Optimum Performance Under a Statistical Specification

Richard M. Weed, New Jersey Department of Transportation, Trenton

Statistical construction specifications are based on a desired end result and usually employ graduated pay schedules to award payment in proportion to the extent that the end result is achieved. Considerable latitude is given to the contractor in deciding how to best meet the requirements of the specification, and those contractors exercising good quality control can often realize economic benefits by working close to one of the specification limits. How close to this limit the target level should be set is a matter of strategy, and often an optimum strategy does exist. A procedure is presented for selecting the target level most satisfactory to both the consumer seeking a quality product and the producer anticipating a fair profit. An example is given by using a statistical specification for concrete compressive strength. The outcome reveals a previously unrecognized fact: A producer who bids on and produces a concrete that is exactly at the acceptable quality level can expect to suffer a financial loss. The desired profit is achieved by increasing the bid price and the target strength to optimum levels. Specifying agencies must consider the cost of this higher level of quality and, if necessary, modify their specifications accordingly.

Statistical specifications are based on a desired end result. A specification for concrete compressive strength, for example, might define the acceptable quality level (AQL) as a lot of concrete in which no more than 10 percent of the material is below design strength. When the acceptance procedure indicates that this requirement has been met, the lot of concrete is eligible for 100 percent payment. If the tests indicate that more than 10 percent of the lot is below design strength, a graduated pay schedule as shown below is used to determine the appropriate reduced pay factor for the lot. The philosophy of this approach has been explained in several recent publications (1-3).

A typical graduated pay schedule is shown in the table. Material at or above the AQL receives a pay factor of 100 percent whereas seriously deficient material (rejectable quality level or RQL) receives only 50 percent payment. Most agencies reserve the option to require removal and replacement of RQL material. An explanation of the development of acceptance procedures by which the quality of a product is estimated is available from various sources (4-6).

Quality Level	Percentage Below Design Strength	Pay Factor (%)
AQL	≤10	100
–	11-20	95
–	21-30	90
–	31-40	75
RQL	>40	50

Because there is always some level of risk associated with a statistical acceptance procedure, AQL lots may occasionally receive less than 100 percent payment. However, as illustrated in a recent paper (7), the average pay factor for any large quantity of AQL lots will be very close to 100 percent.

An objection to this approach that is frequently raised by concrete producers concerns the manner in which these pay factors are applied. Most specifying agencies apply the pay factors to the in-place cost of construction items. When pay reductions occur, it apparently is customary for general contractors to pass these reductions back to the producer in their entirety, a necessity if the general contractor also happens to be the producer of the concrete. For example, if a producer quotes \$39.24/m^3 (\$30/yd^3) on concrete that becomes worth \$261.60/m^3 (\$200/yd^3) when placed in a structure, even a small pay reduction of only 5 percent would mean a loss to the producer of 5 percent of the in-place cost, or \$13.08/m^3 (\$10/yd^3), a very undesirable result from the producer's standpoint. However, it will be seen that the producer can effectively guard against this with the proper mix design and bidding strategy.

The importance of a producer's degree of quality control is illustrated in Figure 1. Assuming that a normal distribution adequately describes a population of concrete compressive strengths, producers with coefficients of variation (V) of 10 percent and 15 percent must achieve mean strengths of 23.7 MPa (3441 lbf/in^2) and 25.6 MPa (3714 lbf/in^2), respectively, to just meet the requirements of this specification (i.e., produce AQL concrete). Based on curves established by the Portland Cement Association (PCA) (8), this difference in mean strength of 1.9 MPa (273 lbf/in^2) corresponds to about 0.33 sack of cement/m^3 (0.25 sack/yd^3). At a cost of \$2/sack, this amounts to a savings of about \$0.70/m^3 (\$0.50/yd^3) realized by the producer with better quality control. When multiplied by the total production quantity for a project or a construction season, the substantial magnitude of this savings becomes apparent.

However, the question that must be addressed is this: Should either of these producers attempt to produce concrete that is exactly at the AQL or would it be better to aim for a somewhat higher (or lower) level?

