Equilibrium Trip Assignment: Advantages and Implications for Practice

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During the past 10 years the problem of assignment of vehicles to large, congested urban transportation networks according to the principle of equal travel times has been solved and an efficient, convergent computer algorithm devised. Although the algorithm is available in the Urban Transportation Planning System, many practitioners continue to use the heuristic trip-assignment algorithms devised in the early 1960s. As in many other cases, this slow implementation of a new, improved algorithm appears to come from (a) a lack of understanding of its basic concepts, (b) an unfamiliarity with the computer program for applying the algorithm, and (c) a lack of evidence concerning the new algorithm's performance in large-scale applications. These three issues are addressed in this paper. Based on the experience with its implementation on a large network, it is recommended that equilibrium trip assignment should always be used instead of iterative assignment. Better results, as judged by the criterion of equalizing travel times for alternative paths between each origin-destination pair, will always be obtained with the equilibrium algorithm for any given amount of computational effort. Which method best replicates the observed vehicle flows may depend on the detail of the network, the adequacy of the capacity-restraint functions, and the time period of the assignment (24 h or peak period).

Assignment of vehicles to large, congested urban transportation networks has been a problem of interest to transportation planners and researchers for over two decades. Initially, heuristic or approximate solution techniques were developed for the problem. Later, several convergent algorithms were devised and some were tested, culminating in an International Symposium on Traffic Equilibrium Methods at the University of Montreal in 1974 (1-3).

Despite these theoretical and practical developments, relatively few applications are being made of equilibrium assignment, despite its availability in the Urban Transportation Planning System (UTPS) (4) and its desirable attributes. Two reasons are apparent for this situation.

1. Practitioners have experienced difficulty in understanding the formulation of the equilibrium-assignment problem and the algorithms devised to solve it, and
2. Practitioners were uncertain about whether the algorithms were superior to competing algorithms, such as iterative and incremental assignment, for large networks.

This paper will explain the equilibrium-assignment problem and the algorithm in terms that are familiar to practitioners and report on a large-scale, prototype implementation of the model. The implementation provides convincing evidence that equilibrium assignment is the method of choice for congested networks. The shortcomings of existing capacity-restraint functions and the weaknesses of 24-h assignments are evident from this application.

The problem of trip assignment in the sequential urban travel-forecasting process is how to assign (or allocate) a specified number of vehicles (or persons) to the paths taken from each origin to each destination. The path chosen by each traveler is generally assumed to be the path that minimizes his or her journey time, or some combination of time and cost. All travelers are assumed to have identical perceptions of travel time and cost. If the network is congested, that is, if each link's travel time depends on the flow of vehicles on that link, then the following equilibrium problem results: Find the assignment of vehicles to links such that no traveler can reduce his or her travel time from origin to destination by switching to another path. These equilibrium conditions were stated by Wardrop (5) and are commonly referred to as the Wardrop conditions.

The user-equilibrium problem has been stated mathematically in several forms: the conceptually simplest form is stated below. Let

v_a = number of vehicles per unit time on link a of the network;
$S_a(v_a)$ = generalized travel time on link a, which increases with flow v (a typical congestion function is $t_a[1 + 0.15(v_a/c_a)^4]$ where t_a is the travel time with zero flow, and c_a is a measure of the capacity per unit time of link a);
X^r_{ij} = number of vehicles of i to j on path r; and
δ^{ar}_{ij} = 1 if link a belongs to path r from i to j, 0 otherwise.

If the trip matrix (T_{ij}) is given, then the equilibrium assignment of trips to links may be found by solving the following nonlinear programming problem:

$$\min \sum_a \int_0^{v_a} S_a(x)dx \tag{1}$$

subject to

$$v_a = \sum_i \sum_j \sum_r \delta^{ar}_{ij} X^r_{ij} \tag{2}$$

$$\sum_r X^r_{ij} = T_{ij} \tag{3}$$

$$X^r_{ij} \geq 0 \tag{4}$$

For all links a in the network; i = 1, ... N; j = 1, ... N; and N = number of zones.

This is a nonlinear programming problem with a convex objective function subject to two sets of linear constraints and two sets of nonnegativity conditions. Constraint set Equation 2 states that the flow of vehicles v_a on link a is equal to the sum of the flows from all zones i to all zones j that use that link. Constraint set Equation 3 states that the number of vehicles from zone i to zone j over each path used must sum to the specified number of trips (T_{ij}). Constraint set Equation 4 ensures that no flow is negative.

