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# Validation and Application of an Equilibrium-Based Two-Mode Urban Transportation Planning Method (EMME)

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The purpose of this paper is to report on the validation and application of the two-mode urban transportation planning technique called EMME. This method may be characterized as an integrated two-mode traffic equilibrium method. Roughly speaking, this method combines a zonal aggregate-demand model with an equilibrium-type road assignment and a transit-assignment method. We describe the validation and application of the model by using data from the city of Winnipeg, Manitoba, Canada.

The purpose of this paper is to report on the validation and application of the two-mode urban transportation planning technique équilibre multimodal-multimodal equilibrium (EMME). This method may be characterized as an integrated two-mode traffic equilibrium method. It was suggested by Florian (1). Roughly speaking, this method combines a zonal aggregate-demand model (which may be a direct-demand model or an origin-destination table coupled with a suitable modal-split function) with an equilibrium-type road assignment and a transitassignment method. The method has been described previously (2) and some of its theoretical properties have been studied by Fisk and Nguyen (3). The model was validated by using data from the city of Winnipeg, Manitoba, Canada. The equilibrium-type route-choice model for travel by private automobiles in congested urban areas was validated by Florian and Nguyen (4) in the Winnipeg road network. The transit-assignment model is essentially a shortest-route choice coupled with the diversion mechanism among sections served by common lines, which was devised by Chriqui and Robillard (5).

For the purpose of transportation planning, the city

of Winnipeg is subdivided into 147 zones. The road network has 1040 nodes and 2836 one-way lines; observed link flows and link times were available for most of the links. The transit network has 56 lines, 1755 line segments, 500 egress-access links, and 800 nodes; 575 of the road network nodes are used in the coding of the transit network as well.

In the summer of 1976 the city of Winnipeg performed a speed-delay study, which consisted of measuring link volumes and link automobile travel times for 80-90 percent of the street system. In addition, bus travel times were measured for 446 transit line sections. These data served to recalibrate the volume-delay curves that were used in the road assignment and to calibrate the busautomobile travel-time relationship required by EMME.

Since the city of Winnipeg had not previously used a transit-assignment model, the transit network was coded according to the EMME specifications, described by Achim and Chapleau (6), that permit the interface between the road and transit networks.

During the summer of 1976, the city of Winnipeg also performed an origin-destination survey of trips taken from home to work. A 17 percent sample of households was sampled and a separate survey of 23 percent of student trips was performed at about the same time. Since all of the analysis is done for the 7:30-8:30 a.m. peak hour, one of the first tasks considered was to define the departure codes, that is, the starting time of trips that will be using the road and transit networks during the peak hour. The departure codes were determined by the city of Winnipeg staff and were specified by origin,

by using a subdivision of origins into 36 super zones. By using the departure codes, the corresponding trips are extracted from the survey data and multiplied by the appropriate expansion factors to obtain an estimate of the total person work trips taken in the peak hour by each mode. Then the total automobile work-trip matrix is scaled by appropriate automobile occupancy factors in order to obtain the total automobile work-trip matrix.

This matrix was then assigned to the road network and compared with the observed link volumes. Since only the work trips are sampled during the origindestination survey, it was necessary to develop a set of adjustment factors that multiply the number of trips in the total automobile work-trip matrix in order to reflect automobile trips that are taken for purposes other than work and a certain amount of truck traffic. These factors are specified by origin to subdivide origins into 10 super zones. The determination of the most appropriate factor is a trial-and-error procedure. Where a factor is tried, the resulting assignments are compared to observed link flows and then a new factor is determined, which, it is hoped, is more appropriate. Five factors were tried until satisfactory results were obtained. In addition, trips to the University of Manitoba required special departure codes, which were specified for the subdivision of origins into 10 super zones, since this zone is relatively more distant from most origins. In the EMME computer system, the factors are converted into a vehicle-adjustment trip matrix, which is added to the total automobile work-trip matrix for the purpose of the assignment.

