

# Analysis of Freeway Traffic Time-Series Data by Using Box-Jenkins Techniques

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This paper investigates the application of analysis techniques developed by Box and Jenkins to freeway traffic volume and occupancy time series. A total of 166 data sets from three surveillance systems in Los Angeles, Minneapolis, and Detroit were used in the development of a predictor model to provide short-term forecasts of traffic data. All of the data sets were best represented by an autoregressive integrated moving-average (ARIMA) (0,1,3) model. The moving-average parameters of the model, however, vary from location to location and over time. The ARIMA models were found to be more accurate in representing freeway time-series data, in terms of mean absolute error and mean square error, than moving-average, double-exponential smoothing, and Trigg and Leach adaptive models. Suggestions and implications for the operational use of the ARIMA model in making forecasts one time interval in advance are made.

In computer-supervised traffic-surveillance systems the control decisions are often based on forecasts of traffic-stream time-series data gathered in real time. One of the many applications of traffic time series in traffic surveillance and control is to urban freeways for determining control strategies for ramp metering, incident detection, and variable message advisory or warning signs. Most vehicle delay on arterial streets, for example, occurs at traffic signals. The sophisticated intersection control strategies that have been developed to alleviate such delay are based on traffic time-series data. These data can also be used to determine changes in traffic demand patterns, onset of peak-period conditions, and occurrence of traffic congestion during special events such as concerts and athletic events.

Computer control strategies usually require forecasts of the traffic variables long before implementation. These forecasts are based on past observations of the variable time series. In freeway surveillance and control systems, a forecast for the next minute is usually needed because changes in traffic flow can occur suddenly. Also, when this forecast is compared with the next observation of the traffic variable, it can signal a possible change in the traffic-stream behavior and can suggest a suitable control response.

The behavior of traffic time series has been the subject of much theoretical and experimental research work in recent years. Two analysis techniques have been commonly used: spectral analysis and discrete time-series analysis. Spectral analysis of time series as discussed by Jenkins and Watts (1) has been applied by Nicholson and Swann (2) to make short-term forecasts of traffic flow volumes in tunnels. Lam and Rothery (3) used the same technique to study the propagation of speed fluctuations on freeways. Also, Darroch and Rothery (4) used cross-spectral analysis of car-following data to explain the dynamic characteristics of a freeway traffic stream. Discrete time-series analysis has been used by Hillegas, Houghton, and Athol (5), who proposed a Markovian first-order autoregressive model when traffic occupancy exceeded 15 percent, and by Breiman and Lawrence (6), who explored short- and long-term fluctuations in traffic flow.

## PURPOSE

The purpose of this paper is to investigate the applica-

tion of the techniques developed by Box and Jenkins (7) to freeway traffic time series. Polhemus (8) previously applied them to a description of local fluctuations in air-traffic operations; Der (9) applied them to Chicago freeway occupancy data; and Eldor (10) applied them to Los Angeles freeway and ramp traffic data, although Eldor's data consisted of 5-min aggregations of volume time-series data.

In this paper, Box-Jenkins techniques are used to develop a forecasting model based on traffic volumes and occupancies by using data from three freeway surveillance systems in Los Angeles, Minneapolis, and Detroit. A total of 166 time series representing more than 27 000 min of observation were used in the development and evaluation of the model. Table 1 summarizes the data sources and types. The data from Los Angeles and Minneapolis are described by Payne and Heifenbein (11), while the data from Detroit are described by Cook and Cleveland (12). The Los Angeles data are 20-s volumes and occupancies per lane, and the data from Minneapolis and Detroit are volumes and occupancies aggregated over all lanes at 30- and 60-s intervals, respectively. Figure 1 shows representative plots of volume and occupancy time series at detector station 7 of I-35 in Minneapolis.

The performance of the model is tested and evaluated in comparison with three other ad hoc smoothing models: the moving-average model, the double-exponential smoothing model, and the Trigg and Leach adaptive model. Performance evaluations are based on the forecasting errors caused by each model.

## BOX-JENKINS APPROACH TO TIME-SERIES ANALYSIS

The Box-Jenkins approach (7) is used here to construct a predictor model for freeway traffic-stream variables. Let  $X_t$  represent a nonseasonal time series of observations taken at equally spaced time intervals.  $X_t$  is either stationary or reducible to a stationary form  $Z_t$  by computing the difference for some integer number of times  $d$  such that

$$Z_t = (1 - B)^d X_t \quad (1)$$

where  $B$  is backshift operator defined as  $BX_t = X_{t-1}$ .

Mathematically, a stationary time series is one for which the probability distribution of any  $(K + 1)$  observations  $(Z_t, \dots, Z_{t-K})$  is invariant with respect to  $t$ . Any set of observations from a stationary series will have the same mean value,  $\mu$ .

Many real-time series can be represented by the following general class of linear models:

$$\Phi_p(B)(1 - B)^d(X_t - \mu) = \Theta_q(B)a_t \quad (2)$$

where

$p, d, q$  = nonnegative integers,

$\mu$  = mean of the series,

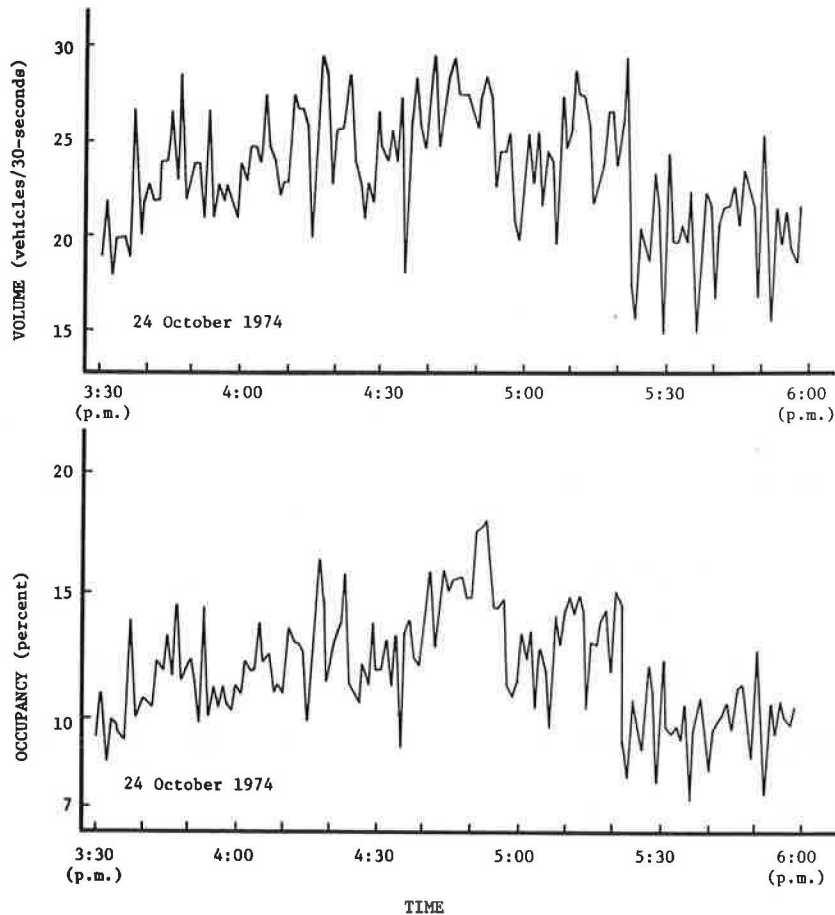
$\Phi_p(B)$  = autoregressive operator of order  $P$  or

Table 1. Data sources and types.

Freeway Location	Detection Hardware	Data Description			
		Type	Aggregation Interval (s)	No. of Intervals per Set	No. of Data Sets
Los Angeles	Induction loops	Volume <sup>a</sup>	60	175	10
		Volume <sup>b</sup>	20	175	30
		Volume <sup>b</sup>	60	525	30
		Occupancy <sup>a</sup>	60	175	10
		Occupancy <sup>b</sup>	20	175	30
		Occupancy <sup>b</sup>	60	525	30
Minneapolis	Induction loops	Volume <sup>a</sup>	30	150	10
		Occupancy <sup>a</sup>	30	150	10
Detroit	Ultrasonic	Volume <sup>a</sup>	60	260	2
		Occupancy <sup>a</sup>	60	121	2
				260	2

<sup>a</sup> Aggregated over lanes.    <sup>b</sup> Per lane.

Figure 1. Freeway traffic volume and occupancy series, Minneapolis, I-35, station 7.



$1 - \phi_1 B \dots - \phi_p B^p, |\phi| < 1,$   
 $\Theta_q(B) =$  moving-average operator of order  $q$  or  
 $1 - \theta_1 B - \theta_2 B^2 \dots - \theta_q B^q, |\theta| < 1,$  and  
 $a_t =$  random disturbances (known as white noise)  
 assumed to be independently distributed as  
 $N(0, \sigma_a^2).$

The models in Equation 2 are autoregressive integrated moving-average (ARIMA) models of order  $(p, d, q)$ .

ARIMA models are fitted to a particular data set by a three-stage iterative procedure: preliminary identification, estimation, and diagnostic check. In preliminary identification, the values of  $p$ ,  $d$ , and  $q$  are determined by inspecting the autocorrelations and partial autocorrelations of the series or its differences, or both, and by comparing them with those of some basic

stochastic processes. The sample autocorrelation function is given by

$$r_K = \frac{\sum_{t=1}^{n-K} [(X_t - \bar{X})(X_{t+K} - \bar{X})]}{\sum_{t=1}^n (X_t - \bar{X})^2}, K = 1, 2, \dots \quad (3)$$

where  $\bar{X}$  is the sample mean and  $n$  is the number of observations. The autocorrelation function of a stochastic process provides a measure of how long a disturbance in the system affects the state of the system in the future.

In general, the autocorrelation function of a moving-average process of order  $q$  has a cutoff after lag  $q$  (memory of lag  $q$ ), while its partial autocorrelation function tails off. Conversely, the autocorrelation function of an autoregressive process of order  $p$  tails off in

Figure 2. Sample autocorrelations and partial autocorrelations, volume data, Minneapolis, I-35, station 7.