Now consider the objective function (Equation 1). $S_a(x)$ is the link-congestion or capacity-restraint function for link a. The integral term is the area under the link-congestion function from zero flow to flow v_a. In Figure 1, S_a is the average travel time. The area under curve S_a has no (known) interpretation. Why, then, should we be interested in minimizing the sum of these areas over all links? The answer to this question is conceptually simple. The link flows for which this objective function achieves its minimum value are those that satisfy the equilibrium conditions stated by Wardrop.

This point can be readily grasped if we consider a highly simplified example (6). Let A and B be two links that connect node 1 to node 2, as shown in Figure 2a. A total of 8000 vehicles travel from node 1 to node 2.

To assign these vehicles to the two links, plot the congestion for link A, mark off the required flow (8000), and plot the second function in the reverse direction. The intersection of these two functions gives the equilibrium travel time of 63.3; the equilibrium flows are 2153 vehicles on link A and 5847 vehicles on link B.

This graphical solution may be stated mathematically as follows:

Figure 1. Congestion function for a given link.

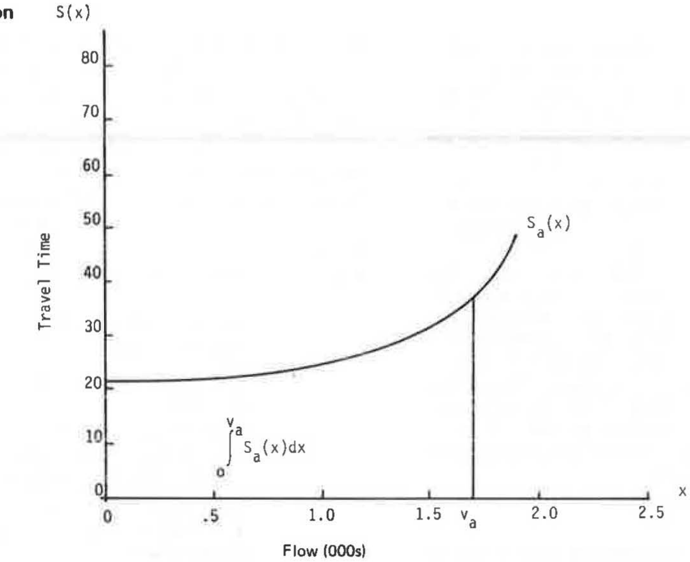
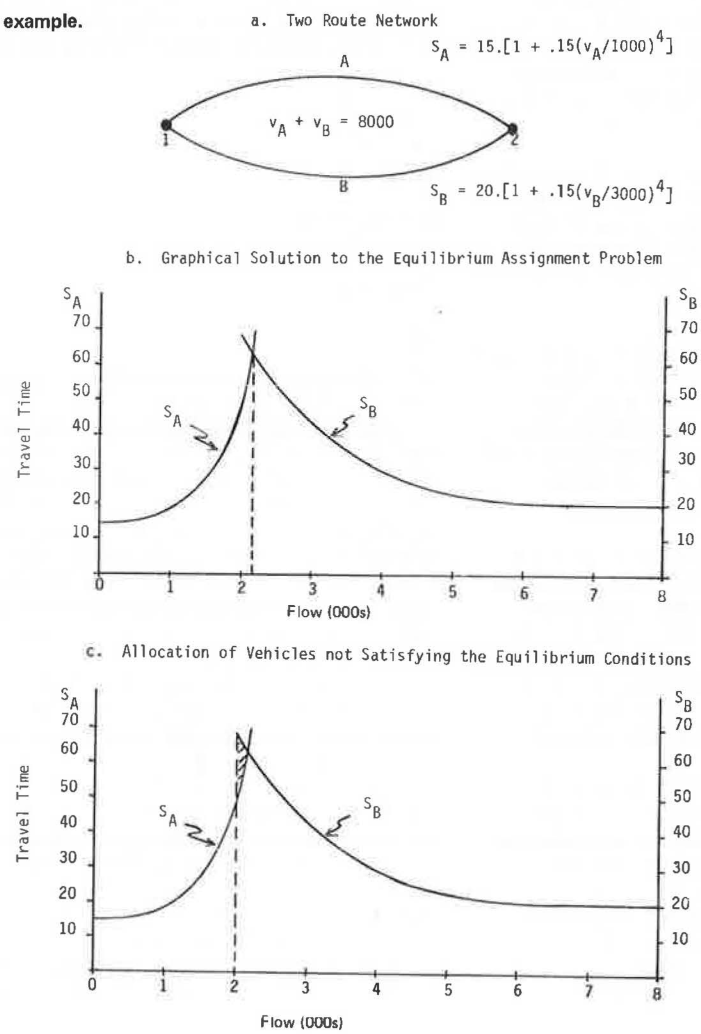


Figure 2. A two-link example.



$$\min \int_0^{v_A} S_A(x)dx + \int_0^{v_B} S_B(x)dx \quad (5)$$

subject to

$$v_A + v_B = 8000 \quad (6)$$

$$v_A, v_B \geq 0 \quad (7)$$

Note that the area under the congestion functions in Figure 2b is equal to 220 674, which is the value of the objective function.

