ducting the experimental work reported in this paper.

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Mathematical Model for Lateral Thermal Buckling and Displacement of Curved Track

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One disadvantage of continuously welded rails is that the possibility of track buckling because of temperature increases is increased significantly by the elimination of rail joints. Many mathematical models have been developed for the buckling of tangent tracks, but there are very few that deal with curved tracks. The objective of this paper is the development of methods for the prediction of both the lateral thermal-buckling load and the corresponding displacement of curved tracks so that criteria for track design, maintenance, and evaluation can be formulated. This objective has been achieved by using a two-dimensional finite-element model that simulates the lateral stability of a track subjected to temperature increases and train wheel loads. This paper illustrates only the basic applications and the potential of the model. A parameter investigation was made that included tracks that had curvatures varying from 0 to 10° and studied the effects of various track parameters on the buckling temperature and the lateral track displacement. The results indicate that the buckling temperature and lateral displacement of a curved track are significantly affected by changes in lateral ballast resistance, misalignment and curvature, and by the presence of ineffective ties. The model provides a promising new approach to the track-buckling problem; however, test data are needed to validate it.

Continuously welded rail is being increasingly used in railway track construction in the United States. A well-known disadvantage of such rails is that the possibility of track buckling because of temperature increases is increased significantly by the elimination of rail joints. Derailments attributed to track buckling have been reported (1). This track-buckling problem—also called the track-stability problem—is consequently of great importance on continuously welded tracks.

Track stability can be subdivided into two main categories according to the plane in which buckling occurs: lateral and vertical. Lateral stability refers to buckling that occurs in the plane of the track, and vertical stability refers to the uplift of the track. Vertical buckling is unlikely to occur, because the initial uplift of the track reduces the lateral ballast resistance and usually causes lateral buckling.

Many mathematical models have been developed for the lateral stability of tangent track, but there are very few that deal with curved track. The objective of this paper is the development of methods for the prediction of both the lateral thermal-buckling load and the corresponding displacement of curved track so that criteria for track design, maintenance, and evaluation can be formulated. This objective is achieved by using a two-dimensional finite-element model that simulates the lateral stability of a track subjected to temperature increases and train wheel loads.

The model was first developed by So and Martin (2) to solve the problem of the lateral stability of tangent tracks. Reasonably good agreement was obtained between the model results and test data. There are no other known applications of finite-element models in this respect. Previous applications of the finite-element method in the analysis of tracks were primarily for the calculation of stresses in the rails under wheel loads.

The finite-element model is quite powerful and efficient in simulating track stability because it uses standard structural-analysis computer programs for elastic frames. A remarkable advantage of the model is its versatility in incorporating all the main parameters that govern the lateral stability of track (3): (a) condition of lateral rail support, (b) rotational resistance of rail fasteners, (c) flexural rigidity of rails, (d) track curvature, (e) track irregularities (such as misalignments and ineffective ties or rail fasteners), and (f) loading on the track (such as thermal loads due to heating of the rails; vertical, lateral, and longitudinal loads due to normal traffic; dynamic vibrations; and train braking and acceleration). Longitudinal loading here refers to loading along the rails. The model uses geometrically nonlinear-beam-deflection theory (large-deflection theory). Geometrically linear-beam-deflection theory (small-deflection theory) has been used for track-stability problems in most previous research. As demonstrated by Kerr (4), this approach yields inaccurate results because buckling deflections are not small.

The finite-element model presented here has not been thoroughly validated because of the lack of test data. Once its validity is fully established, the model should provide a useful approach to track-stability problems. In this paper, only the basic applications and the potential of the model will be illustrated.

REVIEW OF LITERATURE

Experiments on track stability have been conducted in other countries (5, 6, 7, 8); however, no available test data on curved tracks are complete enough to be used for the validation of the finite-element model presented here. Because of the importance of the track-buckling problem, it is recommended that tests be conducted in the United States.

