7	0.282 608 9	0.251 794 8	0.227 091 1
0.33	0.394 567 9	0.330 195 1	0.294 263 4	0.264 618 4	0.240 694 5
0.34	0.401 022 9	0.340 185 1	0.305 992 3	0.277 608 7	0.254 561 9
0.35	0.407 428 9	0.350 174 8	0.317 790 7	0.290 755 6	0.268 678 8
0.36	0.413 790 1	0.360 164 3	0.329 653 6	0.304 049 0	0.283 030 3
0.37	0.420 110 3	0.370 153 5	0.341 576 5	0.317 478 8	0.297 601 5
0.38	0.426 392 9	0.380 142 5	0.353 554 5	0.331 035 0	0.312 377 3
0.39	0.432 641 5	0.390 131 2	0.365 583 1	0.344 707 7	0.327 342 6
0.40	0.438 859 4	0.400 119 9	0.377 658 0	0.358 487 2	0.342 481 9
0.41	0.445 049 8	0.410 108 3	0.389 774 6	0.372 363 3	0.357 779 6
0.42	0.451 215 6	0.420 096 6	0.401 928 4	0.386 326 6	0.373 220 4
0.43	0.457 359 9	0.430 084 7	0.414 115 4	0.400 367 0	0.388 788 5
0.44	0.463 485 4	0.440 072 9	0.426 331 3	0.414 475 0	0.404 468 1
0.45	0.469 595 2	0.450 060 8	0.438 571 9	0.428 640 8	0.420 243 4
0.46	0.475 691 7	0.460 048 7	0.450 832 9	0.442 854 8	0.436 098 7
0.47	0.481 777 7	0.470 036 5	0.463 110 3	0.457 107 3	0.452 017 9
0.48	0.487 856 0	0.480 024 2	0.475 400 0	0.471 388 8	0.467 985 2
0.49	0.493 929 2	0.490 011 9	0.487 697 8	0.485 689 5	0.483 984 5
0.50	0.500 000 0	0.500 000 0	0.500 000 0	0.500 000 0	0.500 000 0

[a]The value I_x (α,β) for x greater than 0.50 is the complement of that for 1 - x. For example, when $\alpha = \beta = 2.25$, the value I_x (α,β) for 0.61 is obtained by subtracting the value 0.327 342 6 for 0.39 from 1; i.e., 1 - 0.327 342 6 = 0.672 657 4.

Table 5 (3, pp. 103-105), a table of the incomplete beta function ratio for some commonly used parameters of the range method, can now be used to solve the following example. If an asphalt content sample of size n = 5 yields a sample mean $\overline{X}$ = 6.0 percent and a sample range R = 0.6 percent for a specification that has L = 5.6 percent and U = 6.4 percent, then an estimate of PWL′ can be computed from Equation 13. Using ν = 3.828 and d_2^* = 2.474, the estimate becomes

$$PWL/100 = 1 - \left[\int_0^{\max(0,0.027)} d\beta(1.414) + \int_0^{\max(0,0.027)} d\beta(1.414)\right] \quad (14)$$

Thus, PWL/100 = 1 - (0.0094 + 0.0094) = 0.9812, or PWL = 98.12 percent. This estimate is shown in Figure 4.

Equations 11 and 13 can now be used to develop tables that will simplify the estimating process. These tables are based on the fact that the PWL′ estimate for case 4 is constant for a given sample size n and given values of either $(U - \overline{X})/s$ and $(\overline{X} - L)/s$ for Equation 11 or $(U - \overline{X})/R$ and $(\overline{X} - L)/R$ for Equation 13. If it is designated that $Q_U = (U - \overline{X})/s$ and $Q_L = (\overline{X} - L)/s$ in Equation 11 and $Q_U = (U - \overline{X})/R$ and $Q_L = (\overline{X} - L)/R$ in Equation 13, then tables such as Table 6 (3, pp. 68-69), for the Equation 11 standard deviation method, and Table 7 (3, pp. 56-57), for the Equation 13 range method, can be developed.