Once the departure codes, and hence the fixed origin-destination matrices, were determined, the modal-split function was calibrated. Due to the large size of the sample, it was possible to calibrate a zonal-aggregate logit modal-split function. We were then provided by the city of Winnipeg with a road-improvement scenario and a transit-improvement scenario. We first analyzed the base-year calibration by using the bimodal model and then proceeded to analyze the impact of the scenarios.

#### THE BUS-AUTOMOBILE TRAVEL-TIME FUNCTION

The purpose of this task was to develop a model that relates the travel time of a transit vehicle on a road link to the corresponding travel time for private automobiles. The model is used to take into account the change of transit travel times as a result of a change in the congestion level of a road link.

The data needed to develop this model are road link lengths, observed automobile travel times on those links, and the corresponding bus travel times. The road link lengths and automobile times were obtained from the road network data. The city of Winnipeg provided us with observed bus travel times for line sections (a line section is defined as the sequence of the corresponding road links). (The model was designed for U.S. customary units only; therefore, values are not given in SI units.)

We first created a data file that, for each line section, contains the following information:

- 1. Starting node,
- 2. Ending node,
- 3. Direction (inbound or outbound),
- 4. Line number,
- 5. Observed bus time,
- Observed automobile time (for complete sequence of links),
  - 7. Minutes per mile for the bus on the section,
  - 8. Minutes per mile for automobiles on the section,

and

# 9. Number of road links in the section.

The file contains observations for 470 line sections. On 25 segments, the observed transit time was smaller than the observed automobile time. Since this problem seems to be related to the accuracy of the data, these observations were not considered in the calibration of the model.

We first introduce some notation:

Let

TA = automobile time on the line section (min),

TB = bus time on the line section (min),

TMA = automobile time per mile on the line section (min/mile), and

TMB = bus time per mile on the line section (min/mile).

First, we plotted TB as a function of TA. Figure 1 shows the resulting scatter diagram; a linear function was fitted, resulting in an R<sup>2</sup> of 0.87. However, some contemplation of this relationship reveals that, over long sections, both the bus and the automobile times are relatively long, and, of course, on short sections, both times are relatively small (that is, they are both correlated to link length). Evidently, such a model would not capture any effect of congestion.

We proceeded then to analyze the inverse of speed (time per mile) (which is used in the formulation of volume-delay curves). A simple linear model of TMB versus TMA resulted in a poor fit of  $R^2 = 0.2$ . A linear model of TMB versus TMA and TA increased the  $R^2$  to 0.49, which also was not satisfactory. In both of the above cases, we tried different models for the inbound and outbound direction but the fits, reflected in the  $R^2$  values, were not improved.

A plot of (TMB/TMA) versus TMA showed that a non-linear model could be more appropriate (Figure 2). An exponential model of the form

$$ln[(TMB/TMA) - 1] = a_0 + a_1 TMA$$
 (1)

was estimated by linear regression. Again, with an  $R^2$  = 0.09, the model was rejected. We then attempted to use a polynomial model of the form

$$(TMB/TMA) - 1 = a_1 (TMA)^{-1/2} + a_2 (TMA)^{-1} + a_3 (TMA)^{-3/2} + a_4 (TMA)^{-2}$$
 (2)

which was estimated with a stepwise linear regression. The only term that entered in the regression was  $a_1$  (TMA)- $\frac{1}{2}$  and it resulted in an  $R^2$  of 0.62, which, considering the accuracy of the data, was the first satisfactory result obtained. The analytical form of this model (M1) is

$$(TMB/TMA) - 1 = 1.97\sqrt{1/TMA}$$
 (3a)

or

$$TMB = TMA + 1.97 \sqrt{TMA}$$
 (3b)

As an alternative, we considered a linear model of the form TMB =  $m(t_0)$  + TMA, where  $t_0$  is the inverse of the free-flow speed of the road link. Values of  $t_0$  were obtained from the road network data. A linear regression gave an  $R^2$  of 0.62; the model (M2) is as follows:

$$TMB = TMA + 1.43 t_0 \tag{4}$$

where 1.43 to is a constant penalty in minutes per mile

for transit vehicles, which is related in some way to the link type.