Many theories have been developed for analysis of the buckling problem of the continuously welded track (9). Some of the published theories and test results have been critically reviewed by Kerr (10). Most of the published theories were developed for the lateral track-stability problem. Several important ones are reviewed here. Prud'homme and Janin (11, 12) assumed a beam that had continuous lateral elastic support from the ballast and continuous rotational resistance from the rail fasteners and formulated a set of differential equations. By using a semiempirical method, Bartlett (7) obtained a similar solution. Bijl (13) and Amans and Sauvage (14) formulated the differential equations for nonlinear rail-fastener and ballast characteristics. Using the finite-difference method, Bijl (15) assumed the rail to be a beam with axial loading and discrete elastic supports and the ballast and rail-fastener behavior to be nonlinear, and formulated the equations of equilibrium. Kerr (16) derived the equilibrium equations for a buckling model by using the principle of stationary total potential energy. Three different assumptions were used for the lateral resistance: constant, linear, and a combination of constant and linearly varying resistance. The results indicated that the simplifying assumption of constant lateral ballast resistance was more suitable for use in the analysis of lateral track buckling. Furthermore, by assuming constant lateral ballast resistance, negligibly small longitudinal ballast resistance in the buckled zone, constant longitudinal ballast resistance in the adjoining regions, and negligible fastener rotational resistance, Kerr (17) derived the equilibrium equations for a track beam representing the rail-tie structure by the principle of virtual displacement. The solutions for four buckled configurations were presented. By using the energy method, Numata (18) formulated the strain energy in the ballast and in the bending of the rails and the potential energy of the external loads, assuming constant lateral ballast resistance and certain buckling wave patterns for both curved and straight tracks.

The lateral track-stability problem is more complicated on curved tracks than on straight tracks. The radial displacement of the curved track will decrease the compressive force in the rails and influence the buckling load or temperature. Calculations with regard to this effect have been published by Numata (18), Engel (19), and Nemesdy (20); all indicated that, for track curvature up to about 3.5°, this phenomenon might be neglected.

None of the models reviewed above possess all the following capabilities: simulations of discrete tie supports, track curvatures, track irregularities such as misalignments and ineffective ties or rail fasteners, nonlinear ballast resistance, nonlinear fastener rotational resistance, lateral wheel loads, and geometrically nonlinear track deflections. The finite-element model presented here has the advantage of possessing all of them.

FORMULATION OF MODEL

The finite-element model uses a general-purpose computer program for the linear analysis of elastic frames (21). The program is modified by using the incremental approach for geometrically nonlinear deflection theory (22). Thus, the modified program can be used to determine large buckling deflections of rails and the corresponding buckling loads.

Figure 1 shows the general formulation of the model. The track structure is represented by a finite number of beam elements. The track curvature is simulated by piecewise linear approximation. The ties are assumed to be rigid and fixed by the ballast against any rotation so that both rails will have exactly the same response in the lateral plane. Hence, the two rails are combined into one for simplification. The ends of the track are assumed to be fixed; however, a parameter investigation of the length of the track will determine how the track model approximates the real track in the field. The lateral ballast resistance and the fastener rotational resistance are simulated respectively by the axial and flexural stiffnesses of the radial elements. The longitudinal ballast resistance is simulated by the axial stiffnesses of the elements that are tangent to the rail elements but shown as on one side of the rail elements for clarity. The simulation of thermal and wheel loads can be achieved by the input of fixed-end compressive forces in the rail elements and concentrated quasi-static loads respectively. Piecewise linear approximation is used to simulate track misalignments, nonlinear rail-fastener behavior, and nonlinear ballast resistance.

The definition of buckling load or temperature used here is based on the curve of the relationship between the thermal load or temperature increase and the maximum track deflection (see Figure 2). When a slight increase in thermal load or temperature increases the maximum track deflection appreciably, buckling is said to occur in a sudden manner and the buckling load or temperature increase (P) is defined as a single value. The slope of the load-deflection curve is smallest in the buckling region (see Figure 2a). However, this region is not always distinct. The variation in the slope of the curve may be gradual over a range of load levels (see Figure 2b). The buckling load can then be specified only as a range of values, and buckling is said to occur in a gradual manner. The change in the slope of the curve may be so gradual that the buckling load becomes undefined. In such a case, the track deflection is more meaningful than the buckling load, and the curve is used to predict track deflection rather than buckling load or temperature.

It should be noted that other criteria have been used [such as the definition of a safe buckling temperature formulated by Kerr (4) and discussed by So and Martin (2)]. Moreover, it should be emphasized that the load-deflection curve shown in Figure 2 is based on the thermal load or temperature increase and not on the equilibrium thermal load or equilibrium axial compressive force in the rails. As shown by Kerr and also as obtained by the finite-element model here, the equilibrium thermal load (or equilibrium axial compressive force) decreases in the buckled parts of the rails when buckling occurs; therefore a plot of this against the track deflection would not be similar to Figure 2.