Tables 6 and 7 are different from those that are currently used by state highway agencies that have the PWL type of acceptance plans. First, to avoid potential problems of interpretation, the tables are accurate to four decimal places (the tables commonly used by state highway agencies are accurate to two decimal places and may result in two different estimates from the same Q_U or Q_L value). Second, the only tables that have until now been readily available to state highway agencies are tables based on the range method. The biggest advantage of the range method is the ease of calculating R from the sample data. The advent of pocket calculators and computer programs developed to determine the contractor's payment is, however, increasing the attractiveness of the standard deviation method, which requires the calculation of s from the sample data. The two methods may give slightly different estimates of PWL′; the standard deviation estimate is the more accurate. For this reason, and because a smaller sample size can be used to achieve the same accuracy, it is recommended that highway agencies consider using the standard deviation method and Table 6.

Figure 4. Representation of the estimate of PWL′ using double specification limits when PWL is a function of $\overline{X}$ and $\overline{R}$ ($\overline{X}'$ and σ' unknown).

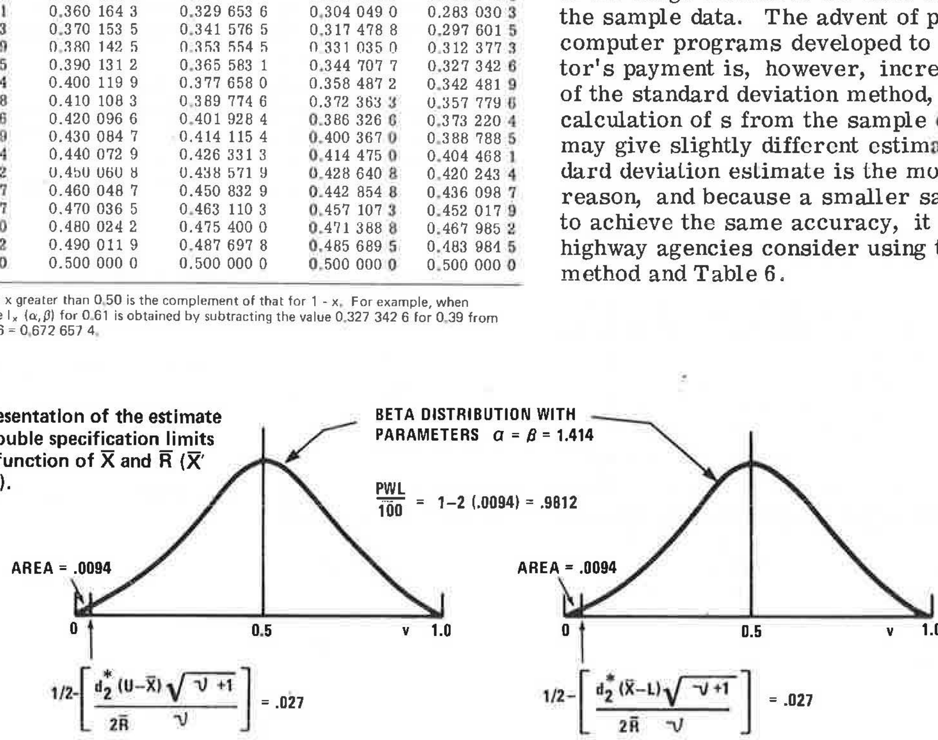


Table 6. Estimation of percentage within specification limits by standard deviation method.