The next step was to make an evaluation of the predictive ability of models M1 and M2. Since we are mainly interested in predicting good transit impedances (origin to destination path times), we decided to com-

pare for each line and each direction in the data file the sum of the predicted travel times on each section against the corresponding observed times. The results were good for most of the lines (within 10 percent) except for express services and for some high-speed regular lines.

It became evident that a natural way to improve the

Figure 1. Bus travel times versus automobile travel times.

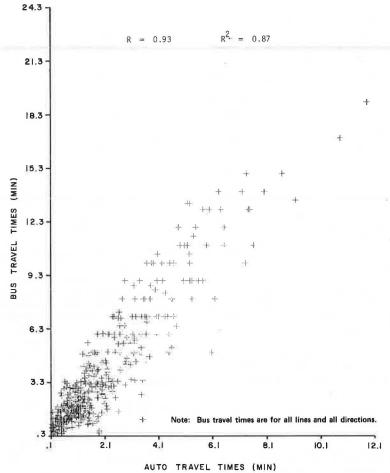
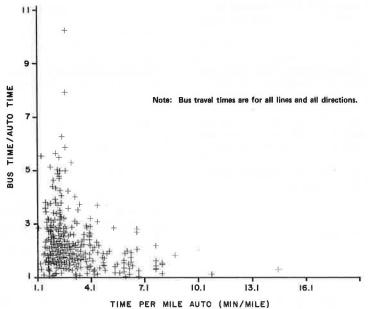


Figure 2. Bus and automobile time versus time per mile by automobile.



models was to stratify the data according to service type. The three types considered were

- 1. Feeder-0 observations in data file,
- 2. Regular-426 observations, and
- 3. Express-44 observations.

Models of the same form as M1 and M2 were estimated for the express service. The  $R^2$  values were close to 0.5 and the models were not significant because of the rather small number of observations available. In the case of regular service, the recalibration of models M1 and M2 resulted in the relations:

$$TMB = TMA + 2.1\sqrt{TMA}$$
 (5a)

$$TMB = TMA + 1.49 t_0$$
 (5b)

The  $R^2$  values improved slightly (0.64), but overall the models did not change significantly. We then subdivided the regular service into two categories by considering the average observed speed of each line. All the lines that ran at less than 10 mph were classified as regular and the others as fast regular.

For the fast-regular lines, the recalibration of model M1 results in

$$TMB = TMA + 2.15 R^2 = 0.84$$
 (6)

and the recalibration of model M2 results in

$$TMS = TMA + 0.9 t_0 R^2 = 0.84 (7)$$

For the regular lines, model M1 becomes

$$TMB = TMA + 9.14/\sqrt{TMA^{1}} \qquad R^{2} = 0.78$$
 (8)

and model M2 becomes

TMB = TMA + 
$$2.12 t_0$$
  $R^2 = 0.73$  (9)

This time the comparison, for each line and direction, of the sum of observed and predicted times on each section showed that TMB = TMA + 2.15 is a good model for fast-regular lines. In the case of regular lines, both models had to be rejected. Our next step in the analysis of the regular lines was to go back to the previous form of the model, that is, to estimate a function of the form

$$TMB = TMA + \alpha \sqrt{TMA}$$
 (10)

that had proved to be satisfactory for regular lines, except for the fast ones. The estimation resulted in an  $\alpha=3.21$  and an  $R^2=0.73$ . Unfortunately, the comparison of the sum of line-section times showed that the previous model ( $\alpha=2.1,\ R^2=0.64$ ), which had been estimated on all regular-lines data (fast regular and regular), gave better results than did the new one, which had been estimated by using data for regular lines (< 10 mph) only.