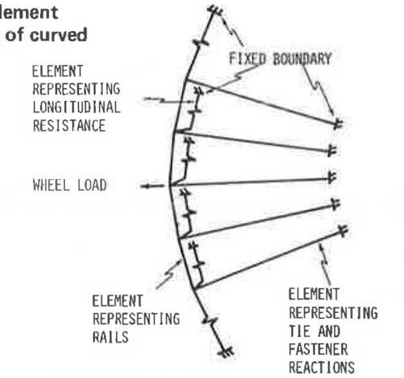


Figure 1. Finite-element model of buckling of curved track.

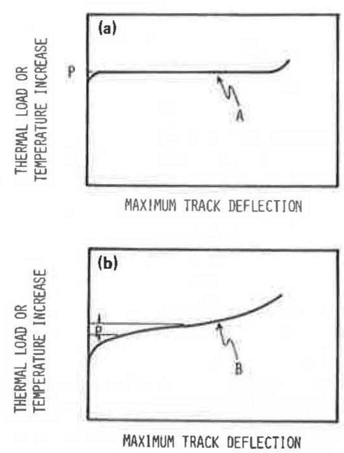


Figure 2. Definition of buckling load or temperature (P).

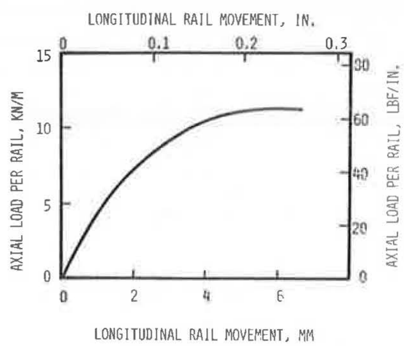


Figure 3. Longitudinal ballast resistance.

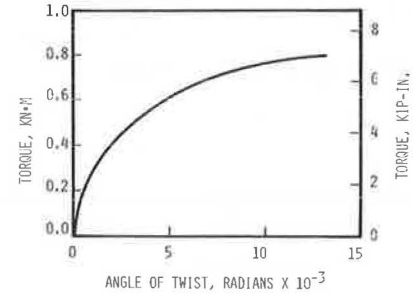


Figure 4. Fastener rotational resistance.

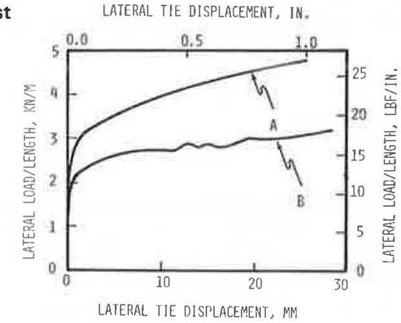


Figure 5. Lateral ballast resistance.

PARAMETER INVESTIGATION

The main parameters that affect the lateral thermal-buckling load of a curved track are investigated here by using the finite-element model. These parameters are track length (L), degree of track curvature (D), initial misalignment (M), ineffective ties, and lateral ballast resistance (R). In this limited investigation, the rail is assumed to be 67.5-kg/m (136-lb/yd) RE specification. The center-to-center spacing between the ties is taken as 0.508 m (20.0 in). An initial misalignment of sinusoidal shape, 12.2 m (40.0 ft) in length, is assumed to exist in the middle of each model track. The longitudinal ballast resistance, the fastener rotational resistance, and the lateral ballast resistance are shown in Figure 3 [test data taken from American Railway Engineering Association (23)], Figure 4 [test data taken from British Railways (7)], and Figure 5 [test data for curve A taken from French National Railways (12) and test data for curve B taken from British Railways (7)] respectively.

Because of symmetry, it is necessary to represent only half of the length of the track by the finite-element model. Figure 6 shows the model configuration of the reference track (no. 1 in Table 1)—a 61-m (200-ft) track that has 2° curvature and 12.7-mm (0.5-in) misalignment at its center. As described above, the track is assumed to be fixed at two ends and the two rails are combined into one and simulated by elements 32 to 47. Each of these elements has a sectional area and a moment of inertia equal to twice those of a single rail. The lateral ballast resistance and the fastener rotational resistance are simulated respectively by the axial and flexural stiffnesses of elements 1 to 16. The longitudinal ballast resistance is simulated by the axial stiffnesses of elements 17 to 31. By symmetry, the center point of the track can move only in the lateral direction. The thermal load, input as fixed-end compressive forces in the rail elements 32 to 47, is started at 89 kN (20 000 lbf) and increased by increments of 89 kN until buckling occurs.