Before analyzing this quantitatively, it might be instructive to consider the consequences of two extreme strategies. First, if the producer chooses to use a very low cement factor, he or she will save on the cost of cement; this, however, will lower the mean strength of the concrete to such an extent that the resulting pay reductions will more than offset these savings. In the other instance, if he or she uses too high a cement factor, the average pay factor will increase but not enough to make up for the extra cost. The fact that both strategies are unsatisfactory (at least from the producer's standpoint) suggests that somewhere between these two extremes an optimum level must exist. In this case it can be found relatively easily by using basic information available to most producers.

The technique requires that the operating characteristics of the acceptance procedure be known. Because one should develop these curves when writing a statistical specification, I believe that it is the specifying agency's obligation to furnish this information. Computer simulation (9) or analytical means (10) will provide several individual points for the upper portion of each operating-characteristic curve that is required. Smooth curves are then fitted through these points, preferably by regression analysis or other mathematical curve-fitting technique, so that an equation is obtained for each curve. Typical operating-characteristic curves are shown in Figure 2.

GENERAL OPTIMIZATION PROCEDURE

The basic procedure for determining the optimum target strength for concrete governed by a statistical specifica-

Figure 1. Mean strength required for AQL concrete for two quality control levels.

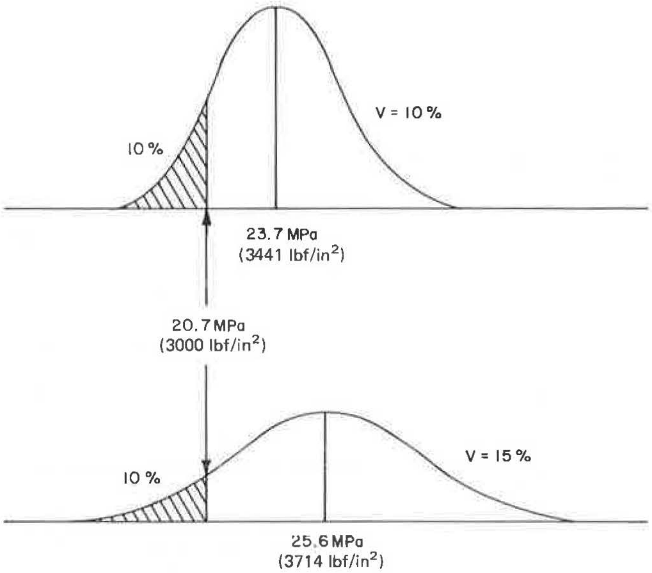
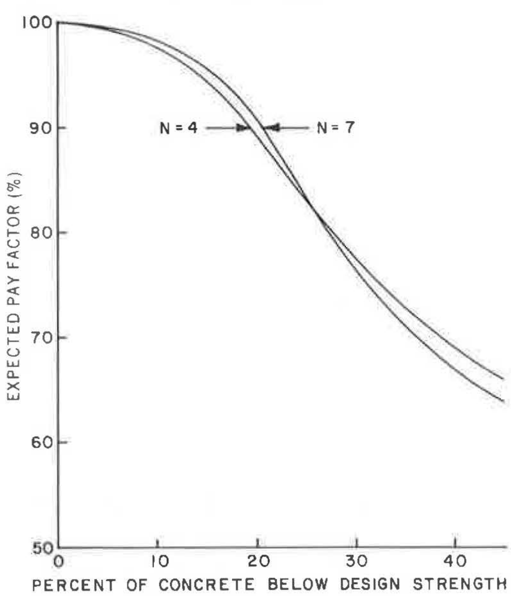


Figure 2. Typical operating-characteristic curves.



