Definition of Alternatives and Representation of Dynamic Behavior in Spatial Choice Models

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This paper considers issues relevant to two important spatial-choice modeling problems: the definition of alternatives and the modeling of dynamic behavior. The definition of alternatives may benefit from the development of a classification scheme that consists of a reasonably small number of categories. This approach could lead to more manageable data requirements and improved model specification through the use of a larger set of alternative-specific constants. Also, spatial alternatives often have characteristics that do not vary from individual to individual. Recognition of this can lead to computational efficiencies and possibly easier use of aggregate data in model estimation. Dynamic behavior is modeled by introducing the effects of previous choices and using an error-components structure in the utility functions for choice models. Four special cases of the dynamic model are considered. It is then possible to identify the assumptions necessary to apply existing choice methodologies to dynamic choice problems and to recommend further research on methodologies that require less restrictive assumptions.

Several features of spatial choice problems have made the conceptual and empirical development of appropriate models challenging. This paper focuses on two of the more important features: the definitions of spatial alternatives and the treatment of the dynamic aspects of spatial choice (1-3). These issues will be discussed in the context of the utility maximization approach to quantal choice problems (4, 5).

Spatial choice problems apparently differ from the more commonly modeled mode choice problems in the identifiability and number of available alternatives. Available transportation modes are easily identified and few in number. Spatial alternatives (e.g., alternative shopping destinations) can be identified in several ways, ranging from individual spatial locations to fairly large geographic zones or other aggregation schemes. Also, in many urban areas, the number of alternatives can be very large.

Another important characteristic of spatial alternatives is that many of their objective characteristics do not vary for different individuals. That is, the characteristics (such as travel times and costs of transportation modes) vary with an individual's location, but objective characteristics of spatial alternatives (such as the number of retail employees at a shopping destination) do not. This property can be used in the exploration of methods that make more efficient use of data in the estimation of spatial choice models.

Dynamic considerations are especially important for short-term spatial choices, such as shopping travel. Although repeated observations of these choices can be obtained during a reasonably short time period, these problems have often been treated empirically in the same manner as longer-range choices (i.e., only a single cross section of observations has been used). By explicit consideration of a time series of cross sections, the dynamic aspects of short-term spatial choices can be studied in detail. In addition, the consequences of improperly ignoring dynamic considerations in the development and application of spatial choice models can be identified.

DEFINITION OF SPATIAL ALTERNATIVES

Research relevant to definition of alternatives can be divided into two categories: (a) classification of alternatives and (b) data requirements for spatial choice models. In general, the definition of spatial alternatives is of both theoretical and practical importance. The validity of the assumptions made for particular model structures [e.g., the independence from irrelevant alternatives property of the multinomial logit model (6)], is closely related to the definition of the alternatives. In addition, specification of independent variables and the resulting data requirements are influenced by the definition of alternatives.

Classification

Choice models have been used most frequently to explain modal choice, in which the modal alternatives are fairly easily identified. For example, the classification of a particular mode as an automobile or bus is relatively easy. Also, the total number of alternatives is small; often only two alternatives (automobile and transit) are considered.

In contrast, there does not appear to be any natural method for classifying spatial alternatives, and the total number of alternatives can be quite large in many problems. Consequently, the actual definition of alternatives has been quite arbitrary and ad hoc, and often there has been no categorization of alternatives or only a very crude classification scheme. Some models, for example, have defined spatial alternatives to be the traffic zones established in metropolitan transportation studies and have made no attempt to classify alternatives, with the exception, perhaps, of the central business district (CBD) (5, 7). Examples of more developed classification systems are the classification of grocery shopping destinations by store type (8) and the classification of shopping centers by distance from home and floor area (9).

Classification is important for two reasons. First, the spatial choice problem can be made more tractable by first assigning an individual to a broad category and then assigning a specific destination within that category (9). For certain regional policy analyses where spatial detail is unnecessary, application of only the first step of this process may be sufficient. Second, even if the specific destinations are used directly, the classification approach allows the use of a fuller set of alternative-specific constants in the utility function of the choice model. The usual approach in specification of destination-choice models has been to exclude constants (5) or to include constants for only special destinations, such as the CBD (7). The classification approach allows the use of an alternative-specific constant for each category. Since the use of constants has been shown to be important in the proper specification of choice models (10), the development and use of a classification scheme is important for improved model specification as well as for problem simplification.