The above analysis suggested that it may be advantageous to define fast-regular lines by using a higher speed value. But further experiments indicated that the results could not be improved in this way.

In consideration of the above analysis and the fact that we did not have sufficient data for feeder and express services, we finally selected and implemented the following bus travel-time relationships. On transit-only links, the user-defined travel times are used. On transit links that correspond to road links, four cases are considered:

- 1. For a feeder service (line type 3) the userdefined line speed is used on all links, independent of automobile speed:
- 2. For an express service (line type 1) the bus speed is the same as automobile speed (TMB = TMA);
- 3. For a fast-regular service (line type 2) we have TMB = TMA + 2.15 (regular lines with an average speed of 10 mph or more were considered as fast lines); and
- 4. For a regular service (undefined line type) we have TMB = TMA +  $2.1 *\sqrt{TMA}$  where TMB = minutes by mile for bus and TMA = minutes by mile for automobile.

The relationships were applied to predict the transit travel time on each of the 1755 transit links of the coded network. On the basis of those predicted times, transit paths between selected origin-destination pairs were calculated. An analysis of the transit times and paths suggested that we should change the classification of some of the lines. After a few iterations of this procedure, we made final classifications for all of the lines.

An important fringe benefit of having included a bus time model in EMME is that the user does not have to define a travel time for each of the transit links; thus the coding of the network is made much easier.

#### RECALIBRATION OF THE VOLUME-DELAY CURVES

The volume-delay curves used by the city of Winnipeg were developed in the early 1960s by Traffic Research Corporation and had the functional form

$$S_{a}(v_{a}) = d_{a}(\delta + \alpha[(v_{a}/\ell_{a}) - \gamma] + {\alpha^{2}[(v_{a}/\ell_{a}) - \gamma]^{2} + \beta}^{\frac{1}{2}})$$
(11)

We modified this functional form by replacing it with the simpler BPR formula:

$$S_{a}(v_{a}) = d_{a}t_{0}[1 + \alpha(v_{a}/c_{a})^{\beta}]$$
 (12)

where

da = the link length,

va = the link volume,

 $\ell_a$  = the number of lanes of the link, and

ca = the practical capacity of the link.

The other parameters are calibrated from the observed data. The initial transformation was done by Branston (7). He estimated a practical capacity for each of the volume-delay curves and then calibrated the constants  $\alpha$ ,  $\beta$  of the BPR formula by using the predicted times of the Traffic Research Corporation functions.

We then recalibrated the BPR curves obtained in this way by using the 1976 data and the following procedure. For each volume-delay curve, the observed data were aggregated by using a subdivision of the link volumes  $(v_a)$  into intervals, and mean values were computed for each interval. The curves and the resulting mean values of the travel times were plotted and analyzed; as a result, new free-flow speeds were determined and then the curves were replotted. This procedure was repeated three times, resulting in a new set of  $\alpha$ ,  $\beta$ , and  $t_0$ . Table 1 shows the values that were actually used.

It was evident from the plots used to determine the free-flow speed that certain links, which exhibited observed times below and to the right of the curves, would be better predicted by delay curves that represent higher-capacity links. In order to assist the city of Winnipeg in this reclassification of links to different curves, a report was produced for all links for which

an observed flow was available. This report gives the time predicted by the currently assigned curve and also other curves that would predict the travel time better and still respect the speed limit. This report was used to reclassify links on a route basis. Links that had large differences between predicted and observed times were plotted on a map in order to determine the links of an avenue or street that had to be reclassified. In some cases the number of lanes was corrected as well. This analysis also resulted in the correction of some observed travel times and volumes. In total, 159 links were reclassified, the number of lanes was changed for 23 links, the observed time was updated for 192 links, and the observed volume was updated for 21 links. Figure 3 shows plots of the origin-to-destination travel times along shortest paths computed by using the volumedelay curves versus the observed times.