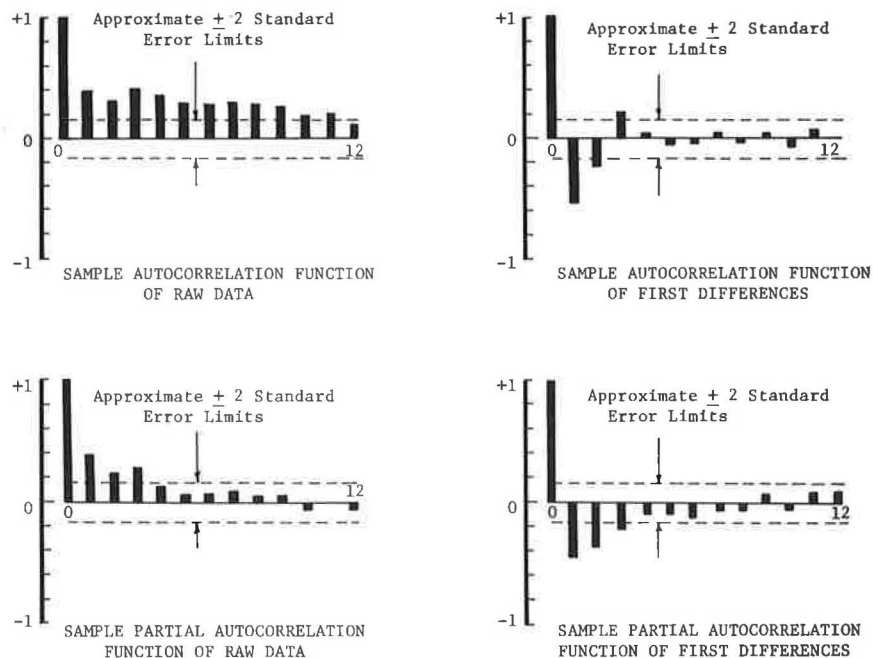
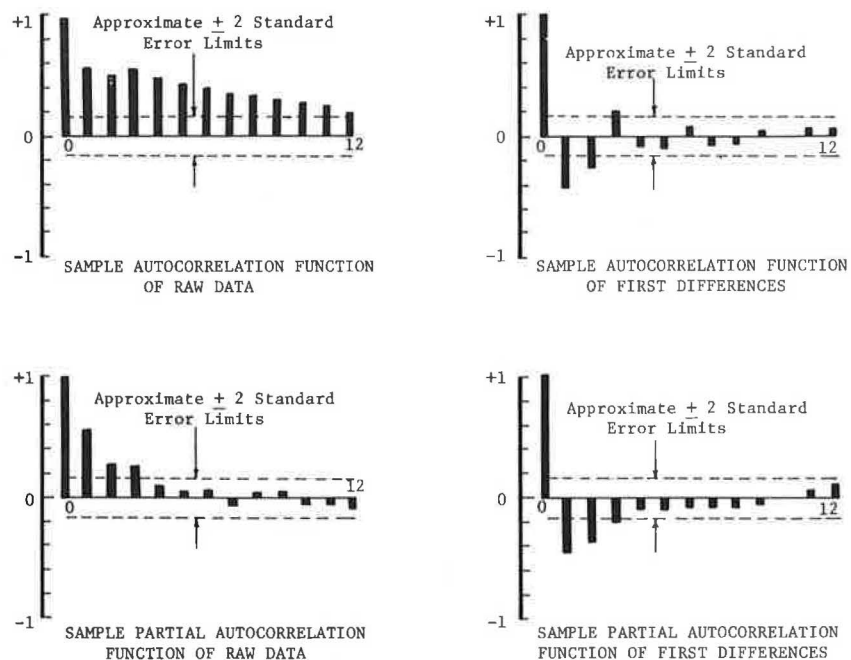


Figure 3. Sample autocorrelations and partial autocorrelations, occupancy data, Minneapolis, I-35, station 7.



the form of damped exponentials or damped sine waves, while its partial autocorrelation function has a cutoff after lag  $p$ . For mixed processes, both the autocorrelations and partial autocorrelations tail off. Failure of the autocorrelation function to die out rapidly suggests that differencing is needed ( $d > 0$ ).

Once the values of  $p$ ,  $d$ , and  $q$  have been determined, the autoregressive and moving-average parameters are estimated by using nonlinear least-squares techniques. Finally, the goodness of the model fit is checked. If the form of the chosen model is satisfactory, then the resulting residuals,  $\hat{a}_t$ , should be uncorrelated random deviations. To test for this, Box and Pierce (13) developed an overall test of residual autocorrelations for lags 1 through  $K$ . They found that the variable

$$Q = n \sum_{i=1}^K r_i^2(\hat{a}) \quad (4)$$

where  $n$  is the number of observations minus the degree of differencing and  $r_i(\hat{a})$  is residual autocorrelation for lag  $i$ .  $Q$  is approximately distributed as a chi-square variable with  $(K-p-q)$  degrees of freedom.

#### MODELING FREEWAY TRAFFIC TIME-SERIES DATA

Three computer programs entitled PDQ, ESTIMATE, and FORECAST (14) were used in this research to perform the computations required by the Box-Jenkins technique. Application to all of the time series listed in

Table 1 resulted in the same ARIMA model, albeit with different coefficients.

Figures 2 and 3 show the sample autocorrelations and partial autocorrelations for the representative volume and occupancy series given in Figure 1 and for their first differences. The sample autocorrelations of the raw data damp off very slowly as lag increases. This suggests that differencing is needed.

The sample autocorrelations of the first differences, however, indicate that only the spikes at lags 1, 2, and 3 are large in relation to their standard error. The partial autocorrelations of the first differences gradually tail off. This confirms that the stochastic process generating the data is ARIMA (0, 1, 3); that is, the first differences of traffic data can be represented by a third-order moving-average model:

$$(1 - B)(X_t - \mu) = (1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3)a_t, \quad |\theta| < 1 \quad (5)$$

or simply

$$\begin{aligned} X_t - X_{t-1} &= Z_t \\ &= a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3} \end{aligned} \quad (6)$$

The model in Equation 6 states that the series of differences  $Z_1, Z_2, \dots, Z_t, \dots$  is a series of moving linear combinations of  $(a_0, a_1, a_2, a_3), (a_1, a_2, a_3, a_4), \dots$ , and  $(a_{t-3}, a_{t-2}, a_{t-1}, a_t), \dots$ , with weight functions  $(-\theta_3, -\theta_2, -\theta_1, 1)$ . It is perhaps more meaningful, however, to view the model as showing that shock  $a_t$  coming into the system at time  $t$  will persist over  $(3 + 1)$  periods ( $t, t + 1, t + 2, t + 3$ ) in proportion to  $(1, -\theta_1, -\theta_2, -\theta_3)$  before dissipation. The vector  $(1, -\theta_1, -\theta_2, -\theta_3)$ , which is the mirror image of the weight function  $(-\theta_3, -\theta_2, -\theta_1, 1)$ , is called the shock-effect function. The coefficients of the volume and occupancy series shown in Figure 1 are