Data Requirements

The most common definitions of spatial alternatives involve numerous destinations that are fairly small in geographic size. Consequently, in order to make the choice problem empirically tractable, it has been necessary to limit the size of the choice set available to each individual or household. This has been done by either assigning a restricted choice set to each individual before estimation of the model (5, 7) or by limiting the number of destinations available to all individuals by focusing on a limited geographical area. For example, the destination-choice project conducted by Northwestern University researchers (11) limited the available shopping destination to a common set. In many applications the definitions of alternative destinations involve some sort of spatial aggregation (12-14).

The necessity for limiting choice sets can be illustrated by considering the data storage requirements for estimating a choice model. When every individual has the same number of alternatives, these requirements (4) are

$$S = N(a-1)v \tag{1}$$

where

S = the number of spaces required to store the independent variables,
N = sample size,
a = the number of alternatives, and
v = the number of independent variables.

The nonlinear nature of the usual logit and probit approaches requires that all of the data be stored simultaneously.

The independent variables include characteristics of the spatial alternatives themselves, characteristics of the individual, and the spatial relationships between the individual and the alternative (e.g., distance). When objective data are used, the characteristics of the spatial alternative do not vary from individual to individual (e.g., the floor area of a shopping center is the same for everyone). In this case, if everyone has the same choice set, the actual storage requirements are

$$S = (a-1)v_1 + N(a-1)v_2 \tag{2}$$

where v_1 is the number of independent variables that do not vary from individual to individual and v_2 is the number of the remaining independent variables. A slight modification of existing logit and probit analysis computer programs would result in the smaller storage requirements of Equation 2. This would lead to greater statistical efficiency by allowing an increase in sample size and number of alternatives per individual or reduced cost for the same level of statistical efficiency.

This modification has the potential of yielding significant computational savings. For example, in a study of housing location choice, Friedman (15) developed a model that had nine communities as alternatives and nine independent variables. Of these variables, only one varied from individual to individual. Consequently, in many spatial choice problems, empirical tractability may not be as large a problem as commonly believed.

For some problems, variables that vary among individuals may not enter directly into the model. For example, a market segmentation approach may result in separate models, which correspond to various combinations of spatial separation and individual characteristics. In this case, v_2 is zero and the problem becomes one of estimating the effects of the characteristics of the spatial alternatives on the aggregate shares. Essentially, the situation is one of repeated observations of a single-choice situation. (Assuming each individual has the same choice set, each individual constitutes a repetition.) Although it was not used for a spatial choice problem, the random-coefficient logit model used to explain market shares of automobile models based on their characteristics is an example of the basic approach (16). Not only is there economy in computation requirements, but data requirements are drastically reduced as well. Only the aggregate shares and the characteristics of alternatives are necessary.

The ability to estimate behaviorally sound spatial choice models by using only characteristics of alternatives and aggregate shares as input data is highly desirable from a practical standpoint; however, the exclusion of independent variables, which indicate the spatial relationships between individuals and destinations and individual or household characteristics, may not be conceptually sound. In this case, it may still be possible to estimate models that have reduced data requirements by using an appropriate procedure for estimating disaggregate models from aggregate data.

Suppose a particular choice model for estimating the probability that a given individual will select a particular alternative is

$$P_i(X_2) = f_i(X_1, X_2) \tag{3}$$

where X_1 represents characteristics of alternatives that do not vary among individuals and X_2 represents independent variables that do vary among individuals. Then the aggregate share is given by

$$\text{Share}_i = \int X_2 f_i(X_1, X_2)\, g(X_2)\, dX_2 \tag{4}$$

where $g(X_2)$ is the probability density function.

In order to estimate the choice model by using aggregate data, it is necessary for Equation 4 to result in the shares being a function of X_1 and characteristics of the distributions of X_2 (e.g., the means and higher moments). If the choice model is multinomial probit, the results of Bouthelier and Daganzo (17) suggest that the means and the variance-covariance matrix that correspond to the variables in X_2 for each alternative are sufficient when X_2 can be approximated by a multivariate normal distribution.

For other choice models, the integral in Equation 4 can be analytically intractable. In these cases, either Monte Carlo integration techniques (18) or the approximation of f_i by a polynomial expansion, such as the Taylor series (12, 19), may yield similar data requirements for the estimation of the model by using aggregate data.