To convert the fixed-end compressive force into a temperature increase in the rails, the following formula is used:

$$T = F/EA\alpha \quad (1)$$

where

- T = temperature increase,
- F = fixed-end compressive force,
- E = Young's modulus of the rail steel [20.7 GPa (30 000 000 lbf/in^2)],
- A = total cross-sectional area of two rails [172.3 cm^2 (26.7 in^2)], and
- α = coefficient of rail-steel expansion [$1.1 \times 10^{-5}/°C$ ($0.61 \times 10^{-5}/°F$)].

To simulate track structures that have parameters different from those of the reference track, other model configurations were constructed in a similar way. The model simulations and the lateral thermal-buckling loads are summarized in Table 1. The following conclusions are based on this limited investigation.

Figure 6. Finite-element model of reference track.

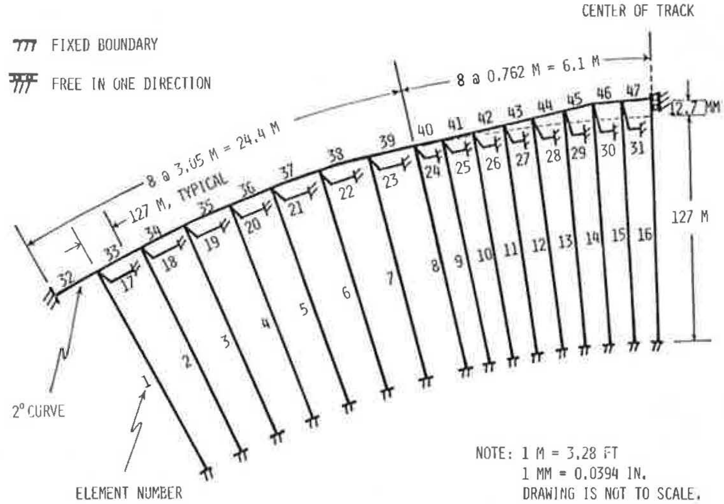


Table 1. Parameter investigation of lateral buckling of curved tracks.

Track No.	Parameter L (m)	M (mm)	D (°)	R	P Load (kN)	Temperature Increase (°C)
1[a]	61	12.7	2.0	Curve 5A	1920	49
2	122	12.7	2.0	Curve 5A	1930	49
3	244	12.7	2.0	Curve 5A	1970	50
4	61	12.7	0.0	Curve 5A	2820	72
5	61	12.7	0.5	Curve 5A	2440	62
6	61	12.7	1.0	Curve 5A	2230	57
7	61	12.7	1.5	Curve 5A	2080	53
8	61	12.7	2.5	Curve 5A	1760-1810	45-46
9	61	12.7	3.0	Curve 5A	1660-1780	42-45
10	61	12.7	3.5	Curve 5A	1570-1730	40-44
11	61	12.7	4.0	Curve 5A	1480-1680	38-43
12	61	12.7	5.0	Curve 5A	None	None
13	61	12.7	7.0	Curve 5A	None	None
14	61	12.7	10.0	Curve 5A	None	None
15	61	38.1	0.0	Curve 5A	1470-1680	38-43
16	61	38.1	1.0	Curve 5A	1390-1600	35-41
17	61	38.1	2.0	Curve 5A	1310-1550	33-40
18	61	38.1	3.0	Curve 5A	1240-1500	32-38
19	61	38.1	4.0	Curve 5A	1160-1450	30-37
20[b]	61	12.7	2.0	Curve 5A	1830	47
21[c]	61	12.7	2.0	Curve 5A	1640	42
22	61	12.7	0.0	Curve 5B	2420	62
23	61	12.7	1.0	Curve 5B	1880	48
24	61	12.7	2.0	Curve 5B	1570	40
25	61	12.7	3.0	Curve 5B	1340-1420	34-36
26	61	12.7	4.0	Curve 5B	1180-1340	30-34

Note: 1 m = 3.28 ft, 1 kN = 225 lbf, and temperature difference in °C = 1.8 x temperature difference in °F.

[a] Reference track.
[b] Simulation of track that has one ineffective tie at center.
[c] Simulation of track that has three consecutive ineffective ties at center.

Figure 7. Relationship between thermal-buckling load and track curvature.

