Measuring Pavement Performance by Using Statistical Sampling Techniques

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A stratified two-stage sampling survey is described that was selected for use in Texas to obtain cost-effective objective information on road-network performance. The sample was obtained by first randomly selecting counties within each highway district and then randomly selecting 3.2-km (2-mile) highway segments within each county. Approximately 1 percent of total statewide centerline kilometers were sampled by using the technique. Various kinds of data were obtained for each of the sampled highway segments; serviceability index and pavement rating score (visual condition) are used as examples to demonstrate the kinds of inferences that can be made. The type and size of sampling survey that should be used are examined. To make these determinations, one highway district was used in conjunction with a simulation procedure. The results of the simulation study and two separate optimization procedures revealed that two-stage sample sizes, generally about 2 percent of total centerline kilometers, provided good estimates for determining roughness, visual condition, deflection, and skid resistance.

To allocate highway rehabilitation and maintenance funds fairly and consistently, a highway administrator needs information about the actual condition of the road network. He or she can get this information in a variety of ways, some of which are more costly than others. This paper presents a methodology that was applied to pavements in Texas for selecting an optimally cost-effective sample size for collecting information on pavement condition and performance evaluation.

There are two broad categories of pavement evaluation information: subjective and objective. Routine or regular visual inspections of roadways are in the subjective category; objective measurements are made with the aid of mechanical devices and include several methods. In addition, combinations of subjective and objective information are often made.

One of the objective methods is the use of mass-inventory surveys (1). These surveys are used to obtain extensive data on all highways in a given area—state, district, county, and so on. The primary advantage of this type of survey is that all segments of the highway system are carefully surveyed so that all the weaknesses in a given highway are indicated. Presumably, the highway with the greater number of weaknesses would receive corrective maintenance sooner than other pavements that serve the same function. This survey method also allows general inferences to be made about the complete highway system. The most obvious problem with this type of survey is the cost associated with the collection and reduction of data and the interpretation of the results.

A method used to obtain both subjective and objective data is the "partial" survey. A partial survey occurs where some type of preliminary, routine visual examination of the highway system is made. The visual examination is used to identify highway segments that require additional, more detailed information. For example, a highway segment is identified as being severely cracked. Some type of deflection survey is then made to determine the load-carrying capability of the pavement. This survey can then be used to assist in making the proper maintenance decision. One advantage of partial surveys is that they are generally low in cost. The disadvantage is that the data obtained do not allow general inferences to be made about the total highway network (state or district).

This leads to the third type of survey—the sampling survey. This method of obtaining objective data on a highway system has a number of characteristics that can be of value to highway departments.

CHARACTERISTICS AND TYPES OF SAMPLE SURVEYS

The purpose of a sampling survey is to make inferences about the sampled "population" (2). The population in this case is the state-maintained highway network.

In any sampling process, two factors affect the usefulness of the data contained in the sample: the size of the sample and the variability of the data within the sample. The goal of most sampling surveys is to keep the sample size as small as possible while keeping the variability of the data below some maximum acceptable limit. To accomplish this goal, careful consideration should be given to the survey design.

Such surveys are generally inexpensive in comparison with other data collection procedures but can still represent a significant investment. Enough emphasis cannot be placed on the design of a sampling survey to minimize costs while maximizing the information gained from the survey. Some of the survey methods available (2-4) are (a) simple random sampling, (b) stratified random sampling, (c) one-stage cluster sampling, (d) multistage cluster sampling (multistage sampling), and (e) systematic sampling.

A brief description and an example of each of these sampling methods follows:

1. In simple random sampling, every sample has an equal probability of being chosen from a population. For example, if all highways in a given geographic area were divided into equal lengths (segments), each highway segment would have an equal chance of being chosen for the required sample size.
2. In stratified random sampling, a population is divided into strata and then random samples are obtained within the described strata. For example, if a given state were divided into a number of highway department districts and data estimates were required for each district, each district could be considered a stratum and individual highway segments randomly selected within each district.
3. In one-stage cluster sampling, elements within a population are first grouped together and then randomly sampled. For example, if data estimates are required for a state, counties can be randomly selected throughout the state. All highway segments in each selected county are sampled. The pavement segments surveyed are considered to be clustered within the selected counties.
4. Multistage cluster sampling (or multistage sampling) is similar to one-stage cluster sampling but takes the process further. Multistage clustering allows for larger areas to be clustered together and then randomly sampled. The elements within these clusters are also randomly sampled. As in one-stage cluster sampling, counties within a district can be randomly selected and then pavement segments within those counties can be randomly selected. Sampling all data within the pavement segment constitutes a two-stage

cluster sample. If only the data within the pavement segment are sampled, this is referred to simply as a two-stage sample. In a three-stage sample, highway department districts within a state, then counties within those districts, then pavement sections within those counties would all be randomly selected.