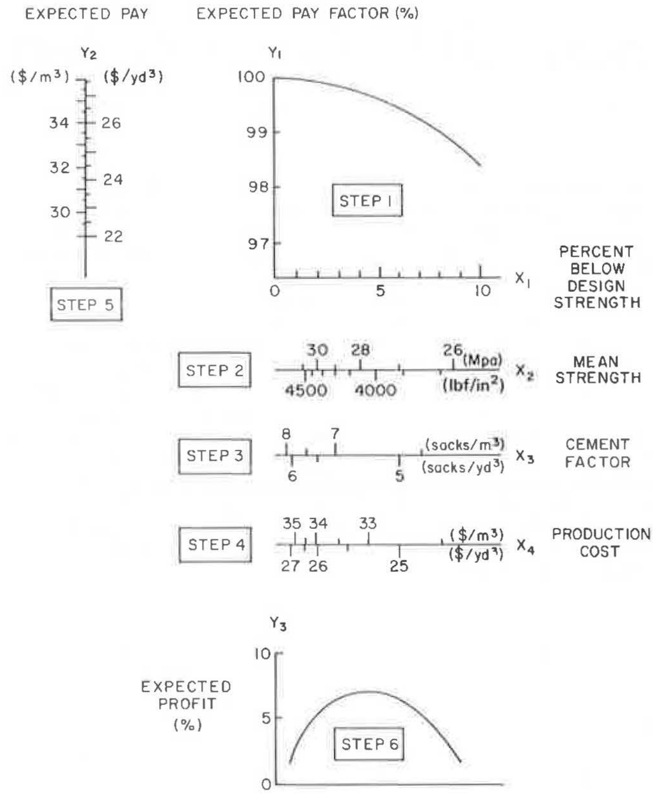
tion follows the six steps illustrated in Figure 3 and listed below.

1. Plot the operating-characteristic curve for the sample size being used and the particular producer's coefficient of variation. (If the specification is of the variability-unknown type, sample size alone determines the curve.) The quality level (percentage of concrete below design strength) appears on axis X_1 and the corresponding expected pay factors are found on axis Y_1. This information would normally be obtained from the specifying agency.

2. Use a table of the standard normal distribution to transform axis X_1 into axis X_2, the mean strength of concrete necessary to produce the amount of defective material given on axis X_1.

3. Using production data available to most producers, transform axis X_2 into axis X_3, the cement factors re-

Figure 3. Diagram of optimization procedure.



quired to produce the mean strengths given on axis X_2.

4. Basic cost data are then used to transform axis X_3 into axis X_4, production cost per unit quantity of concrete.

5. Axis Y_1 must then be transformed into axis Y_2, expected pay per unit quantity of concrete. This must be done in a realistic manner, which will usually mean that pay reductions (a) are based on the in-place bid price of a construction item and (b) are passed in their entirety back to the producer.

6. Because axes X_4 and Y_2 give production cost and expected pay for different quality levels, it is now a simple matter to plot several points to determine which quality level yields the greatest expected profit on axis Y_3. The corresponding target strength is then read from axis X_2.

BASIC INFORMATION REQUIRED

To illustrate the application of this procedure, the following basic information will be assumed.

The concrete design strength is 20.7 MPa (3000 lbf/in^2), and the coefficients of variation for the two producers in this example are 10 and 15 percent. The operating-characteristic curve relating expected pay factor to percentage of concrete below design strength is known, and production cost for a standard 7.85 sacks/m^3 (6.00 sacks/yd^3) mix is \$35.32/m^3 (\$27/yd^3). The change in production cost for other mixes will be based on a cement price of \$2/sack. The desired profit margin is 7 percent. Pay reductions, based on an in-place bid price for concrete construction of \$261.60/m^3 (\$200/yd^3), will be presumed to be passed back in their entirety to the producer of the concrete.

Most producers have data that would enable them to determine the cement factor necessary to achieve any specific level of mean strength. However, because this information is not readily available to me, the PCA curves referred to previously will be used for this step.

Although the optimization procedure could be accomplished by constructing the several transformed axes shown in Figure 3, this is not actually necessary. Instead, the procedure can be carried out in tabular form by using successive values of X_1 and calculating the corresponding values of X_2, X_3, X_4, Y_1, Y_2, and Y_3. Then, because Y_3 represents expected profit, the optimum point can be read directly (or interpolated) from the table along with the appropriate mean strength level.