Now, consider any other solution than the one given by the intersection of S_A and S_B, say $v_A = 2000$ (see Figure 2c). The area under the two congestion functions for this solution is the same as in 2b plus the small triangular-shaped area that lies between 2000 and 2153, which has an area of 1326. Thus all solutions other than the equilibrium solution have a larger value of the objective function than does the equilibrium solution. Hence, the solution that minimizes the sum of the integrals of the congestion functions for all of the links is the equilibrium solution.

ALGORITHM FOR EQUILIBRIUM ASSIGNMENT

Next, consider how we solve the equilibrium-assignment problem for large networks. The equilibrium-assignment algorithm, which is commonly used, has a structure somewhat similar to the version of the iterative assignment in the Federal Highway Administration (FHWA) PLANPAC computer programs (7). To illustrate these similarities and differences each of three algorithms is outlined, and a simple three-link example is solved.

Equilibrium-Assignment Algorithm

Given (a) a network with congestion functions for each link, (b) a trip matrix to be assigned, and (c) a current solution for the link loadings (v_a), perform the following steps:

1. Compute the travel time on each link $S_a(v_a)$ that corresponds to the flow v_a in the current solution;
2. Trace minimum path trees from each origin to all destinations by using the travel times from step 1;
3. Assign all trips from each origin to each destination to the minimum path (all-or-nothing assignment); call this link loading (w_a);
4. Combine the current solution (v_a) and the new assignment (w_a) to obtain a new current solution (v_a') by using a value λ selected so as to minimize the following objective function:

$$\sum_a \int_0^{v_a'} S_a(x)dx \quad (8)$$

where $v_a' = (1 - \lambda) v_a + \lambda w_a$; and

5. If the solution has converged sufficiently, stop; otherwise return to step 1.

Initially, a current solution can be obtained by performing an all-or-nothing assignment based on free-flow times. This initial assignment is then used to compute revised travel times to perform another all-or-nothing assignment (steps 1-3). The two assignments are then combined by using a weight λ selected so as to give a new solution that minimizes the objective function of the nonlinear programming problem. This parameter can be readily determined by use of a one-dimensional search technique.

The change in the value of the objective function provides a measure of the convergence of the algorithm. As the change approaches zero, so does the value of the parameter λ. Thus, the equilibrium assignment is a weighted combination of a sequence of all-or-nothing assignments. The algorithm is not heuristic, that is, a method found to give good solutions. Rather, it is the Frank-Wolfe method for solving nonlinear programming problems applied to the equilibrium-assignment problem. LeBlanc (3) gives a rigorous derivation of the algorithm.

Now, consider a very simple example of the use of the algorithm. A three-link network is defined by adding link C to the network in Figure 2:

$$S_c = 21[1 + 0.15(v_c/1500)]^4 \quad (9)$$

Even this simple problem cannot be solved graphically.

The results of applying the algorithm to this problem are given in Table 1. Five iterations are given after an initial solution. For each iteration, the all-or-nothing assignment is given on the first line followed by the new solution on the second line. The travel times given for each link are the values of the congestion functions for the link flows shown. The values of the objective function and λ were given on the right-hand side of the table.

The initial solution assigns all 8000 vehicles to link A. In the first iteration, all vehicles are assigned to link B, which results in the same combined solution shown in Figure 2. Next, all vehicles are assigned to link C, which results in the first good approximation of the equilibrium solution and has an objective function of 174 807. Iterations 3-5 refine this solution by making small adjustments on the order of 1 percent or less. One could effectively stop the algorithm after iteration 3 since a very small decrease in the objective function and a small value for λ were found. Iterations 4 and 5 are given only to indicate how the algorithm continues to converge.

Table 1. Equilibrium assignment.