There are some potential implications for current practice and future research from these characteristics of spatial alternatives. More research on the classification of alternatives into meaningful categories would be useful in the proper specification of spatial choice models and in the development of models for policy analysis at the regional level. That many of the objective characteristics of spatial alternatives do not vary from individual to individual immediately reduces the computational requirements for the estimation of choice models. Consequently, the use of much larger choice sets may be a possibility. There is also the possibility of estimating disaggregate models with aggregate data. The required data would be the aggregate shares for spatial alternatives, the nonvarying characteristics, and information such as the first and second moments of the distributions of the independent variables, which vary among individuals for each alternative. More research on the development of these procedures for models other than the probit model and on the efficiency and reliability of such methods may yield results that allow the development of practical models that have fairly moderate data requirements.

DYNAMIC ASPECTS OF SPATIAL CHOICE

Most spatial choices are repeated. This is especially relevant for short-run destination choices, such as shopping travel. However, since most models have been estimated by using a single cross section of observations, the dynamic nature of the behavior is not emphasized.

Dynamic spatial behavior was studied by Burnett (20). However, her approach considered only one spatial alternative at a time. Modification of the usual utility maximization approach to choice behavior allows the development of models that consider more than one alternative and the exploration of the consequences of using the assumptions behind static models in dynamic contexts.

This can be seen by considering the typical approach. The utility for a given alternative can be expressed as

$$U_i = X_i \beta + \epsilon_i \tag{5}$$

where

U_i = the utility of the i th alternative,
X_i = the characteristics of the alternative,
β = a vector of coefficient, and
ϵ_i = an error term.

To simplify the discussion, variables that describe individuals will not be identified. A choice model results from the utility maximization assumption and from the assumption of a distribution for the ϵ_i.

The dynamic implications of Equation 5 are not clear. Certainly, if some of the independent variables change in the course of time, the resulting model will produce different selection probabilities. However, during short time periods, these variables are likely to be stable. In this case, any variation in an individual's choice over time is determined by ϵ_i. If the errors are assumed to be the effects of excluded variables rather than pure randomness, then they are unlikely to vary for short time periods for a given individual, resulting in the prediction of a constant choice over time. Since this is clearly unrealistic for some types of spatial behavior (e.g., people do not necessarily limit themselves to one shopping destination), it is necessary to assume random error terms or to respecify the model to consider dynamic behavior explicitly.

A useful approach is to consider a specification analogous to the ones used in linear models that use a time series of cross-sectional data (21, 22).

A general form of such a model would be

$$U_{its} = X_{its}\beta + \sum_j \gamma_{ij}C_{j(t-1)s} + \mu_{is} + \nu_{its} \tag{6}$$

where the subscripts i and j refer to alternatives, t to a time period, and s to an individual. $C_{j(t-1)s}$ is one if individuals chose alternative j in the previous time period and zero otherwise. Although the model could be made more general by considering choices in previous time periods, in linear models a single lag term has often been used. Finally, μ_{is} is an error term that varies among individuals but not time periods and ν_{its} is an error term that varies among both individuals and time periods.

In addition to allowing the use of the time series of cross-sectional data, the revised specification introduces two additional elements. First, the possibility that the choice in one period may influence the following choice is allowed. A positive coefficient for the lag term indicates an increased choice probability in the subsequent time period, and a negative coefficient indicates the opposite influence. Second, the use of a component structure for the error term allows the possibility that some of the unobserved effects may be constant across time periods for particular individuals. An example of such a situation is when the μ_{is} represents the effects of unspecified characteristics of alternatives and the ν_{its} represents pure randomness in the choice process. For sufficiently short time periods, the unspecified characteristics would probably remain fairly constant; therefore, the error components representation would be reasonable.

The discussion in this section is confined to fixed-coefficient models. The development of dynamic choice models, analogous to the random-coefficient linear models (23), will not be considered.

In order to estimate a model that results from Equation 6, the data required are observations of the X_{its} and the C_{jts} for N individuals and T time periods. When one or more of the terms in the model is set equal to zero, several variations are possible.

Case 1

Case 1 is the ordinary utility-maximization model applied to the time-series data: $\gamma_{ij} = 0$ for all i, j and $\mu_{is} = 0$ for all i, s. The basic assumption is that a static choice model can be applied directly to the dynamic problem. In estimating the model, the observations for a given individual over time would be treated as independent (i.e., in the same way as an observation of a different individual that has similar characteristics is treated). The standard static model is a special case when only one time period is observed. If the variables in X vary over time, the estimation of a choice model from the repeated observations of a single individual is another special case.