5. In systematic sampling, every K th element of a set of data is sampled. For example, if data estimates are required for a state that is assumed to have 100 counties, then every 10 th county from a listing of all counties is selected for a total of 10 counties. All highway segments in each selected county would be sampled in the data collection effort.

Combinations of these five methods can also be created—for example, a stratified two-stage cluster sample.

A properly designed highway sampling survey can provide

1. Inexpensive indication of the condition and performance of statewide, district, or county pavements;
2. Year-to-year differences in pavement condition and performance;
3. A valuable research tool for various statistical pavement experiments;
4. Expansion or reduction to accommodate changing needs; and
5. More detailed objective data since the amount of pavement surveyed is much smaller than that surveyed by mass-inventory methods.

TEXAS SAMPLE SURVEY

A sampling survey has been and is continuing to be done in Texas under the sponsorship of the Texas State Department of Highways and Public Transportation (TSDHPT) and the Federal Highway Administration (FHWA) through the Texas Transportation Institute.

A statistically random selection of 3.2-km (2-mile) long Interstate, U.S. and state, and farm-to-market (FM) highway segments was made during 1973. A stratified two-stage sample was used. The stratification involved dividing the highway network into the 25 TSDHPT districts. This was done because separate data estimates were required for each district since each is considered to have its own unique characteristics (e.g., soils or traffic). The two-stage sample was obtained by first randomly sampling counties in each district and then randomly sampling the 3.2-km-long highway segments in each county. This was done for the three types of state-maintained highways by considering each type to be a separate population. Currently, the percentages of centerline kilometers sampled for the three types of highways are as follows: Interstate, 1.8 percent; U.S.-state, 1.0 percent; and farm-to-market, 0.6 percent. These percentages reflect the importance attributed to each kind of highway and are the result of the sampling method used. A total of 250 highway segments were initially selected by using this process.

Several kinds of data have been collected on the highway segments selected. Most of the data are updated annually by using the same highway segments each year. The following kinds of data are collected:

1. Construction information, including layer thickness and width and available material properties as well as the dates and types of all major maintenance that currently represents the highway segment cross section;
2. Traffic histories, including average daily traffic and 80-kN [18 000-lb (18-kip)] equivalent axle loads applied with time;
3. Climatic data, including monthly rainfall and temperatures, freeze-thaw cycles, and Thornthwaite indices;
4. Roughness, in the form of serviceability indices obtained by using the Mays road meter (5);
5. Visual condition, in the form of distress manifestations obtained primarily by a visual process (6);
6. Deflection measurements obtained by using the Dynaflect;
7. Rut-depth measurements; and
8. Skid number (SN) at a speed of 64.4 km/h (40 mph).

Examples of estimates that can be produced from such data are given in Tables 1 and 2. These two tables indicate statewide and district estimated serviceability index (SI) means and standard errors for data obtained in 1974 and 1976, respectively, for Interstate, U.S.-state, and FM highways. The standard errors can be used to estimate the precision of the survey. Estimated means and standard errors for visual condition and deflection data can also be presented in this way.

Table 1, which gives data obtained in 1974, indicates for the statewide condition that Interstate highways have an average SI of about 4.0, which represents a relatively smooth condition. U.S.-state highways have a mean value of about 3.6 and FM highways a value of about 2.9. The data summarized in Table 1 were obtained at about the same time as one district in Texas conducted a mass-inventory survey. This is discussed in more detail later in this paper.

Table 2 gives the estimated mean values of SI data obtained in 1976. Note that both means and standard errors have decreased with respect to the 1974 data by approximately 0.1 unit for two of the three highway types.

The following questions arise:

1. How "good" are the various estimates based on the current highway segment sample with respect to other (larger and smaller) sample sizes?
2. What is the least costly sample size to achieve adequate estimates?
3. Will some other sampling procedure yield better precision?

An approach toward answering these questions is presented below.

SIMULATION STUDY TO EVALUATE SAMPLING PROCEDURE

To begin to answer the questions posed above, a simulation study was done for one of the 25 Texas highway districts to determine the precision of various highway segment sample sizes. This approach was used because direct experimentation on the highway network was too expensive and direct computation of consistently accurate two-stage sampling errors for various sample sizes was not possible.

The highway district studied was district 21, which is located in the southernmost part of the state. For 1974 and 1975, virtually a complete mass inventory of four major kinds of data was performed on all highway types. Since this district has only 53 km (33 miles) of Interstate highways, Interstate highways were not considered in the simulation study.