	Negative Values of Q_U or Q_L						Positive Values of Q_U or Q_L				
PWL	n = 3	n = 4	n = 5	n = 6	n = 7	PWL	n = 3	n = 4	n = 5	n = 6	n = 7
50	0.0000	0.0000	0.0000	0.0000	0.0000	99	1.1510	1.4701	1.6719	1.8016	1.8893
45	0.1806	0.1500	0.1406	0.1364	0.1338	98	1.1476	1.4401	1.6018	1.6990	1.7615
40	0.3568	0.3000	0.2823	0.2740	0.2689	97	1.1439	1.4101	1.5428	1.6190	1.6662
39	0.3912	0.3300	0.3106	0.3018	0.2966	96	1.1402	1.3801	1.4898	1.5500	1.5868
38	0.4252	0.3600	0.3392	0.3295	0.3238	95	1.1367	1.3501	1.4408	1.4892	1.5184
37	0.4587	0.3900	0.3678	0.3577	0.3515	94	1.1330	1.3201	1.3946	1.4332	1.4562
36	0.4917	0.4200	0.3968	0.3859	0.3791	93	1.1263	1.2901	1.3510	1.3813	1.3990
35	0.5242	0.4500	0.4254	0.4140	0.4073	92	1.1170	1.2601	1.3091	1.3328	1.3465
34	0.5564	0.4800	0.4544	0.4426	0.4354	91	1.1087	1.2301	1.2683	1.2866	1.2966
33	0.5878	0.5101	0.4837	0.4712	0.4639	90	1.0977	1.2001	1.2293	1.2421	1.2494
32	0.6187	0.5401	0.5131	0.5002	0.4925	89	1.0864	1.1701	1.1911	1.2001	1.2045
31	0.6490	0.5701	0.5424	0.5292	0.5211	88	1.0732	1.1401	1.1538	1.1592	1.1615
30	0.6788	0.6001	0.5717	0.5586	0.5506	87	1.0596	1.1101	1.1174	1.1196	1.1202
29	0.7076	0.6301	0.6018	0.5880	0.5846	86	1.0446	1.0801	1.0819	1.0813	1.0798
28	0.7360	0.6601	0.6315	0.6178	0.6095	85	1.0286	1.0501	1.0469	1.0437	1.0413
27	0.7635	0.6901	0.6619	0.6480	0.6395	84	1.0118	1.0201	1.0125	1.0073	1.0032
26	0.7905	0.7201	0.6919	0.6782	0.6703	83	0.9940	0.9901	0.9782	0.9718	0.9673
25	0.8164	0.7501	0.7227	0.7093	0.7011	82	0.9748	0.9601	0.9453	0.9367	0.9315
24	0.8416	0.7801	0.7535	0.7403	0.7320	81	0.9555	0.9301	0.9123	0.9028	0.8966
23	0.8661	0.8101	0.7846	0.7717	0.7642	80	0.9342	0.9001	0.8798	0.8693	0.8626
22	0.8896	0.8401	0.8161	0.8040	0.7964	79	0.9122	0.8701	0.8479	0.8363	0.8290
21	0.9122	0.8701	0.8479	0.8363	0.8290	78	0.8896	0.8401	0.8161	0.8040	0.7964
20	0.9342	0.9001	0.8798	0.8693	0.8626	77	0.8661	0.8101	0.7846	0.7717	0.7642
19	0.9555	0.9301	0.9123	0.9028	0.8966	76	0.8416	0.7801	0.7535	0.7403	0.7320
18	0.9748	0.9601	0.9453	0.9367	0.9315	75	0.8164	0.7501	0.7227	0.7093	0.7011
17	0.9940	0.9901	0.9782	0.9718	0.9673	74	0.7905	0.7201	0.6919	0.6782	0.6703
16	1.0118	1.0201	1.0125	1.0073	1.0032	73	0.7635	0.6901	0.6619	0.6480	0.6395
15	1.0286	1.0501	1.0469	1.0437	1.0413	72	0.7360	0.6601	0.6315	0.6178	0.6095
14	1.0446	1.0801	1.0819	1.0813	1.0798	71	0.7076	0.6301	0.6018	0.5880	0.5846
13	1.0597	1.1101	1.1174	1.1196	1.1202	70	0.6788	0.6001	0.5717	0.5586	0.5506
12	1.0732	1.1401	1.1538	1.1592	1.1615	69	0.6490	0.5701	0.5424	0.5292	0.5211
11	1.0864	1.1701	1.1911	1.2001	1.2045	68	0.6187	0.5401	0.5131	0.5002	0.4925
10	1.0977	1.2001	1.2293	1.2421	1.2494	67	0.5878	0.5101	0.4837	0.4712	0.4639
9	1.1087	1.2301	1.2683	1.2866	1.2966	66	0.5564	0.4800	0.4544	0.4426	0.4354
8	1.1170	1.2601	1.3091	1.3328	1.3465	65	0.5242	0.4500	0.4254	0.4140	0.4073
7	1.1263	1.2901	1.3510	1.3813	1.3990	64	0.4917	0.4200	0.3968	0.3859	0.3791
6	1.1330	1.3201	1.3946	1.4332	1.4562	63	0.4587	0.3900	0.3678	0.3577	0.3515
5	1.1367	1.3501	1.4408	1.4892	1.5184	62	0.4252	0.3600	0.3392	0.3295	0.3238
4	1.1402	1.3801	1.4898	1.5500	1.5868	61	0.3912	0.3300	0.3106	0.3018	0.2966
3	1.1439	1.4101	1.5428	1.6190	1.6662	60	0.3568	0.3000	0.2823	0.2740	0.2689
2	1.1476	1.4401	1.6018	1.6990	1.7615	55	0.1806	0.1500	0.1406	0.1364	0.1338
1	1.1510	1.4701	1.6719	1.8016	1.8893	50	0.0000	0.0000	0.0000	0.0000	0.0000