TYPICAL CALCULATIONS

A typical calculation for a coefficient of variation of V = 10 percent and a single quality level of X_1 = 5 percent will follow these five steps.

First, although the operating-characteristic curve could be used graphically, it will be more convenient to use in equation form. The equation may be of any form that provides a good fit of the points on the curve from $X_1 = 0$ out to the AQL value. An expression that seems to satisfy this requirement quite well is

$$Y_1 = \sin(a - bX_1) \qquad (1)$$

where

Y_1 = expected pay factor expressed as a decimal,
X_1 = percentage of concrete below design strength expressed as a decimal,
a = $\pi/2$ = 1.571, and
b = a factor to be determined for each sample size for a variability-unknown specification.

The sine argument is in units of radians, and, for this example, a coefficient of b = 1.821 will be used.

Second, calculate the required mean strength from the equation

$$X_2 = D/(1 + ZV) \qquad (2)$$

where

D = design strength in megapascals,
Z = z-score corresponding to an area of X_1 under the normal curve, and
V = coefficient of variation expressed as a decimal.

For values of D = 20.7 MPa (3000 lbf/in^2), Z = -1.645 corresponding to X_1 = 0.05, and V = 0.10, this yields X_2 = 24.8 MPa (3591 lbf/in^2).

Third, here again it will be convenient to put the relationship of strength to cement factor in equation form. This may be done by regression analysis or other mathematical curve-fitting technique. The PCA relationship can be expressed as follows:

$$X_3 = a - (b - cX_2)^{1/2} \qquad (3)$$

in which the coefficients are a = 10.63 (8.13), b = 66.147 (38.663), and c = 1.8221 (0.007 344 8) and 20.7 MPa (3000 lbf/in^2) $\leq X_2 \leq$ 34.5 MPa (5000 lbf/in^2).

For the mean strength of X_2 = 24.8 MPa (3591 lbf/in^2) obtained in the second step, the equation yields a cement factor of X_3 = 6.05 sacks/m^3 (4.62 sacks/yd^3).

In the fourth step the unit cost to produce the mix is calculated from the equation

$$X_4 = a - b(c - X_3) \qquad (4)$$

where

X_3 = cement factor obtained in the third step,
a = cost of standard mix of \$35.32/m^3 (\$27/yd^3),
b = cost of cement of \$2/sack, and
c = standard cement factor of 7.85 sacks/m^3 (6.00 sacks/yd^3).

This yields X_4 = \$31.72/m^3 (\$24.25/yd^3).

Fifth, expected pay (Y_2) is obtained from the expression

$$Y_2 = P_1 - P_2(1 - Y_1) \qquad (5)$$

where

Y_1 = expected pay factor expressed as a decimal,
P_1 = bid price of concrete per cubic meter, and
P_2 = in-place cost of concrete per cubic meter.

However, before this calculation can be made, a trial bid price must be assumed by selecting a cement factor that seems adequate and then calculating the appropriate bid price. (If the final outcome is unsatisfactory, a different selection would be made and this portion of the procedure repeated.) Based on previous trials, suppose that the producer has selected a trial cement factor of 6.38 sacks/m^3 (4.88 sacks/yd^3) on which to bid.

By substituting this value for X_3 in Equation 4, the cost to produce this mix is found to be \$32.38/m^3 (\$24.76/yd^3). To obtain a 7 percent profit margin, this value is multiplied by 1.07 to yield a bid price of \$34.65/m^3 (\$26.49/yd^3). The in-place cost of the concrete is \$261.60/m^3 (\$200/yd^3), and the expected pay factor is determined from Equation 1 to be 0.9959. Substituting these values into Equation 5, the expected pay is found to be Y_2 = \$33.58/m^3 (\$25.67/yd^3).