Data	Coefficient	Standard Error
Volume		
$\theta_1$	0.7823	0.0825
$\theta_2$	0.0557	0.0105
$\theta_3$	0.0844	0.0082
Occupancy		
$\theta_1$	0.6852	0.0825
$\theta_2$	0.0627	0.0099
$\theta_3$	0.0741	0.0082

Diagnostic checking was carried out by inspecting the residuals  $(\hat{a}_t)$ . The autocorrelation functions of the residuals and the residual plots for volume and occupancy data are shown in Figures 4 and 5, where the autocorrelations exhibit no systematic pattern and are all quite small. For the volume series, the average of the residuals  $(\bar{a})$  is 0.0221, and the estimated standard error of  $\bar{a}$  is 0.2362. This strongly suggests that the  $a_t$  have zero mean. Similarly, the average of the residuals for the occupancy series is 0.0196 and has an estimated standard error of 0.1402, which supports the same conclusion.

The values of  $Q$  for  $K = 24$  lags (a value set in the Box-Jenkins programs in this study) are 27.6 and 21.8 for the volume and occupancy series, respectively. When these values of  $Q$  are compared with tabulated chi-square values with 21 degrees of freedom, they indicate that the residuals are white noise at the 0.05 level of significance.

Der (9), in his analysis of two occupancy series from the Dan Ryan Expressway in Chicago, suggested an ARIMA (1, 0, 1) process to describe traffic occupancies.

However, he reported that a higher-order ARIMA process such as (0, 1, 3) may be a possible candidate process. The problem with an ARIMA (1, 0, 1) process is that it assumes that the raw traffic time series is stationary, which is not always true. Eldor (10) evaluated the ARIMA series (0, 1, 1), (0, 1, 0), and (0, 2, 1).

Some freeway surveillance systems have detectors in all lanes, while other systems have detectors only in some lanes. Also, surveillance data are generally aggregated over different time intervals, usually 20, 30, or 60 s before processing. The transferability of the ARIMA (0, 1, 3) model under these conditions was studied by applying the model to different time series from the three different freeway systems in Table 1.

Tables 2 and 3 show the range of values of the moving-average parameters for 46 series of volume and occupancy aggregated over lanes at a detector station. Although there are some differences in the parameter estimates between or within the different freeway systems, it is emphasized that it is the form of the ARIMA model that is transferable. The differences in parameter estimates arise from variations in flow characteristics and, probably, variations in geometrics and similar factors. Eldor (10) also noted that no universal parameters could be identified with his data aggregated to 5-min intervals.

In addition, the data from Los Angeles, which consist of 20-s compilations of volume and occupancy per lane, provided an opportunity to compare individual lane data with data aggregated across all lanes at a detector station. The ARIMA (0, 1, 3) model was applied to 60 series of 20-s lane volumes and occupancies. The model process was found representative in all these cases. Tables 4 and 5 give the range of values of the moving-average parameters for lane volumes and occupancies. The effect of sampling interval was also investigated by aggregating the 20-s observations to 60-s observations, which also confirmed the model. Therefore, it is concluded that the model can be successfully used in a variety of freeway surveillance configurations to provide short-term forecasts of traffic volumes and occupancies.

## COMPARATIVE EVALUATION OF FORECASTING PERFORMANCE

This section presents a comparative evaluation of the forecasting performance of the model in Equation 6 against three ad hoc smoothing models: the moving-average model, the double-exponential smoothing model, and the Trigg and Leach adaptive model. To facilitate the discussion, these smoothing models are briefly reviewed.

### Moving-Average Model

The moving average at time  $t$  defined over the  $N$  previous observations is given by

$$m(t, N) = (1/N) \sum_{k=1}^N X_{t-k} \quad (7)$$

This model weights each of the previous  $N$  observations by  $1/N$ , while other earlier observations have zero weight. The forecast of  $X_t$  is

$$\hat{X}_t = m(t, N) \quad (8)$$

Five values of  $N$  ( $N = 5, 10, 20, 50, \text{ and } 100$ ) were used in the evaluation of the moving-average model in this study.

### Exponential Smoothing Model

It is assumed that the observation  $X_t$  can be described by a model of the form

$$X_t = F_t + \epsilon_t \quad (9)$$

where  $F_t$  is a deterministic function of time and  $\epsilon_t$  is a stochastic component. Single exponential smoothing as proposed by Brown (15) assumes that  $F_t$  represents some equilibrium level; the corresponding smoothing function is given by

$$S_1(t) = \alpha x_t + (1 - \alpha) \times S_1(t - 1) \quad (10)$$

Figure 4. Residual plots and sample autocorrelations, volume data, Minneapolis, I-35, station 7.