		Link A		Link B		Link C		Equilibrium Objective	
Iteration	Step	Flow	Time	Flow	Time	Flow	Time	Function	λ
Initial solution		8000	9231.0	0	20.0	0	21.0	14 864 600	
1	3	0		8000		0			
	4	2153	63.3	5847	63.3	0	21.0	220 674	0.731
2	3	0		0		8000			
	4	1598	29.7	4341	33.2	2060	32.2	174 807	0.258
3	3	8000		0		0			
	4	1666	32.3	4296	32.6	2039	31.8	174 697	0.011
4	3	0		0		8000			
	4	1659	32.0	4277	32.4	2065	32.3	174 687	0.004
5	3	8000		0		0			
	4	1666	32.3	4273	32.3	2062	32.2	174 686	0.001

Iterative Assignment

As a further basis for understanding the equilibrium-assignment algorithm, the FHWA version of iterative assignment is now sketched (7, pp. 189-193). The algorithm requires the same input information as does equilibrium assignment. To execute the algorithm, perform four iterations of the following sequence and compute the mean of the four all-or-nothing assignments.

1. Compute the travel time on each link $S_a(v_a)$ corresponding to the flow v_a in the current solution;
2. Compute a weighted mean travel time (S_a''), which consists of the current travel time $[S_a(v_a)]$ and the travel time (S_a') from the previous iteration:

$$S_a'' = 0.75 S_a' + 0.25 S_a(v_a) \tag{10}$$

3. Trace minimum path trees from each origin to all destinations by using the weighted travel times S_a'' from step 2;
4. Assign all trips from each origin to each destination to the minimum path (all-or-nothing assignment); call this link loading v_a'; and
5. Return to step 1 and replace v_a with v_a'.

The use of a weighted mean travel time is an attempt to prevent the method from oscillating widely in computing minimum paths. Note, however, that the link loadings are not averaged until the final step, although the link travel times reflect implicitly the all-or-nothing assignments at each iteration.

The same three-link example is solved by using this algorithm in Table 2. The new travel times are given in each iteration as a basis for determining the next assignment. Following four all-or-nothing assignments, the mean flow is computed. The objective function of the equilibrium-assignment problem is computed for each iteration and for the final solution. This function provides a useful measure for comparison of the equilibrium and iterative assignments. The final value of the objective function for the iterative assignment has a somewhat higher value than for the equilibrium assignment. Thus the iterative assignment is not as close to true equilibrium. This conclusion can also be drawn by comparing the travel times that correspond to the final link loading in Tables 1 and 2. At equilibrium, these travel times should be equal.

Another weakness of the iterative-assignment algorithm is that there is no reliable rule about how many iterations to perform or what weights to use in computing the mean travel times. Had one more iteration of the algorithm been performed (or one less), the result would have been much different. With the equilibrium procedure, the overall result always improves with each iteration; the number of iterations depends only on how much improvement is desired.

Incremental Assignment

Another heuristic assignment procedure that has been widely used is incremental assignment. There are two types of incremental loading of a network. In the first type each origin-destination flow is divided into n equal parts, typically four. Each part is assigned by using all-or-nothing assignment; the link-loading and travel times are updated following the assignment of each increment. Following the assignment of the nth part, the link loadings are summed to determine the final loading. An alternate method developed by the Chicago Area Transportation Study (CATS) is the tree-by-tree method. In this case each row of the trip table is assigned completely by all-or-nothing assignment; the travel times are updated following each assignment.

Table 3 gives the results of the first incremental method applied to the three-link example. Four increments are used. By coincidence the final result happens to be the same as that given by the iterative method. The objective function value applies only to the final solution in this case. As with iterative assignment, the number of increments is an important determinant of the quality of the solution. In this case, however, the quality tends to improve as the number of increments increases.

In all three methods, a similar number of all-or-nothing assignments are performed to obtain a solution. No conclusions should be drawn about the relative quality of the solutions among the three methods, since such a small example could be quite misleading. The purpose here is only to educate and to compare the actual calcu-

Table 2. FHWA iterative assignment.

Iteration	Step	Link A Flow	Link A Time	Link B Flow	Link B Time	Link C Flow	Link C Time	Equilibrium Objective Function
Initial solution		8000	9231.0	0	20.0	0	21.0	14 864 000
1	2		2319.0		20.0		21.0	
	4	0	15.0	8000	171.7	0	21.0	402 726
2	2		1743.0		57.9		21.0	
	4	0	15.0	0	20.0	8000	2570.0	4 245 796
3	2		1311.0		48.4		658.3	
	4	0	15.0	8000	171.7	0	21.0	402 726
Mean flows and corresponding travel times		2000	51.0	4000	29.5	2000	31.0	177 967

Table 3. Incremental assignment.

Increment	Step	Link A Flow	Link A Time	Link B Flow	Link B Time	Link C Flow	Link C Time	Equilibrium Objective Function
1	Assignment	2000		0		0		
	Sum, time	2000	51.0	0	20.0	0	21.0	
2	Assignment	0		2000		0		
	Sum, time	2000	51.0	2000	20.6	0	21.0	
3	Assignment	0		2000		0		
	Sum, time	2000	51.0	4000	29.5	0	21.0	
4	Assignment	0		0		2000		
	Sum, time	2000	51.0	4000	29.5	2000	31.0	177 967

lations performed in each case.