The kinds of data used are as follows:

1. Serviceability index, which was obtained every 0.32 km (0.20 mile) by use of the Mays road meter;
2. Pavement rating score (PRS), which ranged be-

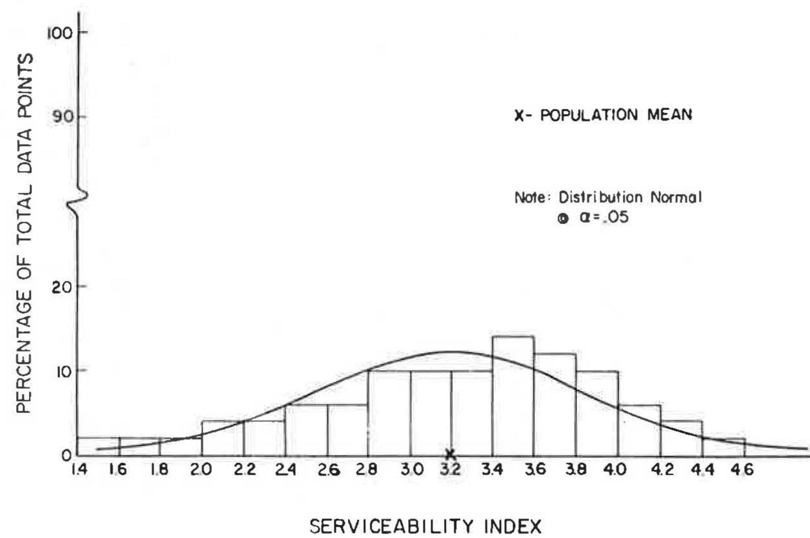
Table 1. Estimated 1974 district and statewide SI means and standard errors for randomly located highway segments by type of highway.

	Interstate		U.S.-State		FM	
District	Mean	Standard Error	Mean	Standard Error	Mean	Standard Error
1	3.4	-	3.6	0.1	2.5	0.2
2	3.1	-	3.7	0.1	2.4	0.2
3	-	-	3.5	0.3	3.2	0.2
4	4.4	0.1	3.8	0.3	3.2	0.2
5	-	-	3.2	0.1	3.2	0.2
6	4.3	-	4.3	0.2	3.6	0.3
7	-	-	3.9	0.2	3.2	0.1
8	4.6	-	2.9	0.2	3.0	0.3
9	4.7	-	3.7	0.4	2.8	0.3
10	-	-	2.9	0.2	2.7	0.3
11	-	-	3.3	0.2	2.0	0.2
12	4.2	-	4.2	0.1	3.4	0.3
13	-	-	3.8	0.2	2.4	0.4
14	-	-	3.9	0.1	2.8	0.2
15	3.4	0.3	3.2	0.2	2.9	0.2
16	3.8	-	3.5	0.1	3.1	0.2
17	-	-	3.2	0.1	2.5	0.3
18	3.4	-	3.9	0.1	2.9	0.3
19	-	-	3.5	0.1	3.1	0.2
20	4.6	-	3.6	0.1	3.3	0.1
21	-	-	3.6	0.1	2.8	0.4
22	-	-	3.3	0.2	3.4	0.1
23	4.3	-	4.0	0.3	2.6	0.1
24	4.4	-	3.5	0.3	2.5	0.4
25	-	-	2.9	0.6	3.0	0.3
Statewide	4.0	0.2	3.6	0.2	2.9	0.2

Table 2. Estimated 1976 district and statewide SI means and standard errors for randomly located highway segments by type of highway.

	Interstate		U.S.-State		FM	
District	Mean	Standard Error	Mean	Standard Error	Mean	Standard Error
1	3.4	-	3.7	0.2	2.2	0.4
2	3.7	-	3.7	0.2	2.1	0.1
3	-	-	3.3	0.4	3.0	0.4
4	4.3	0.3	4.0	0.4	3.0	0.4
5	-	-	2.9	0.3	3.3	0.3
6	4.4	-	4.5	0.3	3.7	0.3
7	-	-	3.9	0.2	3.3	0.2
8	4.6	-	2.7	0.3	2.5	0.5
9	4.5	-	3.5	0.4	2.5	0.3
10	-	-	2.7	0.2	2.4	0.4
11	-	-	3.0	0.4	1.3	0.2
12	4.3	-	4.2	0.1	3.7	0.2
13	-	-	4.0	0.3	2.2	0.6
14	-	-	3.7	0.1	2.8	0.2
15	3.4	0.3	3.4	0.3	2.9	0.3
16	3.5	-	3.4	0.2	2.9	0.3
17	-	-	3.2	0.1	2.0	0.3
18	3.4	-	4.0	0.1	2.8	0.3
19	-	-	3.7	0.1	2.6	0.4
20	4.7	-	3.4	0.1	3.3	0.2
21	-	-	3.7	0.1	3.1	0.6
22	-	-	3.8	0.1	3.9	0.2
23	4.5	-	4.0	0.3	2.2	0.2
24	4.4	-	3.2	0.4	2.4	0.6
25	-	-	2.6	0.7	3.1	0.5
Statewide	4.0	0.2	3.5	0.3	2.8	0.4