# CALIBRATION AND VALIDATION OF THE ROAD-NETWORK ASSIGNMENT

The calibration of the road network was achieved by comparing the observed link volumes with the link volumes predicted by the traffic-assignment model. The comparison is performed by using specially written

Table 1. Volume-delay functions.

CAĈO	Speed Limit (mph)	ø	β	Free-Flow Times (t <sub>0</sub> )	Minutes per Mile
1	0-30	0,7312	3.6596	15.0	4.00
2	0-30	0.6218	3.5038	17.0	3.53
3	0-30	0.8774	4.4613	20.0	3.00
4	0-30	0.6846	5.1644	23.0	2.61
5	0-30	1.1465	4,4239	25.0	2.40
6	31-40	0.6190	3.6544	30.0	2.00
7	31-40	0,6662	4.9432	32.4	1.85
8 9	31-40	0.6222	5.1409	32.4	1.85
9	31-40	1.0300	5.5226	35.3	1.70
10	41-50	0.6609	5.0906	41.4	1.45
14	41-50	0.5423	5.7894	41.4	1.45
15	41-50	1.0091	6.5856	41.4	1.45
15	+50	0.8776	4,9287	55.0	1.09
16	+50	0.7699	5.3443	55.0	1.09
18	+50	1.1491	6.8677	55.0	1.09

programs and by manually comparing screen-line totals for the observed and predicted flows. Discrepancies between observed and predicted values may be caused by the errors introduced in the total automobile origin-destination matrix or by improper coding of the road network. Since the 1976 road network differs little from the 1971 network, which was carefully calibrated, the corrections necessary to the coding of the road network were all found during the recalibration of the volumedelay curves and most of the adjustments made involved the total automobile origin-destination matrix.

This matrix is calculated from the total person worktrip matrix by using the observed modal-split and automobile-occupancy matrices and a set of adjustment factors that serve to add other-purpose trips and truck trips; that is

$$g_{pq}^{au} = (g_{pq} * r_{pq}/\gamma_{pq}) * f_{pq}$$
 (13)

where

(p, q) = an origin-destination pair of zones,

gpq = the total person work trips between q and p,

 $r_{pq}$  = the proportion of trips by automobile,

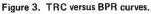
 $\gamma_{pq}$  = the automobile occupancy, and

 $f_{pq}$  = the factor for other trips and truck trips.

The factors  $f_{pq}$  are given as a matrix of values for 10 groups of zones (super zones). The essence of the calibration procedure was a trial-and-error process that was aimed at finding the most appropriate factors based on the comparison of observed and predicted link volumes. While this was carried out, 19 errors in the observed link volumes were detected and corrections were made.

All the factors in the calibration procedure were determined by the staff of the city of Winnipeg by using screen-line counts. The screen lines chosen divide the city into three quadrants by using natural geographic subdivisions. A specially written program selects the links that cross each of these lines and provides the observed and predicted volumes, which are then totaled for each screen line.

First, an assignment was produced by using only the



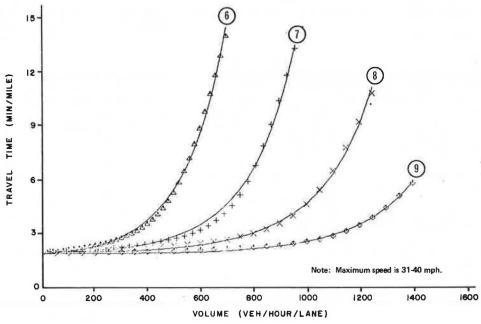


Table 2. Parameters assigned for the analysis.