Table 7. Estimation of percentage within specification limits by range method.

	Negative Values of Q_U or Q_L						Positive Values of Q_U or Q_L				
PWL	n = 3	n = 4	n = 5	n = 6	n = 7	PWL	n = 3	n = 4	n = 5	n = 6	n = 7
50	0.0000	0.0000	0.0000	0.0000	0.0000	99	0.5895	0.6574	0.6642	0.6611	0.6534
45	0.0970	0.0672	0.0573	0.0515	0.0477	98	0.5879	0.6440	0.6387	0.6264	0.6124
40	0.1911	0.1343	0.1149	0.1034	0.0957	97	0.5863	0.6307	0.6166	0.5983	0.5811
39	0.2093	0.1477	0.1265	0.1139	0.1055	96	0.5847	0.6173	0.5966	0.5744	0.5550
38	0.2274	0.1611	0.1382	0.1243	0.1152	95	0.5830	0.6039	0.5777	0.5530	0.5319
37	0.2451	0.1747	0.1497	0.1349	0.1252	94	0.5814	0.5905	0.5600	0.5330	0.5110
36	0.2625	0.1881	0.1614	0.1455	0.1351	93	0.5797	0.5771	0.5431	0.5143	0.4916
35	0.2798	0.2015	0.1732	0.1562	0.1450	92	0.5762	0.5638	0.5267	0.4968	0.4735
34	0.2965	0.2149	0.1835	0.1668	0.1549	91	0.5719	0.5504	0.5108	0.4800	0.4564
33	0.3131	0.2283	0.1968	0.1777	0.1649	90	0.5677	0.5370	0.4955	0.4640	0.4402
32	0.3293	0.2417	0.2086	0.1884	0.1752	89	0.5621	0.5236	0.4808	0.4485	0.4249
31	0.3450	0.2551	0.2206	0.1995	0.1854	88	0.5564	0.5101	0.4657	0.4337	0.4099
30	0.3604	0.2685	0.2325	0.2104	0.1957	87	0.5499	0.4967	0.4514	0.4191	0.3957
29	0.3754	0.2820	0.2446	0.2215	0.2061	86	0.5432	0.4833	0.4373	0.4050	0.3817
28	0.3901	0.2954	0.2567	0.2327	0.2166	85	0.5355	0.4699	0.4234	0.3913	0.3683
27	0.4041	0.3088	0.2689	0.2440	0.2273	84	0.5275	0.4565	0.4097	0.3778	0.3552
26	0.4179	0.3223	0.2811	0.2554	0.2380	83	0.5189	0.4431	0.3962	0.3647	0.3424
25	0.4311	0.3358	0.2935	0.2669	0.2489	82	0.5098	0.4297	0.3829	0.3517	0.3300
24	0.4439	0.3492	0.3059	0.2785	0.2599	81	0.5001	0.4162	0.3697	0.3391	0.3177
23	0.4560	0.3626	0.3184	0.2902	0.2712	80	0.4889	0.4028	0.3567	0.3266	0.3058
22	0.4679	0.3760	0.3311	0.3023	0.2825	79	0.4791	0.3894	0.3438	0.3144	0.2941
21	0.4791	0.3894	0.3438	0.3144	0.2941	78	0.4679	0.3760	0.3311	0.3023	0.2825
20	0.4899	0.4028	0.3567	0.3266	0.3058	77	0.4560	0.3626	0.3184	0.2902	0.2712
19	0.5001	0.4162	0.3697	0.3391	0.3177	76	0.4439	0.3492	0.3059	0.2785	0.2599
18	0.5098	0.4297	0.3829	0.3517	0.3300	75	0.4311	0.3358	0.2935	0.2669	0.2489