Sixth, and finally, expected profit (Y_3) for the quality level and bid price chosen for this series of calculations is obtained from the equation

$$Y_3 = 100[(Y_2 - X_4)/X_4] \qquad (6)$$

where Y_2 is expected pay of \$33.58/m^3 (\$25.67/yd^3) and X_4 is production cost of \$31.72/m^3 (\$24.25/yd^3). This yields Y_3 = 5.86 percent.

These calculations would be repeated for several different values of X_1 in order to locate the quality level that optimizes expected profit. If the maximum attainable profit is found to be unsatisfactory, the producer selects a different cement factor, calculates a new bid price, and repeats the procedure.

In actual practice, this would be a laborious and time-consuming task if it were necessary to perform these steps manually. Fortunately, it is not difficult to computerize this procedure. The data in Tables 1 and 2 were obtained in this manner and establish the conditions necessary for the two producers to realize the desired 7 percent profit margin. Several interesting observations can be made from the data in Tables 1 and 2.

The last column (Y_3) lists the expected profit at various quality levels and indicates that neither producer should attempt to operate close to the AQL (10 percent of the concrete below design strength). The appropriate levels are 3 and 4 percent below design strength for the better (V = 10 percent) and the poorer (V = 15 percent) producer, respectively. In this example, both producers would actually lose money by producing concrete exactly at the AQL.

The better producer can achieve a 7 percent return by bidding on a mix of 6.38 sacks/m^3 (4.88 sacks/yd^3) and then producing a mix of 6.20 sacks/m^3 (4.74 sacks/yd^3) with an expected mean strength of 25.5 MPa (3695 lbf/in^2). In order to achieve the same return, the poorer producer

Table 1. Calculation results for a 10 percent coefficient of variation.

Percentage Below Design (X_1)	Mean Strength (X_2) (MPa)	Cement Factor[a] (X_3) (sacks/m^3)	Production Cost[b] (X_4) (\$/m^3)	Expected Pay Factor (Y_1) (¢)	Expected Pay (Y_2) (\$/m^3)	Expected Profit (Y_3) (¢)
1	27.0	6.51	32.63	99.98	34.61	6.05
2	26.0	6.30	32.24	99.93	34.48	6.96
3	25.5	6.20	32.01	99.85	34.27	7.05
4	25.1	6.11	31.85	99.74	33.97	6.65
5	24.8	6.05	31.72	99.59	33.58	58.6
6	24.5	5.99	31.61	99.41	33.09	4.70
7	24.3	5.95	31.52	99.19	32.54	3.22
8	24.1	5.91	31.44	98.94	31.89	1.41
9	23.9	5.87	31.38	98.66	31.16	-0.70
10	23.7	5.85	31.31	98.35	30.33	-3.12

Note: 1 MPa = 145 lbf/in^2; 1 m^3 = 1.3 yd^3.

[a]Bid mix = 6.38 sacks/m^3 (4.88 sacks/yd^3).
[b]Bid price = \$34.65/m^3 (\$26.49/yd^3).

Table 2. Calculation results for a 15 percent coefficient of variation.

Percentage Below Design (X_1)	Mean Strength (X_2) (MPa)	Cement Factor[a] (X_3) (sacks/m^3)	Production Cost[b] (X_4) (\$/m^3)	Expected Pay Factor (Y_1) (¢)	Expected Pay (Y_2) (\$/m^3)	Expected Profit (Y_3) (¢)
1	31.8	7.77	35.15	99.98	36.13	2.77
2	29.9	7.22	34.06	99.93	36.00	5.68
3	28.8	6.95	33.51	99.85	35.77	6.78
4	28.1	6.76	33.13	99.74	35.47	7.05
5	27.5	6.62	32.86	99.59	35.08	6.76
6	27.0	6.51	32.65	99.41	34.61	6.02
7	26.6	6.42	32.46	99.19	34.05	4.88
8	26.2	6.34	32.31	98.94	33.41	3.37
9	25.9	6.28	32.18	98.66	32.67	1.52
10	25.6	6.23	32.06	98.35	31.85	-0.66

Note: 1 MPa = 145 lbf/in^2; 1 m^3 = 1.3 yd^3.