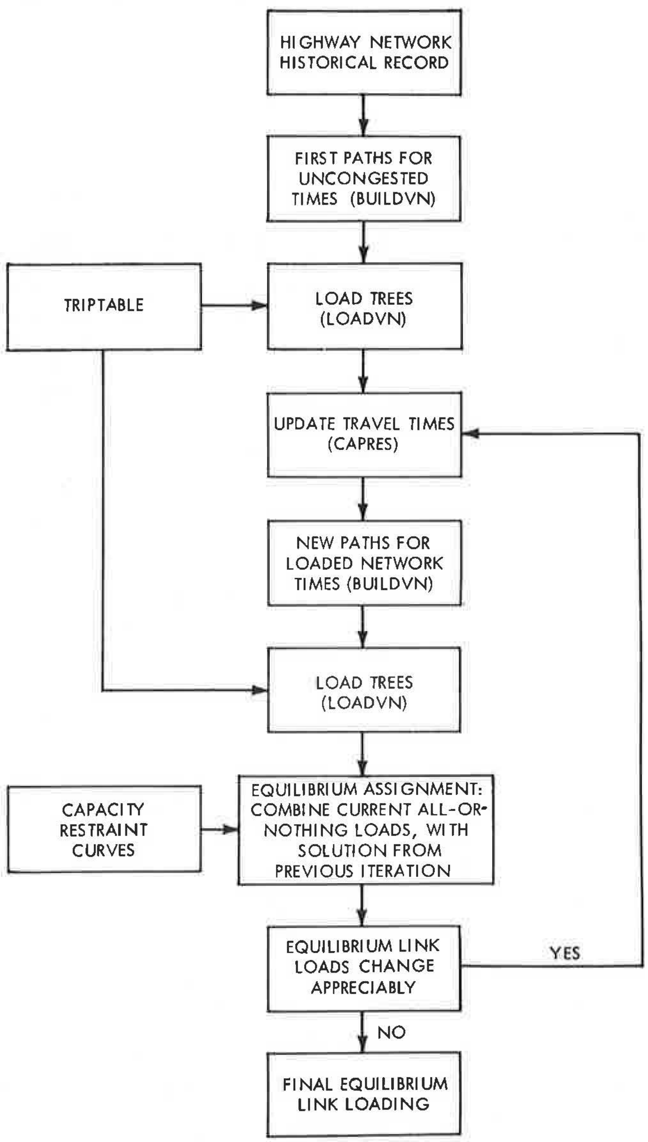
APPLICATION OF AN EQUILIBRIUM-ASSIGNMENT ALGORITHM

This section presents an application of equilibrium assignment to a large-scale trip table and network by CATS. The only other report of such an application was made by Florian and Nguyen (8) for a medium-sized network for Winnipeg. Applications have also been made by the Los Angeles Regional Transportation Study, but no results have been published.

The Equilibrium-Assignment Program

A program to perform equilibrium assignment was developed cooperatively by CATS and the University of Illinois, Urbana-Champaign. This program uses modules of the FHWA System-370 PLANPAC program battery, including programs for tree building, network loading, and network travel-time updating. The equilibrium-assignment program, which incorporates the existing PLANPAC programs, is illustrated in Figure 3. Analysts familiar with PLANPAC will recognize that the sequence of program steps shown in this figure differs only slightly from the usual application of the PLANPAC programs. The equilibrium-assignment program simply replaces the program VOLAVG, which is used to average loadings from separate assignments. But whereas the analyst must arbitrarily select how the two sets of link volumes are to be weighted in VOLAVG, the equilibrium-assignment routine internally determines the weighting of the link loadings that most nearly results in an equilibrium assignment.

Figure 3. Equilibrium assignment combined with PLANPAC programs.



Note: PLANPAC program names in parenthesis.

There is one feature of the PLANPAC programs that greatly simplifies the use of the equilibrium-assignment algorithm—this is the format of the highway network file. In the PLANPAC battery, highway network files are maintained in a binary file, called the network historical record. For each iteration the new link volume and recomputed link travel time are successively added at the end of a link record. Thus, all of the information needed for the calculation of a new equilibrium link volume (except λ) can be stored in one link record. New equilibrium link volumes can then be tagged at the end of the historical record (just like any other link volume) and passed directly into the CAPRES program to recompute link travel times.

