OPTIMAL TIMING FOR PAVING LOW-VOLUME GRAVEL ROADS

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This paper examines the economics of upgrading low volume gravel roads with particular emphasis upon construction postponement. The concept of break-even analysis is re-examined and a case presented for consideration of construction deferment in light of the opportunity cost of capital. This consideration is particularly important for developing countries where capital is scarce and the opportunity cost high. Simplified expressions are developed to determine both the break-even year and the optimal year in which to pave a given gravel road. Their application is illustrated by means of a numerical example.

Gravel surfaced roads are generally adequate for most situations of low volume traffic. However, as traffic increases, the maintenance and vehicle operating costs also increase, making it necessary to consider paving the gravel surface. The economic viability of such an action is easily established whenever the reduction in maintenance and vehicle operating costs is substantial in comparison to the construction cost. The concept of break-even analysis (1,2,3) has often been applied in such cases to determine the cut-off volume above which it is economically feasible to pave the road. This cut-off volume, generally referred to as the break-even volume represents the minimum volume above which the net present value of paving is in excess of zero.

If, on the other hand, there are alternative opportunities competing for the same capital, then the break-even criterion is clearly not the most efficient way to allocate scarce resources. Under such circumstances, it is economically more advantageous to postpone construction beyond the break-even volume until such time when the net present value is maximized. If the opportunity cost of capital is at least as high as the discount rate, and the annual net benefits increase monotonically with time, then the net present value is maximized by paving the road in the year in which the first year benefits are equal to the opportunity cost of capital invested. The volume of traffic in this year has been referred to as the "optimal volume" and the concept as the "first-year-benefits" criterion (4).

Therefore, in most instances, when the base year volume is low but increasing with time, it is possible to invest the capital elsewhere in the economy to obtain returns that are in excess of the benefits foregone during the years that construction is deferred. To pave the road a year before the optimal volume would mean foregoing excess benefits that could be obtained from an alternative investment in the economy, while paving the road a year after, would mean losing excess benefits that could have accrued had the road been paved a year earlier. This situation is illustrated diagrammatically in Figure 1. The top portion of the figure shows the stream of annual net benefits arising from paving the road in the year, say n^1. The net benefits of paving the road in any other year would be the same as shown, except for the years before paving when the net benefits are zero. For example, the net benefits corresponding to construction in the break-even year (n_{be}) and the optimal year (n_{opt}) would correspond to the lines $on_{be}pq$ and $on_{opt}q$, respectively. The lower portion of the figure shows the net present value of the entire project, given the year of construction. The points n_{be} and n_{opt} correspond to zero and maximum net present values, respectively.

Design Life, Costs and Benefits

Design Life

In order to make a valid economic assessment of a project, it is necessary to know the time horizon over which the evaluation is to be made. For most road projects, this has been the design life of the project, taken to vary from 15 to 40 years. However, while it is true that a newly constructed facility will have a fixed physical life, it is conceivable, and often the case, that major rehabilitation will be done at the end of this period and in subsequent periods to perpetuate the useful life of that facility. For road projects, this is often done in the form of major reconstruction and overlays, whenever the serviceability of the pavement has fallen below an acceptable level. Under this assumption, benefits may be assumed to accrue indefinitely, for all practical purposes. The physical life of a pavement is a function of the design standard and the traffic volume that it is subjected to. In this paper, no attempt is made to relate these factors, but instead, the frequency of major rehabilitations is assumed to be planned in advance.

Costs and Benefits

The primary costs associated with road projects consist of the construction, routine maintenance, planned rehabilitation and vehicle operating costs.

Construction costs are assumed to occur only in the first year of construction while routine maintenance costs at the end of each year. Maintenance costs are assumed to remain constant over time. Major rehabilitation costs are planned to occur every N years, where N is presumably close to the physical life of the pavement.

Vehicle operating costs are a function of the vehicle types and their relative mix in the traffic. Therefore, a weighted average unit cost per vehicle-mile will be used, derived as follows:

$$c = \sum_{i=1}^{k} c_i \, p_i \qquad (1)$$

where,

- c = average cost per veh-km (0.6 veh-mile), for all vehicle types,
- c_i = unit operating cost of the i^{th} vehicle type, per km (0.6 mile),
- p_i = fraction of total traffic that is of type i,
- k = number of different vehicle types in the traffic mix.