[a]Bid mix = 7.09 sacks/m^3 (5.42 sacks/yd^3).
[b]Bid price = \$36.17/m^3 (27.65/yd^3).

must bid on a mix of 7.09 sacks/m^3 (5.42 sacks/yd^3) and then produce a mix of 6.76 sacks/m^3 (5.17 sacks/yd^3) with an expected mean strength of 28.1 MPa (4069 lbf/in^2).

To achieve the desired profit margin, the better producer must use a cement factor 0.35 sacks/m^3 (0.27 sacks/yd^3) higher than that required for AQL concrete, while the corresponding increase for the poorer producer is 0.54 sacks/m^3 (0.41 sacks/yd^3). This will raise the mean strengths by approximately 1.7 MPa (250 lbf/in^2) and 2.4 MPa (350 lbf/in^2) and increase the production costs by about \$0.70/m^3 (\$0.50/yd^3) and \$1.05/m^3 (\$0.80/yd^3), respectively, over those for AQL concrete.

It was observed when Figure 1 was discussed that the better producer could produce AQL concrete about \$0.70/m^3 cheaper than the poorer producer. Comparing the two rows for 10 percent below design strength in Tables 1 and 2, the actual difference in production cost is seen to be \$32.06 - \$31.31 = \$0.75/m^3 (\$24.51 - \$23.94 = \$0.57/yd^3). However, since neither producer should operate at the AQL, it is more appropriate to make the comparison at their respective optimum levels. The somewhat larger difference in production cost in this case is \$33.13 - \$32.01 = \$1.12/m^3 (\$25.33 - \$24.47 = \$0.86/yd^3).

From the standpoint of the consumer, who must ultimately foot the bill for the concrete, it is more meaningful to compare the bid prices that enable these two producers to just achieve the desired profit margin. The difference in this case is \$36.17 - \$34.65 = \$1.52/m^3 (\$27.65 - \$26.49 = \$1.16/yd^3). Because of superior quality control, the better producer can either (a) settle for a return of 7 percent and substantially underbid the other producer or (b) raise the bid somewhat, realize a greater profit, and still remain competitive. (The typical calculations described previously appear as the fifth row in Table 1.)

IMPLEMENTATION BY PRODUCERS

Producers must recognize that the optimum level is not a level that will eliminate pay reductions altogether. It is instead a level at which the likelihood of pay reductions and the cost of production are balanced in a way that enables the producer to realize the greatest profit margin. Although large departures from the optimum must be avoided, this balance is not delicate enough to present a serious problem to the knowledgeable producer.

Ideally, the optimum level would be determined by a computer program such as that used to generate Tables 1 and 2. If this is not feasible, the typical calculations described previously illustrate how the method can be practiced with a hand calculator. (Inexpensive, programmable models that can simplify this task considerably are now available.) Because of the many factors to be taken into account, it is unlikely that a simpler method that is accurate in all cases will be found. However, for the benefit of those producers who might have difficulty with this procedure, the following rule of thumb is offered as a guide.

1. Determine the mean strength necessary to produce concrete that is exactly at the AQL. This can be accomplished by using Equation 2, a z-score corresponding to the AQL, and a coefficient of variation representing the producer's established level of quality control.
2. Bid on a mix that has a target strength approximately 3.4 MPa (500 lbf/in^2) higher than that of AQL concrete.
3. Produce a mix that has a target strength approximately 1.7 MPa (250 lbf/in^2) higher than that of AQL concrete.

This recommendation is based on the results of a sensitivity analysis by using a computer program developed for this study and is appropriate under conditions generally similar to those assumed in the example. Very costly construction work or a large coefficient of variation might warrant still greater increases in bidding and production target strengths, whereas, for less costly work or very good quality control, smaller increases may suffice. In addition, the sensitivity analysis indicated that this approach becomes more favorable to the producer as the design strength increases. This is so because the strength versus cement factor curve tends to level off at higher cement factors, the difference in cement factor between the bid mix and the production mix increases, the bid price is correspondingly higher, and, as a result, so is the profit margin. In keeping with the philosophy of end-result specifications, the final decision regarding bidding and production strategies should remain the responsibility of the supplier of the concrete.