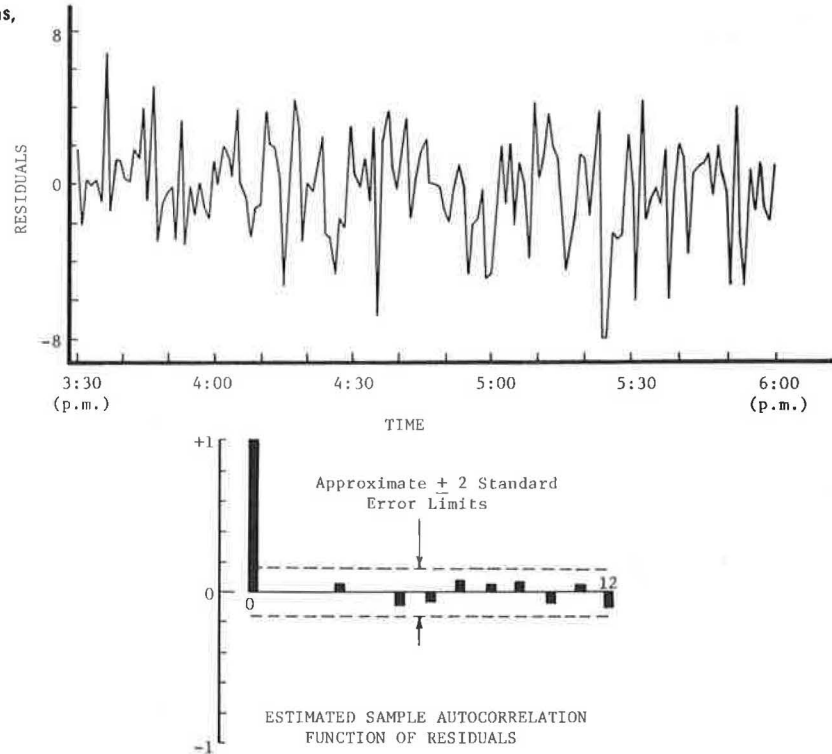
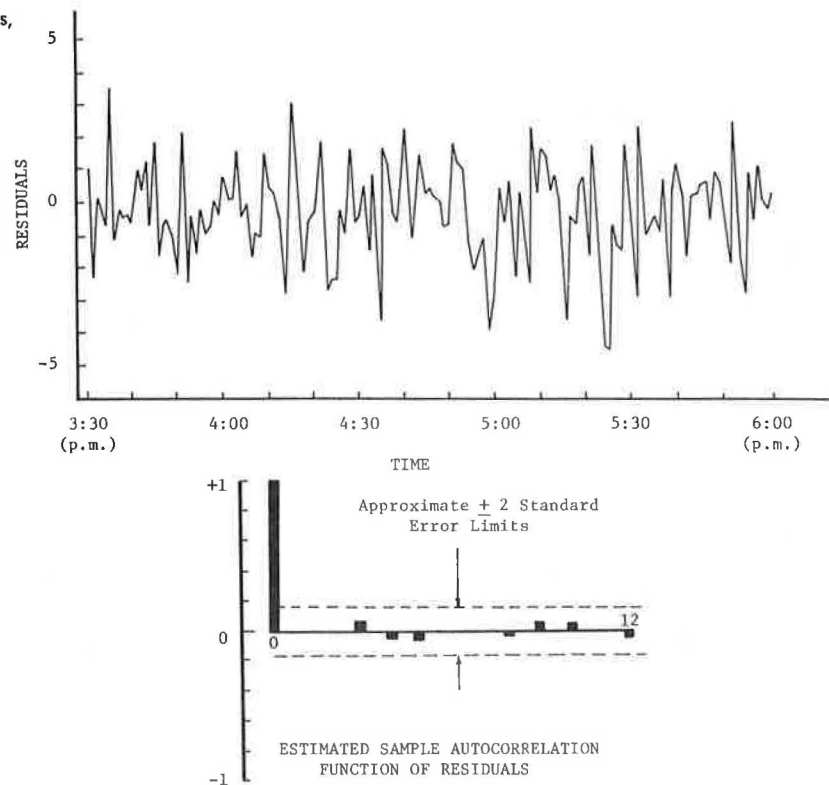


Figure 5. Residual plots and sample autocorrelations, occupancy data, Minneapolis, I-35, station 7.



**Table 2. Moving-average parameters for volume series aggregated over lanes.**

Freeway Location	No. of Data Sets	No. of Observations	Moving-Average Parameters		
			$\Theta_1$	$\Theta_2$	$\Theta_3$
Los Angeles	10	1750	0.7301 ± 0.1885	0.1777 ± 0.5765	0.0391 ± 0.1398
Minneapolis	10	1500	0.7553 ± 0.1375	0.1519 ± 0.5249	-0.1530 ± 0.1438
Detroit	2	381	0.7420 ± 0.0732	0.0403 ± 0.0671	0.0012 ± 0.0143

**Table 3. Moving-average parameters for occupancy series aggregated over lanes.**

Freeway Location	No. of Data Sets	No. of Observations	Moving-Average Parameters		
			$\Theta_1$	$\Theta_2$	$\Theta_3$
Los Angeles	10	1750	0.5611 ± 0.3541	0.1145 ± 0.2711	0.2596 ± 0.3507
Minneapolis	10	1500	0.4710 ± 0.2160	0.1307 ± 0.2515	-0.0848 ± 0.2249
Detroit	2	762	0.6121 ± 0.1649	0.0859 ± 0.1398	0.0704 ± 0.1678

**Table 4. Moving-average parameters for 20- and 60-s lane volumes, Los Angeles.**

Lane No. <sup>a</sup>	Moving-Average Parameters					
	20-s Series (15 750 observations)			60-s Series (5250 observations)		
	$\Theta_1$	$\Theta_2$	$\Theta_3$	$\Theta_1$	$\Theta_2$	$\Theta_3$
1	0.8081 ± 0.1263	0.0752 ± 0.1145	0.0426 ± 0.0742	0.8280 ± 0.1720	0.0311 ± 0.1786	0.0085 ± 0.1129
2	0.8701 ± 0.1245	0.0404 ± 0.1133	0.0401 ± 0.0738	0.7860 ± 0.1127	0.0056 ± 0.1788	0.0229 ± 0.1222
3	0.8131 ± 0.1582	0.0296 ± 0.1253	0.0074 ± 0.0933	0.8180 ± 0.1414	-0.0389 ± 0.3364	0.0412 ± 0.1466
4	0.8811 ± 0.1130	0.0569 ± 0.0573	0.0278 ± 0.0622	0.4526 ± 0.3510	0.1250 ± 0.2358	0.0437 ± 0.0904

<sup>a</sup>Numbering begins with the lane closest to the median and increases toward the right shoulder.