The program to compute the equilibrium assignment has an uncomplicated linear structure. Logic of this program is as follows:

1. The capacity-restraint curves are read into memory;
2. The control card that identifies the location in the network historical record of the current solution and the current all-or-nothing assignment is read;
3. The network historical record is read, and the link capacities and both sets of link volumes and times are loaded into arrays;
4. A one-dimensional search procedure is executed to find the value of λ that minimizes the objective function computed from the current solution and the current all-or-nothing loading; and
5. The historical record is reread and a new historical record is written, which contains the new current solution.

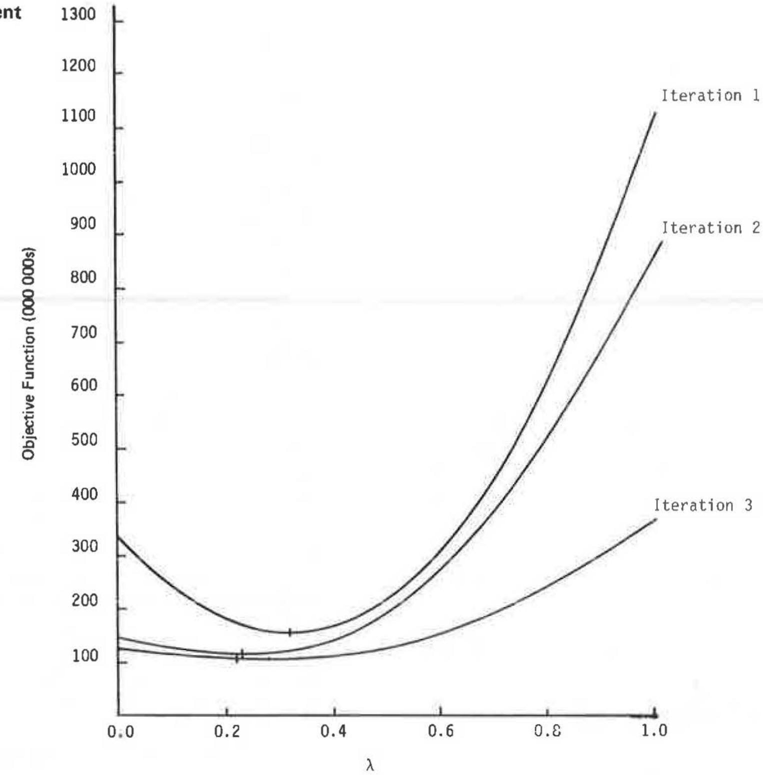
BASE DATA

Network and trip-table data for the application were obtained from a subarea transportation study for DuPage County, Illinois. This is a suburban county in northeastern Illinois, which covers an area of approximately 900 km^2 (350 miles^2) directly west of Cook County and the city of Chicago. The eastern half of the county is quite developed and has several major retail and employment centers. Current county population is about 500 000 persons; county employment is about 250 000 jobs. A wide range of traffic conditions can be observed in the county, including congestion and delay on many arterials.

Although the DuPage County network is for a subarea study, the assignment network is still quite large. There are nearly 29 000 one-way links and 9400 nodes in the 1975 network. Approximately one-third of the network is contained within the primary study area of DuPage County and a 10-km (6-mile) wide collar around the county. The network in this area is detailed and includes all roads except minor local streets. Outside of the primary study area the network is more aggregated, but it still contains all major and minor arterials.

The zone system has 906 zones in DuPage County plus

Figure 4. The equilibrium assignment objective function versus λ.



an additional 93 zones for the remainder of the northeastern Illinois region.

Definition of Capacity-Restraint Functions

Three different sets of capacity-restraint functions were used to determine their effect on the algorithm's performance: (a) CATS original capacity-restraint curves, (b) the standard FHWA capacity-restraint curves, and (c) a revised set of FHWA capacity curves. Instead of using the actual functions, the curves are entered into the program as a set of data points. The function is then approximated by chords connecting these points.

The CATS capacity-restraint curve used in this application is

$$S = t_0(2^{v/c} + 1)/2 \qquad (11)$$

The standard FHWA capacity function is

$$S = t_0[1 + 0.15(v/c)^4] \qquad (12)$$

where t_0 = free-flow link travel time and v/c = link flow to capacity ratio.

Algorithm Convergence Toward Equilibrium

One of the first questions raised in dealing with the equilibrium-assignment algorithm is, How quickly does the assignment converge to equilibrium? Figure 4 shows how the objective function varies with different values of λ through three iterations by using the CATS' capacity-restraint function. In the first iteration, the objective function is strongly concave and has a minimum at λ = 0.34. The objective functions for the next two iterations flatten out considerably; by the third iteration the optimal value of the objective function differs from the objective function at λ equal to zero by less than 10 percent. Nearly identical results were obtained by use of the standard FHWA capacity-restraint function.