When the independent variables for the individuals do not vary over time, then the model becomes a choice experiment with repeated observations (4). Such choice problems have been treated in three ways:

1. The use of a single time period is the special case just mentioned (models estimated from data from standard transportation surveys are examples of this approach),
2. Actual observations of the repeated choices could be made (this would require travel-behavior diaries or recontact of a survey panel), and
3. Respondents could be asked to give their usual choices or the usual choice is constructed from reported choice frequencies in an attempt to capture the predominant pattern of repeated behavior (8).

If the case 1 assumptions are valid, then either of the first two data collection procedures will allow the estimation of consistent model coefficients. However, it is conjectured that the use of the usual choice as the dependent variable does not result in consistent estimation. This is based on an empirical example (24) in which models that used the usual behavior were quite different from those that used actual choice behavior. In addition, simulated data can be used to show that for some simple binary-choice models, when usual choice is the dependent variable, coefficient estimates do not converge to finite values when the data were constructed by assuming a model with finite coefficients. More research on the consequences of using the usual rather than actual choices would be useful in the determination of whether the apparent inconsistency is generally the case and the magnitude and direction of the bias if it is a problem.

Case 2

The key assumption in case 2 is that previous choices affect current choices: $\mu_{is} = 0$ for all i, s. However, there is no constant component to the error term for a given individual. The effects of factors not explicitly included in the model are treated as completely random.

Since previous behavior is explicitly considered, observations of more than one time period are necessary. However, since the error terms are independent across time periods, existing models (such as the multinomial logit model) could be used directly.

A special case of this model occurs when the $X\beta$ term is zero (i.e., current choice is only a function of previous choices). The model then yields the transition probabilities of a Markov model of spatial choice (25, 26). In general, the model can be viewed as incorporating the effects of learning (27).

Case 3

Case 3 introduces the possibility that there may be unspecified effects that are constant for individuals over time: $\nu_{ij} = 0$ for all i, j. Since it is impossible to distinguish empirically between the two error components when only a single cross section of observations is made, the identification of the variance components specific to individuals requires more than one period of observation. Most of the research on linear models has been concerned with the development of estimates for models analogous to the case 3 model (28-33).

This particular model illustrates the ambiguity of interpreting the selection probabilities estimated from a static model in a dynamic context. If the μ terms are zero (case 1 model), then each individual has a probability of selecting a particular alternative for each time period as determined by the model. At the other extreme, if the ν component is zero, each individual makes a constant deterministic choice. The selection probabilities from the model are the probabilities that individuals who have the same choice situation will make a particular constant choice. For example, in the mode choice case, the case 1 model gives a probability that an individual will use the bus on a particular day, and the extreme version of the case 3 model gives the probability that an individual who faces a particular choice situation will always choose the bus. The intermediate case is when both μ and ν are nonzero, in which case the selection probabilities for an individual lie between those estimated from the model and the deterministic situation.

Estimation of case 3 models introduces correlations in individual behavior over time. Therefore, each time period does not constitute a completely independent observation. As a result, estimation of the model, as in case 1, does not appear to be valid.

A possible estimation approach, which is analogous to that used in linear models, would be to explicitly identify the μ terms. This is referred to as the fixed-effect approach. This would result in a set of alternative-specific constants for each individual. Since this is undoubtedly unwieldy in practice, it may be possible to first classify the sample and have one set of constants for each category. Also, it might be necessary to classify the alternatives, as suggested earlier, in the specification of manageable sets of constants. When this is done, standard choice models can be used directly.

A conceptually more appealing approach is to deal directly with the more complex variance structure, the random-effects approach. This approach would be analogous to the work on correlated error terms among alternatives (34, 35) (e.g., the development of the multinomial probit model). Further, simulation and empirical work with linear models has indicated that models that deal directly with variance components perform better in small samples than do those that identify constant terms (22, 36). This suggests that research on the estimation of case 3 models may be very important.

Case 4

Case 4—the full model—does not appear to introduce any new considerations. However, note that, for linear models, this case is the most sensitive to incorrect assumptions. That is, when a case 4 model is estimated as a case 2 model, inconsistent coefficients result. On the other hand, when a case 3 model is estimated as a case 1 model, the coefficient estimates are consistent but inefficient (21, 22). Further research could be useful in the determination of whether an analogous situation exists with respect to choice models. It could be the case that explicit consideration of the error components is especially important for case 4 models.