**Table 5. Moving-average parameters for 20- and 60-s lane occupancies, Los Angeles.**

Lane No. <sup>a</sup>	Moving-Average Parameters					
	20-s Series (15 750 observations)			60-s Series (5250 observations)		
	$\Theta_1$	$\Theta_2$	$\Theta_3$	$\Theta_1$	$\Theta_2$	$\Theta_3$
1	0.6196 ± 0.2786	0.1971 ± 0.1988	0.0814 ± 0.1261	0.7057 ± 0.2856	0.1666 ± 0.2337	-0.0581 ± 0.0565
2	0.7096 ± 0.2353	0.1037 ± 0.1234	0.0674 ± 0.0776	0.6284 ± 0.2547	0.1330 ± 0.2681	0.0134 ± 0.1046
3	0.6672 ± 0.2394	0.1658 ± 0.1700	0.0211 ± 0.1116	0.6888 ± 0.3111	-0.0261 ± 0.2991	0.0388 ± 0.1448
4	0.6539 ± 0.1802	0.0400 ± 0.0857	0.1094 ± 0.1203	0.5617 ± 0.3655	0.1855 ± 0.1277	0.0431 ± 0.1687

<sup>a</sup>Numbering begins with the lane closest to the median and increases toward the right shoulder.

where  $S_1(t)$  is the smoothed value of  $X$  at time  $t$  and  $\alpha$  is a smoothing constant,  $0 < \alpha < 1$ . The function  $S_1(t)$  is a linear combination of all previous observations weighted by damped exponential weights. The forecast of  $X_t$  is

$$\hat{X}_t = S_1(t) \quad (11)$$

Note that single-exponential smoothing is equivalent to an ARIMA (0, 1, 1) process where the smoothing constant  $\alpha$  is set equal to  $\theta_1$ . The double-exponential smoothing model assumes that  $F_t$  can be described by a linear trend. The corresponding smoothing function is

$$S_2(t) = \alpha[S_1(t)] + (1 - \alpha) \times S_2(t - 1) \quad (12)$$

Brown demonstrated that the steady-state response of exponential smoothing to a linear trend has a constant lag of  $(1 - \alpha)/\alpha$ . Therefore, the forecast of the next observation  $X_{t+1}$  is

$$\hat{X}_{t+1} = \phi(t) + \psi(t) \quad (13)$$

where  $\phi(t)$  is  $2[S_1(t)] - S_2(t)$  and  $\psi(t)$  is  $(\alpha/1 - \alpha)[S_1(t) - S_2(t)]$ . Values of  $\alpha$  used in the evaluation of the double-exponential smoothing model were 0.1-0.9 in increments of 0.1.

#### Exponential Smoothing with Adaptive Response

Adaptive approaches for adjusting the smoothing con-

stant  $\alpha$  have been suggested by many authors, including Chow (16), Roberts and Reed (17), and Trigg and Leach (18). Most of these approaches use the forecasting performance of the smoothing model to determine the proper adjustment of the smoothing constant. The following is the adaptive approach proposed by Trigg and Leach:

$$TS(t) = SE(t)/SAE(t), \quad -1 < TS < 1 \quad (14)$$

$$SE(t) = \gamma \times e_t + (1 - \gamma) \times SE(t - 1) \quad (15)$$

$$SAE(t) = \gamma \times |e_t| + (1 - \gamma) \times SAE(t - 1) \quad (16)$$

$$e_t = X_t - \hat{X}_t \quad (17)$$

where

$$\begin{aligned} TS(t) &= \text{tracking signal at time } t, \\ SE(t) &= \text{smoothed error at time } t, \\ SAE(t) &= \text{smoothed absolute error at time } t, \\ e_t &= \text{forecast error at time } t, \text{ and} \\ \gamma &= \text{smoothing constant, } 0 < \gamma < 1. \end{aligned}$$

Adaptive response of the smoothing constant  $\alpha$  is achieved by setting it to equal the absolute value of the tracking signal. The Trigg and Leach model was tested by using nine values of  $\alpha$  between 0.1 and 0.9 and three values of  $\gamma$ , 0.1, 0.2, and 0.3.

In evaluating the four forecasting models, the following mean absolute error (MAE) and mean square error



Figure 6. Ratio to Box-Jenkins for mean absolute error.