This experience from the DuPage study suggests that, for all practical purposes, equilibrium is reached after four iterations of the equilibrium-assignment algorithm. This corresponds to the building and loading of five sets of minimum-time-path trees since one additional all-or-nothing assignment is needed to find an initial solution. The building of the minimum-time paths is the most expensive operation in each iteration. The one-dimensional search does not significantly increase the computation time as compared with an FHWA iterative assignment. Execution of the BUILDVN program for the DuPage network requires 10 min of central processing unit (CPU) time on an IBM 370/168 computer.

Further documentation of how λ converges is tabulated in Table 4, which lists the values of the objective function for separate runs of four iterations on each of the two capacity-restraint functions. Although the value of the objective function is much different for the two equilibrium-assignment runs, performance of the algorithm is not significantly altered. By the fourth iteration, λ values of less than 0.10 are attained in both examples. Therefore, four iterations would appear to be sufficient for large networks over a reasonable range of capacity-restraint functions.

COMPARISON OF EQUILIBRIUM AND FHWA ITERATIVE ASSIGNMENTS

The equilibrium-assignment objective function was computed for a conventional FHWA iterative assignment to determine how well this heuristic approximates equilibrium link loadings. The results for the iterative assignment are shown in the right-hand column of Table 4. These calculations were made by using the standard FHWA capacity-restraint functions and are directly comparable with the adjacent column. The objective

Table 4. λs and objective functions for two sample runs.

	Equilibrium Assignment				FHWA Iterative Assignment
	CATS Capacity Restraint		FHWA Capacity Restraint		FHWA Capacity Restraint
Iteration	λ	Objective Value	λ	Objective Value	Objective Value
1	0.34	151×10^9	0.34	227×10^9	481×10^9
2	0.23	120×10^9	0.21	177×10^9	917×10^9
3	0.22	112×10^9	0.20	156×10^9	728×10^9
4	0.25	112×10^9	0.07	154×10^8	790×10^9
					255×10^9[a]

[a]Objective value for assignment formed by averaging iterations 3 and 4.

Table 5. FHWA iterative and equilibrium-assignment results for DuPage study (CATS capacity-restraint function).

		FHWA Iterative		Equilibrium	
Counted Volume Group Range	Average Count	Average Volume	RMS Error (%)	Average Volume	RMS Error (%)
0-500	243	1 702	1 012.3	1 750	960.0
500-1 000	706	1 495	226.3	1 556	226.9
1 000-2 000	1 456	2 099	93.2	2 086	89.7
2 000-3 000	2 462	2 535	54.7	2 665	55.0
3 000-5 000	3 971	4 045	47.4	4 014	45.6
5 000-10 000	7 002	6 808	37.1	6 831	41.1
10 000-15 000	12 057	11 623	32.4	11 632	27.2
15 000-20 000	16 780	16 735	27.9	16 270	26.0
20 000-25 000	21 714	19 815	21.5	18 028	23.9
30 000-40 000	36 300	35 644	7.5	27 446	26.7
Entire volume range	6 352	6 383	39.8	6 223	43.2

Table 6. Average assigned volumes by using different capacity-restraint functions after four iterations of the equilibrium-assignment algorithm.

		CATS Capacity Curve		FHWA Capacity Curve		Adjusted FHWA Capacity Curve	
Counted Volume Group Range	Average Count	Average Volume	RMS Error (%)	Average Volume	RMS Error (%)	Average Volume	RMS Error (%)
0-500	243	1 750	960.0	1 688	978.1	1 683	979.8
500-1 000	706	1 556	226.9	1 601	244.9	1 621	247.5
1 000-2 000	1 456	2 086	89.7	2 187	110.0	2 161	105.6
2 000-3 000	2 462	2 665	55.0	2 916	67.7	2 862	65.8
3 000-5 000	3 971	4 014	45.6	4 331	52.9	4 259	50.7
5 000-10 000	7 002	6 831	41.1	7 067	42.6	7 038	42.2
10 000-15 000	12 057	11 632	27.2	11 393	29.4	11 472	28.5
15 000-20 000	16 780	16 270	26.0	14 581	23.0	14 939	23.1
20 000-25 000	21 714	18 028	23.9	17 104	29.8	17 276	28.2
30 000-40 000	36 300	27 446	26.7	28 485	24.0	28 652	23.6
Entire volume range	6 352	6 223	43.2	6 293	45.1	6 291	44.3

function for the mean of the third and fourth FHWA iterations is almost 50 percent greater than the objective function for the equilibrium-algorithm loadings after four iterations. Clearly, the conventional iterative approach produces a rather poor approximation of equilibrium.