Assignment	WALK	WAIT	WFAC	WPEN	WMIN	WMAX
First stage						
1	3.0	3.0	0.5	0.0	0.0	10.0
2	0.0	1.0	0.5	0.0	0.0	10.0
3	0.5	2.0	0.5	4.0	0.0	10.0
Common line section						
4	3.0	3.0	0.5	0.0	0.0	10.0
5	0.0	1.0	0.5	0.0	0.0	10.0
6	0.5	2.0	0.5	4.0	0.0	10.0
7	0.5	3.0	0.5	4.0	0.0	10.0

automobile work-trip matrix, which is obtained by setting  $f_{pq}=1$ . By relating super-zone pairs with screenline crossings it is possible to adjust the various factors to increase or decrease the interchanges across the screen lines. The correspondence is as follows:

Quadrant	Super Zones
1	1, 2, 3
2	4, 5
3	Rest + downtown (0)

Various other considerations were taken into account in determining the factors  $(f_{pq})$ , such as the low production of truck trips by residential areas and the high production of truck trips by industrial zones.

# TRANSIT NETWORK VALIDATION AND CALIBRATION

This part of the project required considerable effort, since prior to this study the city of Winnipeg did not have a transit network model and the work included the definition of the network, its coding, validation, and calibration.

The purpose of the validation is to make sure that the transit system is described properly. The coded network must represent adequately all possible passenger movements and transit vehicle movements. The validation of the network consists, then, of ensuring that the coding rules have been followed correctly and that the representation of the two types of movements is satisfactory. The tools used in validation are

- 1. EMME data bank programs, which perform the syntactic and data consistency checks;
- 2. Network generation programs, which ensure that the rigorous restrictions imposed on the input data in order to realize the interface with the road network and to determine transit travel times are satisfied;
  - 3. Graphical displays of the network;
  - 4. Manual checks of the data; and
- 5. Analysis of the complete printout of the transit assignment.

This task was carried out in cooperation with the staff of the city of Winnipeg.

The calibration deals with the other aspect of the transit system, that is, the behavior of the transit passengers in the selection of paths on the network. Given the shortest-path behavior hypothesis, it is necessary to estimate the value of certain parameters of the transit path algorithm in order for it to produce satisfactory paths between the various origin-destination pairs.

The parameters to be estimated are

- 1. WFAC—a regularity factor relating the waiting time to the headway of the line to be boarded,
  - 2. WMIN-the minimum waiting time,

3. WMAX-the maximum waiting time,

- 4. WAIT—the weight of waiting time used in the calculation of the impedance of a path in generalized time units.
- 5. WPEN—a constant penalty added to the impedance every time the passenger has to wait for the bus, and
- 6. WALK—the weight of walking time (access-egress) used in the calculation of the impedance of a path.

A given path that contains n line sections has an impedance, in generalized time units, that is given by the expression:

IMP = WALK \* (access + egress time) + WAIT \* 
$$\sum_{\varrho=1}^{n} w_{\varrho}$$
  
+  $\sum_{\varrho=1}^{n} T_{\varrho} + n * WPEN$  (14)

where

 $W_{\ell}$  = the waiting time of the  $\ell$ th line defined as W min [max (WMIN, WFAC \* HDW<sub> $\ell$ </sub>), WMAX],

 $HDW_{\ell}$  = the headway of the  $\ell$  th line and transfer time and is considered as being included in waiting time, and

 $T_{\ell}$  = the in-vehicle time spent on the  $\ell$ th line, which is assumed to have a wait of 1.0 in the impedance calculations.

For each origin-destination pair the algorithm selects the path with minimum impedance from origin O to destination D. The best way to calibrate the transit model would be to compare the predicted paths to the actual paths obtained from the origin-destination survey. Unfortunately, in the Winnipeg survey there was no question about the path used by transit riders. The method that we used consisted of analyzing the predictions of a transit assignment by comparing it with the observed volumes on the segments. Analyses were also made on level-of-service statistics (i.e., mean total trip time, mean number of transfers, and distribution of total trip time) and on predicted line volumes. Given the all-ornothing aspect of the assignment, only large volumes may be analyzed. The following volumes were analyzed:

- 1. The volume at the maximum load point of each line in both directions,
  - 2. The location of the maximum load point,
  - 3. The volume profiles on lines, and
- 4. Screen-line volumes [entering and leaving the central business district (CBD), bridges, and other high-volume links].