Figure 1. District 21 1974 mass-inventory SI data for U.S.-state highways.



tween 100 (no distress) and 0 (a large amount of distress);

3. Skid number at 64.4 km/h (40 mph); and
4. Surface curvature index (SCI), which was obtained by use of the Dynaflect.

From this mass inventory of data, Figure 1 shows a typical plot of SI data for U.S.-state highways distributed by use of a histogram. The normality of these data was checked by using the chi-square test. The null hypothesis tested was that the distribution conforms to a normal distribution. The generated normal curve is shown superimposed on Figure 1. At a level of significance of 0.05 (i.e., a probability of 0.05 of rejecting a true hypothesis), the data test approximately normal. Similar plots made for pavement condition, skid, and deflection data also indicated that such distributions were normally or nearly normally distributed.

Since a mass inventory was available for district 21 for both 1974 and 1975, a comparison was made of the summary statistics for each year. This information is given in Table 3 and shows total kilometers and population means and standard deviations for each data type. The numbers of kilometers given vary between the two years. This occurs primarily for SCI data because the Dynaflect survey was not completed until 1975 and only partial data were available in 1974. It should also be pointed out that there was some overlap of data between the two years for SI and SN data, which reduces potential year-to-year differences. This is not true for PRS since independent surveys of these data were conducted during each of the two years.

The differences between the estimated SI means given for district 21 in Table 1 and the population means given in Table 3 are of interest. The estimates given in Table 1 for U.S.-state and FM highways were obtained from the statewide sample survey for which sampling of highway segments was done in district 21 as well as in the other 24 districts. The population means given in

Table 3. District 21 mass inventory: statistical summary.

Highway Type	Year	Data Type	Number of Kilometers	Mean	Standard Deviation
Interstate	1974	SI	61	3.3	0.6
		SCI	0	-	-
		SN	53	0.35	0.06
		PRS	61	83	8
	1975	SI	60	3.6	0.5
		SCI	61	0.2	0.1
		SN	63	0.38	0.06
		PRS	59	91	6
U.S.-state	1974	SI	1760	3.2	0.7
		SCI	600	0.7	0.5
		SN	1630	0.32	0.10
		PRS	1723	82	13
	1975	SI	1722	3.3	0.7
		SCI	1129	0.6	0.4
		SN	1807	0.34	0.10
		PRS	1745	78	14
FM	1974	SI	2214	2.6	0.7
		SCI	720	0.8	0.4
		SN	1983	0.34	0.09
		PRS	2314	78	16
	1975	SI	2361	2.6	0.8
		SCI	1892	0.8	0.4
		SN	2473	0.35	0.09
		PRS	2374	75	16

Note: 1 km = 0.62 mile.

Table 4. District 21 mass inventory: statistical summary for Zapata County.

Highway Type	Year	Data Type	Number of Kilometers	Mean	Standard Deviation
Interstate	1974	SI	0	-	-
		SCI	0	-	-
		SN	0	-	-
		PRS	0	-	-
	1975	SI	0	-	-
		SCI	0	-	-
		SN	0	-	-
		PRS	0	-	-
U.S.-state	1974	SI	127	3.1	0.5
		SCI	88	0.7	0.3
		SN	123	0.32	0.05
		PRS	124	94	4
	1975	SI	129	3.1	0.6
		SCI	89	0.7	0.3
		SN	133	0.34	0.06
		PRS	128	89	6
FM	1974	SI	38	2.3	0.7
		SCI	32	1.2	0.4
		SN	37	0.39	0.10
		PRS	43	89	8
	1975	SI	53	2.3	0.7
		SCI	44	1.0	0.5
		SN	63	0.38	0.08
		PRS	53	75	25

Note: 1 km = 0.62 mile.

Table 3 were obtained from a complete districtwide mass inventory for each highway type. The differences are 0.4 SI unit for U.S.-state highways and 0.2 SI unit for FM highways (1974 comparison). These variations between the means are believed to result primarily from differences between the Mays road meters used to conduct the surveys and from sampling error. This is discussed in more detail in a later section of this paper.

The treatment used for the entire district 21 was also applied to each county in the district. For example, the summary statistics for Zapata County are given in Table 4 for both 1974 and 1975. Of special significance in this table is that PRS decreased significantly from 1974 to 1975, especially for FM highways. As the PRS means decreased, the standard deviations increased for this county. The source of these year-to-year differences is not known. They could be the result of an increase in pavement deterioration, rating error, or a combination of the two.