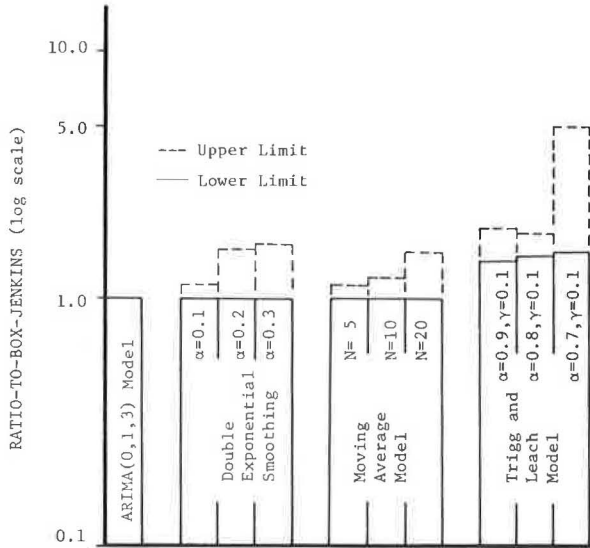
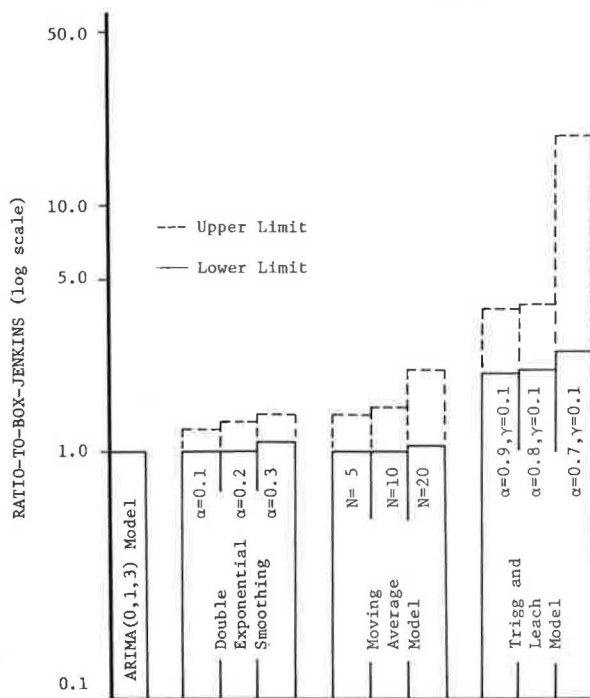


Figure 7. Ratio to Box-Jenkins for mean square error.



MAE indicates the error expected to be associated with each forecast, while MSE detects the presence of frequent large forecasting errors.

For the purpose of comparing the smoothing performance of the different models, values of MAE and MSE of the fitted ARIMA (0, 1, 3) models were chosen as a basis. These values ranged from 1.30 to 6.50 for MAE, and from 2.80 to 91.41 for MSE. Results of the moving-average model indicated that both MAE and MSE increase with increases in  $N$ . When  $N$  equaled five, the ratio to Box-Jenkins varied between 1.00 and 1.27 for MAE and between 1.00 and 1.45 for MSE. Larger values of  $N$  (10-100) resulted in values of ratio to Box-Jenkins of between 1.00 and 2.85 for MAE and between 1.00 and 6.86 for MSE.

The best results of the double-exponential smoothing model were associated with small values of  $\alpha$ . For smoothing constants between 0.1 and 0.3 the ratio to Box-Jenkins ranged from 1.00 to 1.64 for MAE and from 1.00 to 1.43 for MSE.

The Trigg and Leach model did not improve the forecasts. With large initial values of the smoothing constant  $\alpha$  between 0.6 and 0.9 and a smoothing constant ( $\gamma$ ) of 0.1, which gave the best results for this model, the ratio to Box-Jenkins varied between 1.45 and 8.20 for MAE and between 2.08 and 44.34 for MSE. The reason for the poor performance of the Trigg and Leach model could be the abrupt successive changes in  $\alpha$ . Figures 6 and 7 illustrate the ranges of the best values of the ratio to Box-Jenkins for MAE and MSE for the different models. The ARIMA (0, 1, 3) model is seen to be superior: It more accurately represents the stochastic process generating the traffic data.

#### MODEL APPLICATIONS TO SHORT-TERM FORECASTS

To appreciate the operational value of the ARIMA (0, 1, 3) model, one should examine how it can be used in making short-term forecasts in real time.

Let  $\hat{Z}_{t-1}(1)$  be the one-step-ahead forecast made at time  $(t-1)$  for  $Z_t$ , which when observed will be represented by Equation 6. If  $\hat{Z}_{t-1}(1)$  is the minimum mean-square-error forecast, then its value will be determined by the conditional expectation of  $Z_t$ , given the history ( $H_t$ ) of the series up to time  $t$ ; that is,

$$\hat{Z}_{t-1}(1) = E(Z_t/H_t) = -\theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3} \quad (20)$$

Therefore, the forecast error at time  $(t-1)$  is determined by subtracting Equation 20 from Equation 6:

$$e_{t-1}(1) = Z_t - \hat{Z}_{t-1}(1) = a_t \quad (21)$$

Hence, the white noise that generates the process is the one-step-ahead forecast error. In a similar fashion

$$a_{t-1} = e_{t-2}(1) \quad (22)$$

and

$$a_{t-2} = e_{t-3}(1) \quad (23)$$

Consequently, an operational expression for updating the forecasts of the model in Equation 6 is

$$\hat{Z}_t(1) = -\theta_1 e_{t-1}(1) - \theta_2 e_{t-2}(1) - \theta_3 e_{t-3}(1) \quad (24)$$

(MSE) functions were used as evaluation criteria:

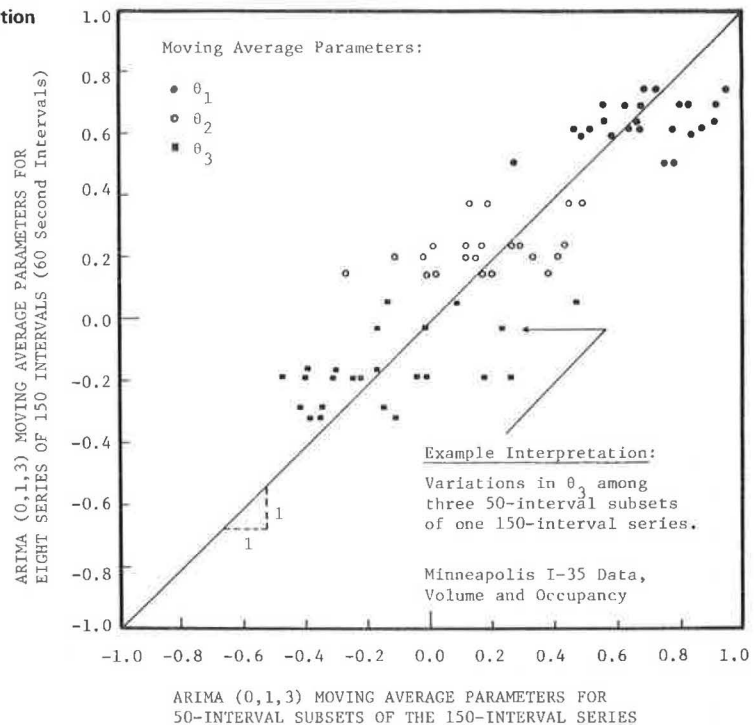
$$MAE = (1/N) \sum_{t=1}^N |X_t - \hat{X}_t| \quad (18)$$