The comparison of equilibrium and FHWA iterative assignment was further investigated by comparing the results of two assignments by using CATS' capacity-restraint functions. The link flows given in Table 5 are those produced by the fourth iteration of the algorithm. Included in this table are approximately 600 links in DuPage County for which traffic counts were available. The data listed in the table come from the output of the PLANPAC program CAPRES. Each flow entry is the average flow assigned on all links in the link's class, and the root mean square (RMS) error column lists the RMS error as a percentage of the average count for the class. For a selected set of links with traffic counts within DuPage County, the two assignments showed significant differences. The equilibrium-assignment flows are generally less than the FHWA assignment flows on higher-flow links. Whether this is a general bias between the two techniques is impossible to tell at this point; the results of Table 5 may just point up the limitations in the capacity-restraint functions.

IMPACT OF DIFFERENT CAPACITY-RESTRAINT FUNCTIONS

In order to examine the above point one step further, different functions were tested to determine how they affected the results of the equilibrium-assignment algorithm. CATS capacity-restraint functions were used in the algorithm first, then the FHWA set of curves was used, and finally an adjusted set of FHWA curves was inserted in the algorithm. The adjusted curves were tested because of an apparent underassignment of high-volume links and overassignment of low-volume links by the algorithm when the FHWA curves were used. The adjusted FHWA capacity curves were set so that the capacity of a high-capacity link is effectively increased by 10 percent and the capacity of a low-capacity link is decreased by 10 percent.

Table 6 provides some results from these three equilibrium-assignment runs, which incorporate different capacity-restraint functions. There are no substantial differences between any of the assignments. The use of CATS original capacity-restraint function provides an assignment slightly closer to actual counts, but the results are not significantly better than the remaining two assignments. All three assignments tend to overpredict traffic on low-volume links, partially because the local street network over which the beginning and ending segments of trips travel is incomplete. Comparison of the second and third assignments shows that the effect of the adjustment to the FHWA curves is almost negligible.

The changes that do occur, however, are in the desired direction, which indicates that some control over the assignment can be exerted through capacity-restraint functions. Since the equilibrium-assignment algorithm produces a convergent series of assignments, it should be possible to calibrate these functions according to route type or location in an urban area.

CONCLUSIONS

Although our experience with applications of equilibrium assignment to large-scale, congested networks is still limited, we believe that the results reported in this paper provide convincing evidence that equilibrium assignment should always be preferred to FHWA iterative assignment for congested networks. We reach this conclusion for three reasons:

1. Equilibrium assignment provides a better assignment in terms of the overall objective of equal travel times over all paths used between each origin and destination pair,
2. The computational effort is similar and may be less in some cases in which the equilibrium algorithm converges quickly, and
3. Equilibrium assignment can be readily incorporated into FHWA's PLANPAC battery; moreover, it is already available in UTPS.

The preliminary results we have presented concerning the ability of equilibrium assignment to reproduce observed 24-h flows are not as convincing. There are two reasons for this result. First, the capacity-restraint functions are probably too crude. This problem has been explored slightly here, but more study and experimentation are needed. Second, the use of equilibrium assignment to produce 24-h assignments may be inappropriate in that only the peak periods have truly congested flow. All-or-nothing assignment may be sufficient for off-peak periods. Additional study of this question is needed to determine the actual cause of these apparent differences between ground counts and assigned flows.

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The basic research on equilibrium assignment, on which this paper is based, has been conducted by many individuals during the past 10 years. Since it was not our purpose to review the development of equilibrium-assignment methods, we have not referred to this literature, except in the few cases in which it was directly pertinent. We are grateful for the advice and encouragement that we received from David Gendell of the Federal Highway Administration and Thomas Hillegass of the Urban Mass Transportation Administration.

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Equilibration Properties of Logit Models

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Despite the importance of supply-demand equilibration in travel-demand forecasting and urban planning, no attention has been paid to the equilibration properties of logit models of travel demand and residential mobility. The preponderance of logit models in travel demand and related fields suggests that these properties are worth examining if these models are to become useful forecasting tools. This paper demonstrates the basic price equilibration properties of logit models for simplified versions of six typical problems encountered in travel-demand and residential-location forecasting. Measures of the differential price of any two alternatives are derived in closed form and shown to reflect the well-known logit property of the independence from irrelevant alternatives as long as the population of travelers and households is one homogeneous group. It is shown that