After the mass-inventory data had been organized into a computer-accessible form, they were reorganized into a format similar to that of data for the statewide random segments. To accomplish this task, a FORTRAN computer program was written that divided all highways in the district into 3.2-km (2-mile) segments. The program also organized the data contained in each of these 3.2-km segments into the form of a summary that consisted of the number of data points, means, and standard deviations for each of the data types. This information was computed and stored for future processing.

An additional computer program was prepared to access these segments, draw samples, and make estimates of the population mean and standard error for various sample sizes. The computer program performed essentially the same task on all the 3.2-km highway segments as was performed manually to select the statewide sample. This selection process was computerized because hundreds of samples would be selected and statistically summarized.

To select a given sample size, total highway kilometers were multiplied by the sample-size percentage desired. This gave the approximate number of kilometers to be sampled. The number of kilometers thus obtained was divided by 3.2 km to obtain the number of required highway segments. Next, the program randomly selected a county from the total number of counties in the district. Highway segments were then randomly selected within the selected county for both U.S.-state and FM highways. The number of highway segments chosen for each highway type depended on county kilometers and the desired sample size. Additional counties and highway segments were selected until the required sample size for the entire district had been achieved.

To further explain this process, for each trial computer iteration the following numbers of 3.2-km (2-mile) U.S.-state and FM highway segments were selected for district 21 for the given sample sizes, which are based on the percentage of centerline kilometers:

Sample Size (%)	Number of Segments
0.5	6
1	12
2	24
3	35
5	59
10	117

The lower and upper bounds for sample sizes were 0.5 and 10 percent, respectively. A 0.5 percent sample size was felt to represent the smallest reasonable sample that should be considered. Conversely, a 10 percent sample size was felt to represent a more than adequate estimate of the population parameters.

Means and standard errors were computed for each of the sample sizes for both the 1974 and 1975 data. The overall district mean was computed by averaging the means obtained from each of the sample estimates calculated. The formula used to compute the stratified two-stage sample mean is

$$\hat{\bar{\bar{Y}}} = \sum_{1}^{n} M_i \bar{y}_i \Big/ \sum_{1}^{n} M_i \qquad (1)$$

Table 5. District 21 means and standard errors for six sample sizes and 300 sample-selection iterations: 1975 data.

Sample Size (%)	Highway Type	SI		PRS		SCI		SN	
		Mean	SE	Mean	SE	Mean	SE	Mean	SE
0.5	U.S.-state	3.33	0.35	78.6	7.8	0.62	0.20	0.34	0.05
	FM	2.62	0.42	75.8	9.3	0.79	0.21	0.36	0.05
1	U.S.-state	3.31	0.28	78.9	5.7	0.61	0.14	0.34	0.04
	FM	2.61	0.27	75.5	5.6	0.80	0.14	0.36	0.04
2	U.S.-state	3.32	0.17	78.6	3.8	0.60	0.09	0.34	0.03
	FM	2.62	0.18	75.1	3.9	0.78	0.09	0.35	0.02
3	U.S.-state	3.30	0.15	78.2	3.5	0.61	0.08	0.34	0.02
	FM	2.66	0.13	75.0	3.2	0.79	0.07	0.35	0.02
5	U.S.-state	3.30	0.11	78.6	2.5	0.61	0.06	0.34	0.02
	FM	2.65	0.11	75.7	2.4	0.79	0.06	0.35	0.02
10	U.S.-state	3.31	0.08	78.4	1.7	0.60	0.04	0.34	0.01
	FM	2.64	0.07	75.2	1.6	0.79	0.04	0.35	0.01

Table 6. District 21 means and standard errors for three sample sizes and 300 sample-selection iterations: 1974 data.

Sample Size (%)	Highway Type	SI		PRS	
		Mean	SE	Mean	SE
0.5	U.S.-state	3.19	0.35	82.1	7.6
	FM	2.59	0.39	80.4	9.9
1	U.S.-state	3.21	0.27	82.5	5.4
	FM	2.59	0.26	79.9	6.0
3	U.S.-state	3.19	0.15	82.9	3.0
	FM	2.61	0.13	78.8	3.2

where

$\hat{\bar{\bar{Y}}}$ = estimate of district mean for a given sample size, highway type, and data type;
n = number of counties selected for a given sample size;
M_i = number of possible 3.2-km (2-mile) highway segments within a county; and
$\bar{y}_i$ = estimate of mean value for the ith county.

Equation 1 was used to compute a sample mean for each highway and data type considered. This was repeated for 300 sample-selection iterations. Each of the 300 district estimates so calculated was used in calculating the overall district mean.