If a concrete producer does elect to use either the optimization procedure or the rule of thumb, several results can be expected. For example, the bid price and mean strength will approach optimum levels, which will enable the producer to realize the desired profit margin, and the number of lots receiving pay reductions will be reduced. There will also be a small increase in cost to compensate for the added strength of the concrete. The magnitude of this increase will depend on the manner in which the strength is raised but would normally be less than 1 percent of the in-place cost.

In the case of concrete pavement, the additional strength would provide an increase in expected service life. In the case of bridge decks and other structures subject to salt intrusion and spalling distress, the higher-strength mix will usually be less permeable and thus will provide greater durability.

The decrease in the frequency of pay reductions resulting from the increase in target strength will promote a smoother implementation of specifications of this type and will create a better working relationship among all parties involved.

By means of calculations similar to those used in the optimization procedure, it is possible to check how the two producers in the example would have fared if they had used the rule of thumb. The better producer would have bid \$35.02/m^3 (\$26.77/yd^3), which is higher than necessary; produced a target strength of 25.5 MPa (3691 lbf/in^2), which is almost exactly at the optimum level; and realized a profit of 8.1 percent, which is greater than the planned value of 7.0 percent because of the higher bid price. The poorer producer would have bid slightly low at \$35.98/m^3 (\$27.51/yd^3), produced a target strength somewhat below optimum at 27.3 MPa (3964 lbf/in^2), and realized a profit of 6.0 percent, down a bit from the desired return of 7.0 percent. In sharp contrast to this, if these two producers had bid on AQL concrete and then produced at that level, they would have incurred losses of -6.8 and -6.4 percent, respectively.

EFFECT ON SPECIFYING AGENCIES

Specification writers will note that the specification used in this example tends to force an increase in quality to a level well above the AQL because this is the most advantageous choice for the producer. It is my belief that this will usually be the case with specifications of this type, although it can be demonstrated that an extremely lenient pay schedule can produce the opposite effect by causing the optimum point to fall below the AQL.

Once contractors begin to understand how their strategies affect their performance, it is likely that they will tend to operate at optimum levels to maximize profits. Specifying agencies will then be receiving (and paying for) the optimum quality level, not the AQL. Because the optimum level appears to be relatively stable over a fairly broad range of conditions, it may be possible for specification writers to exercise some control over its location by making appropriate modifications of the graduated pay schedule. It is suggested that, at the very least, specifying agencies should check the approximate location of the optimum level to assure that it is reasonably close to the desired quality level.

SUMMARY AND CLOSING REMARKS

The optimization procedure can be an effective tool for both contractors and specifying agencies. It allows the contractor to determine in advance what the bid price and target level should be to achieve the desired profit margin. It also provides the specifying agency with knowledge and control of the level of quality a specification will tend to produce. This should lead to a better contractual relationship, in which the consumer obtains quality at a price that enables the producer to realize a fair profit.

The optimization procedure can readily be applied to other statistical specifications, although the application is easiest when there is a single factor that controls quality and cost (such as the cement factor in the example). The method is also applicable to specifications that have both lower and upper limits, such as would be the case for asphalt content of bituminous concrete. In this case, axis X_1 (percentage outside specification limits) would increase in both directions from zero, and the operating-characteristic curve for pay factor would be bell shaped with a maximum at $X_1 = 0$. Also, axes X_2 and X_3 would be identical because the factor controlling cost (asphalt content) is the same factor on which acceptance is based.

The example points out that it will usually be advisable to aim for a level of quality well above what is defined as acceptable. It also illustrates an advantage of good quality control, which enables a better producer to offer a lower bid price and still realize a greater profit margin.