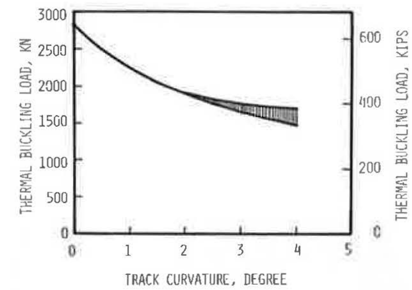
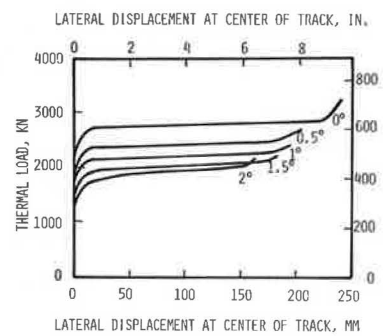


Figure 8. Relationship between thermal load and displacement: 0 to 2° model tracks.



1. Track length: Track lengths of 61, 122, and 244 m (200, 400, and 800 ft) were investigated (nos. 1, 2, and 3 in Table 1). The buckling loads found for these tracks were 1.92, 1.93, and 1.97 MN (432 000, 434 000, and 443 000 lbf) respectively. There is only a negligible effect, an increase of about 0.5 percent, on the buckling load when the length of a 2° curved track is varied from 61 to 122 m. Again, only a 2 percent increase results when the length is increased from 122 to 244 m. This indicates that the 61-m track length can be considered a good approximation to the real track length in the field as far as the buckling load is concerned. Hence, as an approximation, a track length of 61 m was used for the remainder of the parameter investigation.

2. Track curvature: Simulations no. 1 and nos. 4 to 14 in Table 1 represent a set of model tracks that have curvatures that vary from 0 (a straight track) to 10°. Figure 7 shows the effect of track curvature on the buckling load for this set of model tracks. The reduction in the buckling load when the curvature is increased from 0 to 4° is about 44 percent.

For each of the tracks that have curvatures of 0 to 2°, there is a distinct decrease in the slope of the load-deflection curve at a particular load level (see Figure 8). According to the criterion for the buckling load discussed above, that load level is the buckling load of the track. As the track curvature increases from 2.5 to 4° (see Figure 9), the change in the slope of the load-deflection curve becomes more and more gradual and the buckling load of each track can be specified only as a range of load levels. For each of the curved tracks sharper than 4° (see Figure 10), there is hardly any significant change in the slope of the load-deflection curve. This no longer meets the criterion of the buckling load. Hence, the buckling loads of the tracks sharper than 4° are undefined and the track deflections are more meaningful than the buckling loads.

Attention should also be drawn to another important effect related to the track curvature, namely, the radial

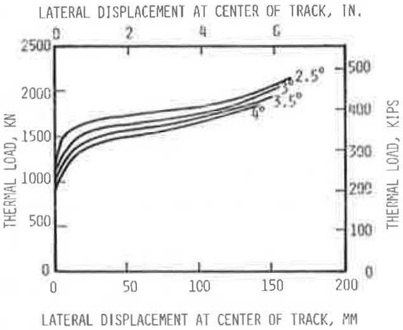


Figure 9. Relationship between thermal load and displacement: 2.5 to 4° model tracks.

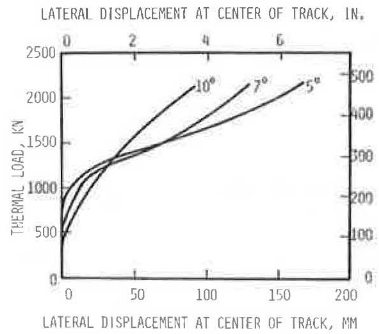


Figure 10. Relationship between thermal load and displacement: 5, 7, and 10° model tracks.

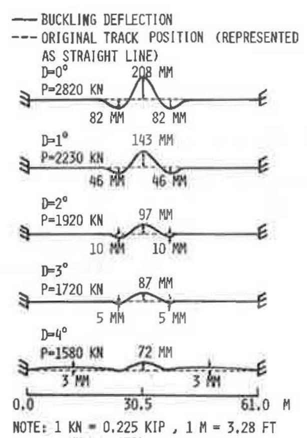


Figure 11. Buckling deflections: 0 to 4° model tracks.

displacement of the track as a whole. The wave patterns and the amplitudes of the postbuckling deflections of the tracks that have curvatures between 0 to 4° (nos. 4, 6, 1, 9, and 11 in Table 1) vary as shown in Figure 11. As the track curvature becomes sharper, the amplitudes of the buckling waves diminish and the track has a tendency to deflect radially. As shown in Figure 11, the postbuckling deflections of the 4° curved track have some radial displacements and, when the curvature is greater than 4° (nos. 12 to 14 in Table 1), the radial displacements increase rapidly (see Figure 12). Consequently, the track deflections are no longer confined to the local phenomenon of buckling. Instead, the whole track is displaced radially.