The simulation standard error (SE) was computed based on the means obtained by using Equation 1. The formula used to accomplish this is

$$SE = \sum_{1}^{t} (\hat{\bar{\bar{Y}}} - \bar{\bar{Y}})^2 / (t-1)^{1/2} \quad (2)$$

where $\bar{\bar{Y}}$ is the average of all district estimates for a given sample size, highway type, and data type and t is the number of sample-selection iterations for a given sample size (300 in all cases). This formula is similar to that used for calculating the standard deviation of a set of data and is calculated differently from the standard error computation for a sample used in Tables 1 and 2.

The overall means and standard errors computed by Equations 1 and 2 are given in Tables 5 and 6. Table 5 gives the overall means and standard errors for six sample sizes for data obtained primarily during 1975, and Table 6 gives the same kind of data for 1974. The data processed for 1974 were not as extensive as those for 1975 because of the incompleteness of 1974 SN and SCI data. In addition, the 1974 data presentation is intended only as a check on the 1975 data. As should be expected, the data contained in both tables indicate that standard error decreases as sample size increases. If all possible highway segments were repeatedly sampled (100 percent sample sizes), the standard error would approach zero.

It is of interest to compare the above method of obtaining standard error with that used in simple random sampling, which would involve sampling the required highway segments by using a completely random pattern throughout a district. The standard error of various sizes of simple random samples can be computed as follows:

$$SE = \sigma_{\bar{y}} = (S/\sqrt{n})\sqrt{1-(n/N)} \quad (3)$$

where

S = standard deviation of the population,
n = number of 3.2-km (2-mile) highway segments sampled for a given sample size,
N = total number of 3.2-km highway segments in the district, and
(n/N) = sampling fraction.

Standard errors for a simple random sampling technique were computed by using Equation 3 and the population standard deviations in Table 3 for the 1975 data. The values so calculated were compared with standard errors obtained from the simulation study for the two-stage sampling technique. Table 7 gives a comparison of both standard errors for different sample sizes and highway and data types.

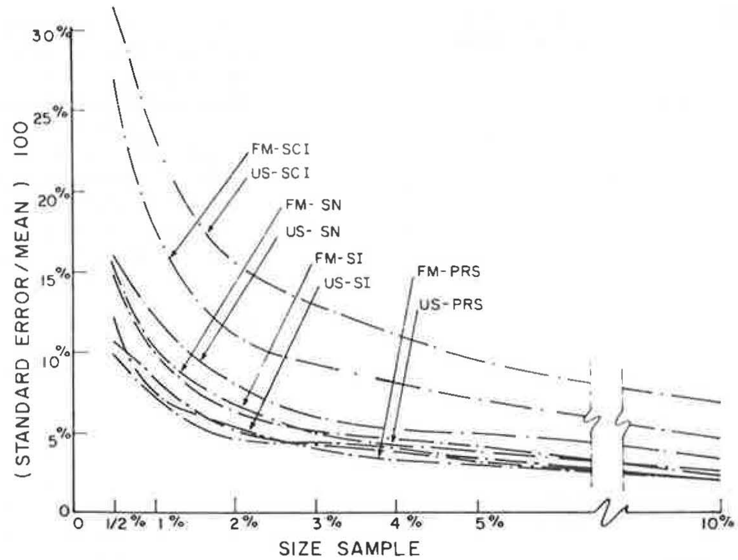
The data given in Table 7 reveal that the standard errors obtained for the two-stage sampling technique are in most cases lower than those calculated by simple random sampling. Of 48 possible comparisons, the two-stage standard errors are lower in 34 cases, the same in 9 cases, and larger in 5 cases. The largest observed difference is 50 percent, in which case the standard error obtained by simple random sampling is the larger.

The primary goal of this study of sample size was to determine the optimum sample size for each combination of highway and data type. Figure 2 is a plot of sample size versus standard error divided by the mean times 100. The ordinate term, called the coefficient of sample variation, is analogous to a coefficient of variation and allows the standard errors for each data type to be compared. The figure shows that the variability of a given sample size decreases rapidly at first and then begins to stabilize at about 10 percent. For SI, PRS, and SN, the coefficient of sample variation at a 0.5 percent sample size ranges from 10 to 15

Table 7. District 21 standard errors for simple random and two-stage sampling techniques.