The weighted average cost will be assumed to remain constant over time, but have different values on paved and gravel surfaces, respectively. It follows, therefore, that the annual growth rate of traffic must be the same for all vehicle types.

The benefits from paving a gravel road comprise largely of the reduction in the total vehicle operating and routine maintenance costs. Net benefits are then obtained after allowing for the construction and rehabilitation costs.

As in all other economic evaluations of this nature, the stream of costs and benefits must be adjusted to reflect the temporal value of money, by applying an appropriate discount rate. The choice of an appropriate discount rate is a continuing topic of debate (5,6,7). For our purpose, the opportunity cost of capital will be used as the discount rate also.

Analytical Framework

Definition of Variables

Let the variables relevant to a gravel road being considered for paving be defined as follows:

- Q_0 : volume of traffic in the base year, in vehicles per day
- m, m': uniform equivalent annual routine maintenance costs per km, before and after paving, respectively
- c, c': weighted average operating costs per veh-km on gravel and paved surfaces, respectively
- C: fixed construction costs per km of paving
- N: frequency of major rehabilitation, in years
- R: cost of rehabilitation, in dollars per km
- i: opportunity cost of capital
- r: traffic growth rate per annum (generally less than i)

Let n denote the year in which the gravel road is paved. The present value (P.V.) of various cost elements per km of roadway, are obtained as follows-keeping in mind that the road is kept in service indefinitely through periodic rehabilitation:

P.V. of construction costs

$$= \frac{1}{(1+i)^n} \, C \qquad (2)$$

P.V. of rehabilitation costs

$$= \frac{1}{(1+i)^n} \cdot \frac{R}{(1+i)^N - 1} \qquad (3)$$

P.V. of savings in routine maintenance costs

$$= \frac{1}{(1+i)^n} \cdot \frac{(m-m')}{i} \qquad (4)$$

P.V. of savings in vehicle operating costs

$$= \frac{1}{(1+i)^n} \cdot Q_n \cdot 365\,(c-c') \cdot \frac{1+r}{i-r}\,; \quad r<i \qquad (5)$$

where,

$Q_n = Q_0\,(1+r)^n$ = volume of traffic in year n

$\frac{1+r}{i-r}$ = the discount factor for present value of a geometric series with growth rate r and discount rate i, over an indefinite period (for r<i).

The net present value (NPV) of paving is then obtained as:

$$NPV = \frac{1}{(1+i)^n}\left[Q_n \cdot 365\,(c-c')\,\frac{1+r}{i-r} + \frac{(m-m')}{i} - C - \frac{R}{(1+i)^N-1}\right] \qquad (6)$$

Break-Even Analysis

If we now define n as the break-even year, then the break-even volume, Q_{be}, may be obtained by setting the NPV equal to zero in equation (6). Hence,

$$Q_{be} \cdot 365\,(c-c')\,\frac{1+r}{i-r} + \frac{(m-m')}{i} = C + \frac{R}{(1+i)^N-1}$$

$$Q_{be} = \frac{C + \frac{R}{(1+i)^N-1} - \frac{(m-m')}{i}}{365\,(c-c')\,\frac{1+r}{i-r}} \qquad (7)$$

Provided the base year volume, Q_0, is less than Q_{be}, the break-even year, n_{be}, may then be obtained from,

$$Q_{be} = Q_0 (1+r)^{n_{be}}$$

$$n_{be} = \frac{Log_e(Q_{be}/Q_0)}{Log_e (1+r)} \quad \text{for } Q_0 < Q_{be}, \text{ otherwise zero} \tag{8}$$

Optimal Year of Paving

The optimal year for paving the gravel road is obtained by maximizing equation (6) with respect to n. However, as seen in Figure 1, with the discount rate equal to the opportunity cost of capital, the net present value is maximum when the undiscounted net benefits in the year of paving are equal to zero. For such a maximum to exist, it is necessary that the net benefits increase monotonically with time. This is ensured as long as the savings in the vehicle operating costs and the traffic growth rate are both non-negative.