This approach to dynamic spatial choice models is similar to the methodology developed by Heckman (37) to explain dynamic labor-force-participation decisions. The models tested the effects of personal, household, and economic characteristics as well as previous participation in the labor force on women's decisions to work. Two variations of a generalization of the case 4 model were used. The first explicitly considered the variance structure (random effects) and the second directly identified the error components that corresponded to individuals (fixed effects). The models described here involve a generalization of Heckman's approach from the binary to the multinomial case and also shift the emphasis to independent variables that describe the characteristics of alternatives.

The specification of models that satisfy Equation 6 can be viewed as a special case of specification analysis that involves the possible exclusion of independent variables (33). That is, the μ_{is} can be treated as independent variables and the consequences of considering or not considering these components can be examined. In this regard, the recent work in specification analysis for choice models is relevant (38, 39). This analysis indicates that exclusion of the μ_{is} component can result in two sources of bias in the coefficient estimates: bias resulting from possible correlations between the error component and the other independent variables and bias resulting from changes in the distribution of the random component of the utility functions.

The bias resulting from excluding μ_{is} can be illustrated by a special case of the binary probit model. Assume that μ_{is} is not correlated with the independent variables, which are further assumed not to vary over time for the individuals. In this case, Equation 6 applies to two alternatives and the μ and ν are independent normal variables that have expected values equal to zero. Let the variance of μ be σ^2 and the variance of ν be one-half. If the inverse standard normal function is applied to the observed proportion that each individual selects the first alternative, and this variable is used as the dependent variable and X is the independent variable, then it can be shown that consistent estimates of β are obtained when ordinary or generalized least squares is applied (38). On the other hand, if the maximum-likelihood method is used, the resulting coefficients converge to

$$b = \beta/\sqrt{1 + 2\sigma^2} \tag{7}$$

Therefore, the ratio of the regression estimators and the maximum-likelihood estimators yields information on σ^2, the variance of the error component that corresponds to individuals. This result follows from the fact that the μ are left-out variables that are uncorrelated with the observed variables and from the fact that uncorrelated, left-out variables result in the above differences between the regression and maximum-likelihood estimators (38, 39).

Further, the specification analysis approach allows explicit consideration of the distribution of μ_{is} in the development of random-effects models. Therefore, initial research on the development of dynamic choice models can be guided by the approach used in the analysis of specification problems.

The four cases of the dynamic choice model have presented a framework for discussing dynamic choice problems. It was noted that certain cases allow the use of existing choice models. In addition, further research on choice models to explicitly consider the variance structure in Equation 6 appears to be important to the development of dynamic choice models.

SUMMARY AND CONCLUSIONS

The definition of spatial alternatives, the efficient use of both disaggregate and aggregate data sources, and the proper specification of models of dynamic behavior have been recognized as important issues. As in the case of mode-choice modeling, the ability to classify alternatives into a reasonably small number of categories would lead to models that are empirically more tractable. Further, classification allows the use of a larger set of alternative-specific constants, which may be important in the proper specification of choice models. Unlike the mode-choice case, however, many spatial choice models have subsets of independent variables that do not vary from individual to individual. Modification of existing programs to account for this feature and exploration of techniques for estimating choice models by using aggregate data would allow greater efficiencies in data collection, computation, and statistical accuracy.

Dynamic choice behavior was considered by modifying the utility function in the choice model to include effects of past behavior and by introducing an error component that is constant for a given individual over time. Several cases were considered. These are useful in understanding how previous models of spatial choice fit into a dynamic context, in exploring the consequences of improper dynamic assumptions, and in indicating necessary research to develop dynamic choice methodology.

Some of the cases allow direct use of existing choice methodology. The use of such methodology, which requires the most careful consideration, is the case in which the error components that are constant for a given individual over time are explicitly specified as constant terms (fixed-effect approach). In order for such an approach to be empirically manageable, both individuals and alternatives should be classified into a reasonably small number of categories. Investigation of the statistical reliability of this approach in small samples by use of empirical and simulated data is an important area for further investigation.

The analytical development of dynamic models that are derived from direct consideration of the components of the variance structure (random-effects approach) is an area for longer-term research activity. In the development of such models, the special features of spatial alternatives, which were discussed earlier, would have to be considered. Based on experience with linear models, this is the most desirable approach to the development of dynamic spatial choice models. Investigation of the small sample properties of such models is also important.

Finally, the prediction accuracies of the dynamic choice models derived from future research should be assessed. This assessment would indicate the extent to which models that have less-restrictive assumptions improve on the prediction accuracies of existing choice models used in dynamic contexts.

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