Sample Size (%)	Highway Type	Simple Random Sample SI	PRS	SCI	SN	Two-Stage Sample SI	PRS	SCI	SN
0.5	U.S.-state	0.49	9.9	0.28	0.07	0.35	7.8	0.20	0.05
	FM	0.40	3.0	0.20	0.04	0.42	9.3	0.21	0.05
1	U.S.-state	0.31	6.2	0.18	0.04	0.28	5.7	0.14	0.04
	FM	0.30	6.0	0.15	0.03	0.27	5.6	0.14	0.04
2	U.S.-state	0.22	4.4	0.13	0.03	0.17	3.8	0.09	0.03
	FM	0.21	4.2	0.11	0.03	0.18	3.9	0.09	0.03
3	U.S.-state	0.18	3.7	0.11	0.03	0.15	3.5	0.08	0.02
	FM	0.17	3.4	0.09	0.02	0.13	3.2	0.07	0.02
5	U.S.-state	0.14	2.8	0.08	0.02	0.11	2.5	0.06	0.02
	FM	0.13	2.6	0.06	0.02	0.11	2.4	0.06	0.02
10	U.S.-state	0.10	2.0	0.06	0.01	0.08	1.7	0.04	0.01
	FM	0.09	1.8	0.05	0.01	0.07	1.6	0.04	0.01

Figure 2. District 21 coefficient of sample variation versus sample size: 1975 data.



percent. At a 10 percent sample size, this coefficient ranges from about 3 to 5 percent. The exception is the coefficient for SCI, which ranges from about 27 to more than 30 percent at a 0.5 percent sample size and less than 10 percent at a 10 percent sample size.

Although the data shown in Figure 2 give a good indication of the precision gained with increasing sample size, a better gauge was sought to answer the question, How large is large enough? To answer this question, a procedure for minimization of variance was used for various levels of fixed survey costs. A simple utility method was also developed as an independent check of this procedure.

The procedure for minimization of variance is described in a number of sampling survey texts (4, 7). This technique minimizes the variance of the estimated mean for a fixed survey cost. The procedure is possible since both the number of sampled counties in a district and the number of 3.2-km (2-mile) highway segments sampled within a county are considered within the variance term. Lagrange multipliers are used to determine a minimum variance as a function of the number of highway segments within a county. The following equation results from this procedure:

$$m_{opt} = S_2/\sqrt{S_1^2 - (S_2^2/M)}\sqrt{c_1/c_2} \quad (4)$$

where

m_{opt} = optimum number of 3.2-km (2-mile) highway segments per county;
S_1^2 = variance among county means in a district;
S_2^2 = variance among 3.2-km highway segments within counties;
M = total number of potential 3.2-km highway segments within a county;
c_1 = costs associated with sampling a county, including travel costs; and
c_2 = costs associated with obtaining a specific type of data within a 3.2-km highway segment.

The optimum number of counties can now be determined for a fixed survey cost by use of the following equation:

$$C = c_1 n + c_2 nm \quad (5)$$

where

C = total available budget for the survey,
n = number of counties to be sampled in a district, and
m = m_{opt}.

The appropriate variances and costs were determined from data available from the district 21 mass inventory and the prior statewide two-stage sample surveys. The optimum sample sizes for a district were determined for both U.S.-state and FM highways and the four data types. This process first involves using Equation 4 to determine the optimum number of highway segments to sample in each sampled county. The resulting optimum numbers of highway segments, given in Table 8, range from a minimum of two to a maxi-

Table 8. Procedure of variance minimization to determine optimum sample size.

Highway Type	Data Type	Ratio of Costs (c_1/c_2)	Highway Segments per County (m_{opt})	Budget ($/district)	Counties per District (n)	Optimum Sample Size (%)
U.S.-state	SI	1.6	2	100	2	0.9
				200	5	2.2
				400	9	3.9
	SCI	1.0	3	250	2	1.3
				500	3	2.0
				1000	7	4.6
	SN	4.0	2	100	2	0.9
				200	3	1.3
				400	7	3.0
	PRS	1.6	2	150	2	0.9
				300	5	2.2
				600	9	3.9
FM	SI	1.6	2	100	2	0.6
				200	5	1.4
				400	9	2.6
	SCI	1.0	2	250	2	0.6
				500	5	1.4
				1000	9	2.6
	SN	4.0	2	100	2	0.6
				200	3	0.9
				400	7	2.0
	PRS	1.6	4	150	2	1.1
				300	3	1.7
				600	6	3.4

Table 9. Comparison of district 21 two-stage random sample and population means: 1974 data.

Original Sample Size (%)	Highway Type	Data Type	Original Sample Mean	Population Mean	Mean Plus One Standard Error	Mean Minus One Standard Error
0.9	U.S.-state	SI	3.6	3.2	3.5	2.9
		PRS	85	82	88	76
0.6	FM	SI	2.8	2.6	3.0	2.2
		PRS	76	78	87	69

mum of four. Then, if the fixed survey budgets are known, the appropriate number of counties per district can be calculated by using Equation 5.