17	0.5189	0.4431	0.3962	0.3647	0.3424	74	0.4179	0.3223	0.2811	0.2554	0.2380
16	0.5275	0.4565	0.4097	0.3778	0.3552	73	0.4041	0.3088	0.2689	0.2440	0.2273
15	0.5355	0.4699	0.4234	0.3913	0.3683	72	0.3901	0.2954	0.2567	0.2327	0.2166
14	0.5432	0.4833	0.4373	0.4050	0.3817	71	0.3754	0.2820	0.2446	0.2215	0.2061
13	0.5499	0.4967	0.4514	0.4191	0.3957	70	0.3604	0.2685	0.2325	0.2104	0.1957
12	0.5564	0.5101	0.4657	0.4337	0.4099	69	0.3450	0.2551	0.2206	0.1995	0.1854
11	0.5621	0.5236	0.4808	0.4485	0.4249	68	0.3293	0.2417	0.2086	0.1884	0.1752
10	0.5677	0.5370	0.4955	0.4640	0.4402	67	0.3131	0.2283	0.1968	0.1777	0.1649
9	0.5719	0.5504	0.5108	0.4800	0.4564	66	0.2965	0.2149	0.1835	0.1668	0.1549
8	0.5762	0.5638	0.5267	0.4968	0.4735	65	0.2798	0.2015	0.1732	0.1562	0.1450
7	0.5797	0.5771	0.5431	0.5143	0.4916	64	0.2625	0.1881	0.1614	0.1455	0.1351
6	0.5814	0.5905	0.5600	0.5330	0.5110	63	0.2451	0.1747	0.1497	0.1349	0.1252
5	0.5830	0.6039	0.5777	0.5530	0.5319	62	0.2274	0.1611	0.1382	0.1243	0.1152
4	0.5847	0.6173	0.5966	0.5744	0.5550	61	0.2093	0.1477	0.1265	0.1139	0.1055
3	0.5863	0.6307	0.6166	0.5983	0.5811	60	0.1911	0.1343	0.1149	0.1034	0.0957
2	0.5879	0.6440	0.6387	0.6264	0.6124	55	0.0970	0.0672	0.0573	0.0515	0.0477
1	0.5895	0.6574	0.6642	0.6611	0.6534	50	0.0000	0.0000	0.0000	0.0000	0.0000

SUMMARY

The complete development of a PWL type of acceptance plan is founded on complex statistical theory. It is not necessary to understand the theory to use a PWL acceptance plan since estimation tables can easily be modified from Military Standard 414. However, if flexibility in adapting the standard to highway construction specifications is desired, a knowledge of the underlying theory is certainly helpful. Although one adaptation of Military Standard 414 plans—the range method—has gained a foothold in statistically based highway construction specifications, we believe that PWL plans are not being used to their fullest potential. It is hoped that the summary presented in this paper of the basic theory that underlies PWL acceptance plans will better equip highway agencies to develop acceptance plans specifically suited to their needs.