The net benefits in the first year after paving are given as:

$$Q_n \cdot 365 (c-c') + (m-m') - Ci - \frac{Ri}{(1+i)^N-1} \tag{9}$$

To obtain the optimal volume (Q_{opt}), we substitute Q_n by Q_{opt} in the above expression and set it equal to zero.

$$Q_{opt} \cdot 365(c-c') + (m-m') - Ci - \frac{Ri}{(1+i)^N-1} = 0$$

from which,

$$Q_{opt} = \frac{C_i + \frac{Ri}{(1+i)^N-1} - (m-m')}{365 (c-c')} \tag{10}$$

Again, if Q_0 is less than Q_{opt}, we obtain the optimal year of paving, n_{opt}, as:

$$n_{opt} = \frac{Log_e (Q_{opt}/Q_0)}{Log_e (1+r)} \quad \text{for } Q_0<Q_{opt}, \text{ otherwise zero} \tag{11}$$

The break-even and the optimal years obtained from equations (8) and (11) are rounded to the nearest whole numbers, in line with the usual assumption of year-end cost outlays.

Numerical Example

In this section a numerical example is considered to illustrate the procedure. Although the following values are realistic, any different set of data would indicate the same direction in results as shown by this example.

Q_0 = variable
m-m' = $500 per km
c-c' = $.022 per veh-km
C = $46,500 per km
N = variable
R = $25,000 per km
i = 10 percent per annum
r = 6 percent per annum

Equations (7) and (10) are used first to determine the break-even and the optimal volumes, respectively. These are shown in Table 1 for three different values of N, the frequency of periodic rehabilitation. With base-year volume known, the break-even and the optimal years for paving the road are computed using equations (8) and (11). Table 2 shows these values for base-year volumes ranging from 150 to 400 vehicles per day. The year of paving is taken as zero whenever the base-year volume is in excess of the break-even and optimal volumes, respectively.

It is clear from these results that the difference in the break-even and the optimal years can be large depending upon the base-year volume. This difference reflects the period during which the economy can benefit more from investing the capital elsewhere. Table 3 shows the net benefit of construction deferment computed as the difference in the net present values of paving the road in the years suggested by the optimal and the break-even criteria. As indicated in Table 3 the benefits tend to decline as the base year volume increases. In those cases where the base year volume is much higher than the optimal volume, any delay in paving will lead only to a decrease in the net present value. However, the type of highway considered in this paper is low-volume gravel roads where the base year volume will not generally exceed 400-500 vehicles per day.

Conclusions

This paper has presented an approach to determine optimal timing for paving low-volume gravel roads based on explicit consideration of the opportunity cost of capital. As there are many competing uses for capital at any given time, selecting one use of capital implies the cost of foregoing the opportunity to earn a return with it elsewhere. This situation is particularly important for developing countries where capital is scarce and the opportunity cost is high.

This paper has indicated that the often used break-even criterion is not a desirable approach in determining the year of paving, and that it is economically advantageous to postpone construction beyond the break-even volume until such time when the net present value is maximized. The net present value is maximum when the undiscounted net benefits in the year of paving is equal to zero. By ignoring the effect of the cost and timing of periodic rehabilitation, the optimal time of paving can be approximated by the "First Year Benefit Rule" which states that the time to pave is when the benefits during the first year (savings in maintenance and vehicle operating costs) expressed as a percentage of the construction cost is greater than the opportunity cost of capital. The results of a numerical example considered in this paper indicated that a considerable net savings can be realized by defering paving beyond the break-even volume, particularly in those cases where the base year daily volume is relatively low.