Three budget levels were selected for each combination of highway and data type to represent the minimum (low), expected (medium), and maximum (high) budget levels that can be expected from TSDHPT funding. The budgets for each data type were the same regardless of highway type. This weights the U.S.-state highways since they have fewer kilometers in a district than farm-to-market highways. In light of these budgets, the range of counties per district to be sampled was calculated (Table 8). The overall optimum sample size for a district can now be calculated by multiplying the number of segments per county by the number of counties to be sampled within a district. This result is multiplied by the 3.2-km (2-mile) length of each segment and is then divided by the appropriate total highway district kilometers. Kilometers of U.S.-state and FM highways for district 21 were used to perform this final calculation.

The resulting sample sizes, given in Table 8, range from a low of 0.6 percent for the low budget to a high of 4.6 percent for the highest budget. More specifically, for U.S.-state highways and the four data types, the mean optimum sample size is 1 percent for the low budget, 1.9 percent for the medium budget, and 3.8 percent for the high budget. For FM highways, the optimum sample sizes for these three budgets are 0.7, 1.4, and 2.6 percent, respectively.

In addition to the procedure of variance minimization, a utility method was developed to provide an independent check on the optimum sample size. That procedure is not described in this paper since it has been reported elsewhere (8). It suffices that the results of the two optimization methods provide similar results for approximately equivalent budgets.

Finally, a comparison between the two-stage random sample means obtained for the highway segments originally selected in district 21 as part of the statewide sample, the district population means, and simulation standard errors is appropriate. Only the SI and PRS data types for each highway type are considered in this case (see Table 9).

The sample sizes given in Table 9 are for the original two-stage samples. For U.S.-state highways, the actual sample size was 0.9 percent; for FM highways, it was 0.6 percent. This consisted of four U.S.-state 3.2-km (2-mile) segments and four FM segments. The population means and the simulation standard errors are compared with the original sample means. It can be seen that all means except one compare favorably.

The population means plus or minus one standard error are also given in Table 9 for the actual sample sizes used. Approximately 68 percent of all possible sample means for the given sample sizes should fall within these ranges. For U.S.-state highways, this range is 0.6 SI units for the 0.9 percent sample, less than 0.4 for a 2 percent sample (not given in the table), and less than 0.2 for a 10 percent sample (not given in the table). By using a different highway and data type, PRS ranges for FM highways are 18 PRS units for a 0.6 percent sample, less than 8 for a 2 percent sample (not given in the table), and slightly more than 3 for a 10 percent sample (not given in the table). This again demonstrates how the range of the standard error decreases with increasing sample size.

SUMMARY AND CONCLUSIONS

The state of Texas has used a stratified two-stage random sample to obtain a limited amount of highway performance data throughout the state. Highway segments 3.2 km (2 miles) long were used, and approximately 1 percent of total statewide centerline kilometers was sampled. Information on construction, traffic, climate, roughness, visually determined condition, deflection, rut depth, and skid resistance was obtained for each of the sampled highway segments. District and statewide estimates of serviceability index for 1974 and 1976 were indicated.

To examine the method and size of sampling survey currently used in Texas, simulation techniques were used on a complete set (mass inventory) of data available for one highway district, district 21. The precision (as measured by standard error) of the two-stage sampling method was shown to be superior to that of simple random sampling. In addition, a procedure of variance minimization and a utility method both indicated that about a 2 percent sample of total centerline kilometers appears to best minimize sampling error. The analysis further shows that, for Texas conditions, approximately two highway segments for each highway type should be sampled in each sampled county. The above information was determined by using four types of data: serviceability index, pavement rating score, surface curvature index (deflection), and skid number. For two of these data types, the estimates provided by the portion of the original statewide sample in district 21 are generally in reasonable agreement with the population means obtained for that district even though the sample sizes used are about half the optimum size.

The information provided by the sample sizes currently used in Texas is most reliable for statewide data estimates and next most reliable for district estimates. Current instrument, personnel, and sampling errors make small year-to-year variations in district data difficult to detect, but reductions in all three error sources are continuing to be made.

Some highway-oriented government agencies may wish to conduct a sampling survey that conforms to a selected precision. Thus, a determination of optimum sample size may not be necessary for such agencies.

A sampling survey will not answer all of the important questions about the condition and performance of a highway network, but it can provide a significant amount of valuable, relatively inexpensive information. To that end, the information contained in this paper could be used by any state or other government agency in planning a sampling survey.

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Laboratory Testing of a Full-Scale Pavement: The Danish Road-Testing Machine

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Full-scale pavements can be tested under controlled climatic conditions and with a controlled groundwater level by using the Danish road-testing machine. The response of the pavement in terms of stresses, strains, and deflections can be monitored during performance tests of a maximum ten-thousand 65-kN wheel loads/day. A qualitative evaluation of pavement response during the first two test series (0.5 million loads) has confirmed