ACKNOWLEDGMENT

The contents of this paper reflect our views, and we are responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Pennsylvania Department of Transportation or the Federal Highway Administration. This paper does not constitute a standard, specification, or regulation.

REFERENCES

1. Military Standard 414—Sampling Procedures and Tables for Inspection by Variables for Percent Defective. U.S. Government Printing Office, 1957.
2. J. H. Willenbrock and P. A. Kopac. A Methodology for the Development of Price Adjustment Systems for Statistically Based Restricted Performance Specifications. Pennsylvania Transportation Institute, Pennsylvania State Univ., Oct. 1976.
3. J. H. Willenbrock and P. A. Kopac. The Development of Tables for Estimating Percentage of Material Within Specification Limits. Pennsylvania Transportation Institute, Pennsylvania State Univ., Oct. 1976.
4. J. H. Willenbrock and P. A. Kopac. The Development of Operating Characteristic Curves for PennDOT's Restricted Performance Bituminous Concrete Specifications. Pennsylvania Transportation Institute, Pennsylvania State Univ., Oct. 1976.
5. E. I. Grant and R. S. Leavenworth. Statistical Quality Control. McGraw-Hill, New York, 4th Ed., 1972.
6. R. V. Hogg and A. T. Craig. Introduction to Mathematical Statistics. Macmillan, New York, 3rd Ed., 1970.
7. Mathematical and Statistical Principles Underlying Military Standard 414. Office of the Assistant Secretary of Defense (Supply and Logistics), 1958.
8. G. J. Lieberman and G. J. Resnikoff. Sampling Plans for Inspection by Variables. Journal of American Statistical Association, Vol. 50, 1955, pp. 457-516.
9. K. S. Miller. Partial Differential Equations in Engineering Problems. Prentice-Hall, New York, 1953.
10. K. Pearson. Tables of the Incomplete Beta Function. Biometrika Office, University College, London, 1934.
11. L. S. Nelson. Use of the Range to Estimate Variability. Journal of Quality Technology, Vol. 7, Jan. 1975, pp. 46-48.

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Contractor Control of Asphalt Pavement Quality

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Warren Brothers Company builds asphalt pavements in seven states that use statistically based end-result specifications that require contractor control of quality. Company experiences with these seven different specifications are described, and control systems developed to comply with the specifications are explained. Problems and their solutions are discussed, and contractor costs and benefits are tallied. On balance, company experience with end-result specifications has been favorable. It is shown that end-result specifications can be workable for contractors, and improvements that would be beneficial to both contractors and agencies are suggested.

Over approximately the past 10 years, several state highway agencies have adopted end-result specifications for asphalt paving that encourage, if not require, contractor control of quality (1). All of these specifications are statistically oriented to some degree. There has been a high degree of interest in statistically oriented end-result specifications for about 20 years, but in spite of that interest implementation has been slow. One reason cited for the slow pace of implementation has been contractor resistance to change (1). This paper is concerned with the experiences and practices of one contractor—Warren Brothers Company, a division of Ashland Oil—with modern end-result specifications and quality control systems for asphalt paving.

HISTORICAL PERSPECTIVE

Contractor control of quality is not a new concept. In fact, early pioneers in bituminous paving such as Abbott, DeSmedt, and the Barber Asphalt Paving Company had their own quality control systems 100 years ago (2, 3). They had to have their own systems because nobody else knew how, but they had learned that control was necessary in order to duplicate successes.

Warren Brothers is no newcomer to quality control.