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Acceptance Sampling of Neoprene Joint Sealers

David A. Law and Gerald L. Anania, New York State Department of Transportation

This paper describes an investigation of acceptance-sampling procedures for 3.1-cm (1.25-in) neoprene joint sealer. Background information is presented on the manufacturing process and quality-control techniques. Historical data dealing with rejection rates and property variability are discussed. Various types of sampling plans are discussed before the appropriate plan is chosen. The proposed acceptance plan is analyzed by using operating-characteristics curves, and the correct sampling location and the variability within a lot are determined.

In the past, acceptance sampling of construction materials was based on sampling plans that placed great emphasis on testing method and less emphasis on the number of samples needed to characterize a lot. Often the number of samples was chosen by rule of thumb, and many of these plans have since been found to be inefficient because the risks taken were large and the quality levels accepted varied greatly.

The New York State Department of Transportation (DOT), by designing new statistically based acceptance-sampling plans, is attempting to eliminate sampling plans that fall into this category. Such plans are a part of quality assurance but are not used for direct control of the production process. Their purpose is to provide a buyer with a consistent procedure, which has known risks and quality levels, for accepting or rejecting lots. This report describes the development of an acceptance-sampling plan for 3.1-cm ($1\frac{1}{4}$-in) neoprene joint sealer. Similar reports have already been published on steel reinforcing bars (1) and structural paints (2).

BACKGROUND

Neoprene joint sealer is made by extruding polychloroprene through a die. After extrusion it is usually cut into convenient lengths for handling and curing. The material is then cured by a process that varies from manufacturer to manufacturer. After curing, the lengths are glued together to form large rolls. These are placed in cartons or wrapped on cable reels to form a lot for shipment to the job site.

At either the manufacturing plant or the job site, an inspector samples the lot and forwards these samples to New York DOT's Materials Bureau for testing. The joint sealer is tested for the following properties: (a) strength (aged and unaged), (b) elongation (aged and unaged), (c) hardness (aged and unaged), (d) recovery [at two temperatures, -29 and 100°C (-20 and 212°F)], and (e) force deflection.

Current test procedures and specifications limits for these properties are listed below (1 MPa = 145 lbf/in^2; t°C = (t°F - 32)/1.8; 1 cm = 0.39 in). If test results are satisfactory the lot is approved for use; if unsatisfactory the lot is rejected and cannot be used.

Property	ASTM Test Method	Requirements Min	Requirements Max
Tensile strength, MPa	D 412	14	
Ultimate elongation	D 412	250	
Hardness, type A durometer	D 2240	50	60
Aged tensile strength, % change			-20
Aged ultimate elongation, % change			-20
Aged hardness, point change			+10
Oil swell (ASTM Oil No. 3, 70 h at 100°C), % change of weight			45
Recovery under 50 percent deflection, %			
After 70 h at 100°C		85	
After 22 h at -29°C		85	
Deflection condition, force, kg/cm			
Compressed to 2.5 cm			0.54
Compressed to 1.5 cm			2.16

In judging the efficiency of this sampling plant, one must evaluate several factors, such as how many samples are being taken, what the risks involved in judging acceptance with that many samples are, where the samples are being taken, and whether sampling from those locations provides a representative sample. In addition, data are needed on the distributional form of the properties tested, variability of those properties, and historical rejection data. All this information must be combined with quality-control theory and engineering judgment to design a statistically based acceptance-sampling plan.

Operating-Characteristics Curve

The efficiency of existing sampling plans must be determined if new plans are to be compared with them. A common method for evaluating efficiency and the risks associated with a sampling plan is the construction of an operating-characteristics (OC) curve for the plan. The general concepts and terms associated with OC curves will be presented, but no attempt will be made to detail how these curves are derived. A complete description of the OC-curve theory can be found in most good texts on quality control (3).

An OC curve of the type considered in this report shows the probability of accepting lots that have varying fractions defective. The curves are based on the assumption that, if one has a series of lots with the same fraction defective, a percentage of the bad lots will