3. Track misalignment: The results of two sets of simulations of model tracks that had different misalignments and 0 to 4° curvatures (simulations no. 1 and nos. 4 to 11 and nos. 15 to 19 in Table 1) are shown in Figure 13. The buckling loads for the tracks that had the larger misalignment are about 17 to 44 percent lower. The effect is more significant for curvatures of less than 2°.

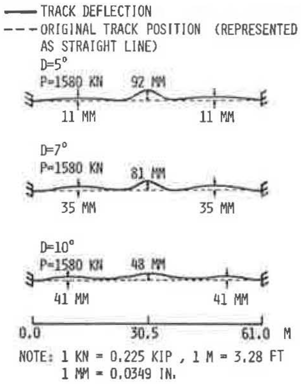


Figure 12. Deflections: 5, 7, and 10° model tracks.

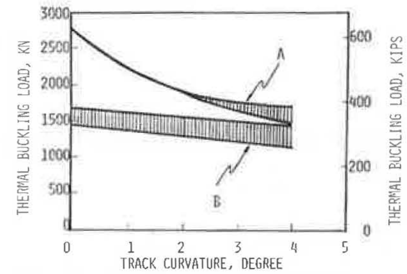


Figure 13. Relationship between thermal-buckling load and track curvature at different track misalignments.

Note: Curve A—misalignment = 12.7 cm (0.5 in); curve B—misalignment = 38.1 cm (1.5 in).

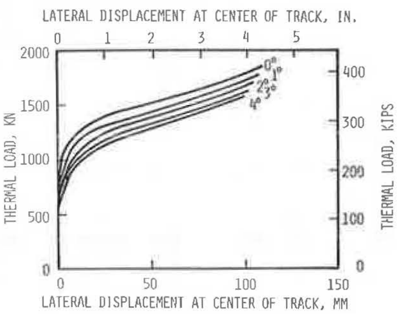


Figure 14. Relationship between thermal load and displacement: 0 to 4° model tracks that have 38.1-mm (1.5-in) misalignments.

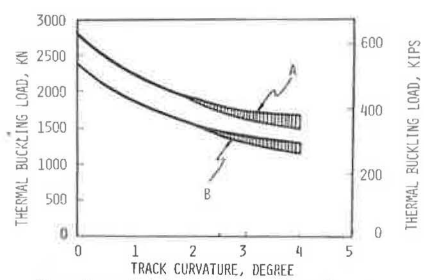


Figure 15. Relationship between thermal-buckling load and track curvature for tracks that have different lateral ballast resistances.

Note: Curve A—ballast resistance as shown in curve A of Figure 5; curve B—ballast resistance as shown in curve B of Figure 5.

Note that the buckling loads of the tracks that had 38.1-mm (1.5-in) misalignments are all specified as ranges of values. This indicates that buckling usually occurs in a gradual manner for those tracks that have relatively large misalignments (see Figure 14).

4. Ineffective ties: Two cases (nos. 20 and 21 in Table 1) that simulated respectively one and three totally ineffective ties located at the center of the track were investigated. When compared with the reference track (no. 1 in Table 1), the reductions of buckling loads were 5 and 15 percent for the tracks with one and three ineffective ties respectively.

5. Lateral ballast resistance: Two sets of model tracks that had different lateral ballast resistances (see Figure 5) and 0 to 4° curvatures were investigated (simulations no. 1 and nos. 4 to 11 and nos. 22 to 26). The results are shown in Figure 15, which indicates that, for the tracks that had the lower lateral ballast resistance, the buckling loads were about 14 to 20 percent lower. The effects are about the same for track curvatures from 0 to 4°.

CONCLUSION

This study has indicated that the finite-element model is an efficient and powerful method for the calculation of the thermal-buckling strength and the corresponding deflection of continuously welded curved tracks. The model uses general-purpose computer programs for structural analyses and incorporates all the main parameters that govern the lateral stability of track. The results of a parameter investigation indicated that the buckling temperature and lateral displacement of a curved track are significantly affected by changes in lateral ballast resistance, misalignment, and curvature and by the presence of ineffective ties. The model appears to be a promising new approach to the track-buckling problem; however, test data are required to validate the model.

ACKNOWLEDGMENT

This research was sponsored by the Research and Test Department of the Association of American Railroads. We wish to thank James R. Lundgren and Allan M. Zarembski at the association for reviewing the manuscript and making valuable suggestions.

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