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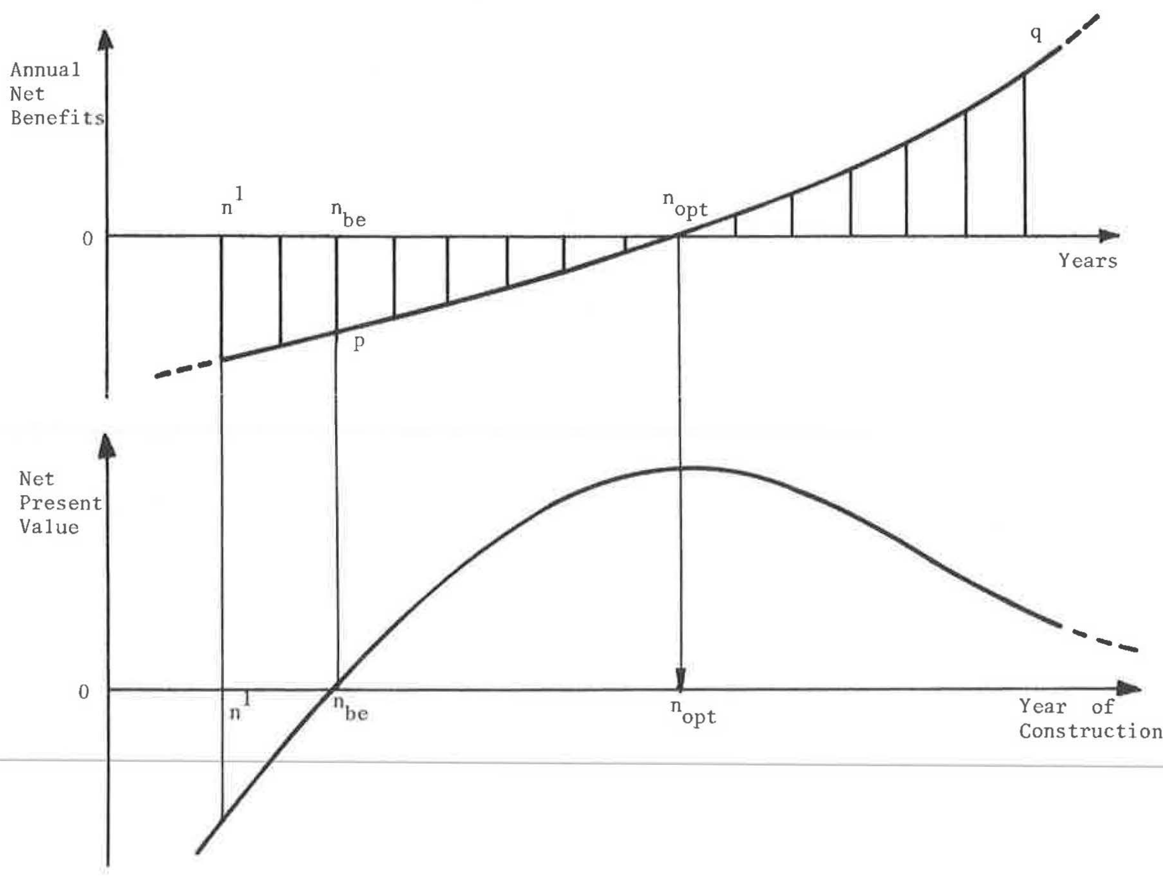

Figure 1. Net Present Value as a Function of the Year of Construction

Table 1. Break-even and optimal volumes (vehicles per day).

Rehabilitation Frequency (N) in Years	10	15	20
Break-Even Volume (Q_{be})	269	232	216
Optimal Volume (Q_{opt})	712	615	571

Table 2. Break-even and optimal year of paving

Base-Year Volume (Q_0)	Rehabilitation Frequency (N) in years 10	15	20
150	10/27*	8/24	6/23
200	5/22	3/19	1/18
300	0/15**	0/12	0/11
400	0/10	0/7	0/6

*Break-even/Optimal year of paving.
**Where base-year volume is in excess of the break-even volume, the break-even year is negative hence taken to be zero.

Table 3. Present value of construction deferment to the optimal year (dollars per km of road)

Base-Year Volume (Q_0)	Rehabilitation Frequency (N) in years 10	15	20
150	7,195	8,274	8,447
200	11,587	13,326	13,605
300	15,928	11,499	8,538
400	8,434	6,073	3,444