$$MSE = (1/N) \sum_{t=1}^N (X_t - \hat{X}_t)^2 \quad (19)$$

where

$X_t$  = observed variable value at time  $t$ ,  
 $\hat{X}_t$  = predicted value of variable at time  $t$ , and  
 $N$  = the number of observations.

Figure 8. Variability of ARIMA (0,1,3) parameters by location and over time.



The computational utility of the above expression stems from the fact that its application requires computer storage only of the latest three forecast errors and the current observation.

The sensitivity of the performance of the ARIMA (0, 1, 3) model to variations in the  $\theta$  parameters over time was tested to a limited extent as follows. A number of volume and occupancy time series, each 150 60-s time intervals from the Minneapolis I-35 data were broken into three 50-interval segments. The ARIMA (0, 1, 3) model was applied separately to each segment.

The variations in the estimated moving-average parameters ( $\theta_1$ ,  $\theta_2$ , and  $\theta_3$ ) for both sets of series are depicted in Figure 8. The horizontal scatter of points indicates that the parameters do vary over time but no consistent pattern in this variation was noted. However, due to the limited number of observations used in estimating the parameters for each 50-interval segment, the conclusion that these parameters vary with time cannot be accurately drawn. It is also important to note that the same form of the ARIMA (0, 1, 3) model that represented the 150-observation series represented the 50-observation segments just as well.

It may be desirable, although not necessarily warranted, to update the moving-average parameters in real time. It is believed that a rapid adjustment in the parameter estimates—each observation interval, for example—may degrade the overall forecasting performance of the ARIMA model. Past experience with adaptive-exponential-smoothing models, particularly the Trigg and Leach model, has shown that successively changing the smoothing constant value over time yielded potentially larger forecasting errors than those resulting from Brown's original exponential-smoothing models (19). The results depicted in Figures 6 and 7 also tend to confirm this belief.

Another important factor that should be taken into consideration when one is contemplating real-time updating of the model parameters is that of computer computational requirements. One way to lower these requirements would be to update the parameters only

occasionally, e.g., at the beginning of peak and off-peak periods. Parameter updating was not explored in this research, in part because the available data sets consisted of afternoon peak-period time series only. Further research along these lines is strongly recommended. Operational expressions for updating the moving-average parameters  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$  can be found in Box and Jenkins (7, pp. 162-164).

#### SUMMARY AND CONCLUSIONS

In this paper an application of the Box-Jenkins approach for modeling traffic time-series data has been presented. An ARIMA (0, 1, 3) model was found to represent volume and occupancy data from three different freeway systems of varying detector configurations and data-aggregation time intervals. The comparative evaluation of the ARIMA (0, 1, 3) model against some other ad hoc smoothing models has indicated the overall superiority of the ARIMA (0, 1, 3) model in providing short-term forecasts of traffic parameters.

The forecasting model described in this paper should be of use in real-time computerized freeway traffic-control systems and may be applicable to traffic-signal networks. At this writing, the model was being used to develop freeway incident-detection algorithms.

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## Automobile Diversion: A Strategy for Reducing Traffic in Sensitive Areas

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In recent years awareness of the negative impacts of motor-vehicle travel has increased. One approach to those impacts is automobile diversion, a strategy for reducing vehicle use in congested areas. This paper reports on a recent study directed toward developing and evaluating the potential for automobile diversion in Denver. General traffic problems are identified and a potential yardstick for locating affected areas—the environmental capacity of city streets approach—is discussed. Benefits and problems of notable U.S. background experience in automobile diversion are summarized. A detailed breakdown is given of the various transportation system management strategy-formation elements applicable to automobile diversion, and several implementation techniques are described. Advantages and disadvantages are also presented to demonstrate the use of automobile diversion as a community-improvement tool. Finally, the study determines that the potential for automobile diversion in Denver relies on the degree of citizen interest, the identification and resolution of issues and problems, and sound decision making in the political forum.

In the fall of 1975, the Urban Mass Transportation Administration (UMTA) and the Federal Highway Administration (FHWA) jointly issued urban transportation planning regulations directing appropriate local agencies

to develop transportation system management (TSM) plans for their respective urban areas (1). TSM plans are intended to document local strategies for improving air quality, conserving energy, and improving transportation efficiency and mobility through management of existing transportation systems. TSM strategies deal with low-capital, short-range, or policy-oriented urban transportation improvements.

Although many TSM strategies have been implemented in the Denver transportation system, only recently has emphasis been placed on directly identifying and pursuing those strategies in an organized and coordinated manner. For instance, Denver now has computerized traffic control and operations, transit operations, carpooling, and various preference and restraint programs. These management concepts and control strategies, and their respective action elements, were developed and implemented only when the need became obvious.

Because of federal emphasis on TSM and the techniques already in use in Denver, the Denver Planning