In the first stage of the analysis, three assignments (assignments 1-3 in Table 2) were performed by use of the parameters given.

The analysis made by the staff of the city of Winnipeg showed that assignment 1 was the best one, but the split of volumes between competing lines was not satisfactory. We then introduced the "common line section" algorithm in the model. With this algorithm the passengers are diverted over common bus lines proportionally to the frequency of each line (i.e., passengers are assumed to board the first line that arrives at the bus stop). We ran four new simulations (assignments 4-7 in Table 2).

Assignment 4, which is similar to number 1, proved to be the best one and the spread of volumes had improved significantly.

# CALIBRATION OF THE MODAL-SPLIT FUNCTION

The basic data that were used for the calibration of the modal-split function are the results of the origindestination survey that was carried out by the city of Winnipeg in the spring and summer of 1976. The survey was carried out in large part by home interviews of a sample of 20 percent of households. The actual sample size obtained was roughly 17 percent, after refusals and rejections have been taken into account. In addition, a survey questionnaire, which was to be returned by mail, was distributed to the students of the three Winnipeg universities; the effective sample of student trips was approximately 23 percent. The total sample, consisting of the individual detailed data, amounted to 52 424 questionnaires and these data were transmitted to us on a magnetic tape by the city of Winnipeg. Then, the departure codes, described earlier, were applied in order to separate the trips that occurred during the 7:30-8:30 a.m. peak. There were 17 761 individual records in the peak-hour subsample.

The number of trips that occurred during the peak hour was expanded by the proportion of the sample in each zone, which was calculated by the city of Winnipeg, in order to obtain the following origin-destination matrices:

Automobile drivers and passengers—1, automobile drivers—1'

Transit passengers-2 Total trips-3 = (1 + 2)Modal split-4 = (1/3)Automobile occupancy-5 = (1/1')

(The automobile drivers and passengers origin-destination matrix shall be referred to as the automobile origin-destination matrix.) The automobile origin-destination matrix was scaled by the appropriate factor to obtain the total automobile origin-destination matrix and this last was assigned to the road network by using the equilibrium traffic assignment of EMME. The resulting origin-to-destination travel times constitute the origin-destination matrix of

Road travel times-6

and by tracing a set of shortest paths on the links that carry flow we obtain the origin-destination matrix of

Distance by road-7

Next, the transit origin-destination matrix was used to calibrate the transit assignment. Other than refinements of the transit network representation, this calibration determines the coefficients of generalized time (or cost) in the expression

Transit impedance = 
$$\alpha$$
(Access time + egress time + wait time) +  $\beta$ (In-vehicle time) (15)

As described earlier, the values for  $\alpha$  and  $\beta$ , determined in cooperation with the city of Winnipeg, are 3 and 1, respectively. Thus we obtained the origin-destination matrix of

Transit impedance-8

and by tracing the shortest paths used we determined the origin-destination matrix of

Number of transit transfers-9

Since our approach is to calibrate a zonal-aggregate modal-split function, we extracted from the survey data (a) the average automobile ownership per household per zone, (b) the proportion of adults who travel at the peak hour, and (c) the proportion of students who travel at the peak hour for each origin-destination pair.

The other socioeconomic variables were obtained by the city of Winnipeg from various sources and transmitted to us. The Statistics Canada 1976 Census provided the average income per household per zone and the origin-destination survey estimated the number of jobs per zone. The parking costs per month per zone and the number of parking spaces per job per zone were evaluated by using 1971 data.

Thus, in all, a file was constructed that consisted of the dependent variable, the modal split, and the independent explanatory variables outlined above. This file contained the records for all origin-destination pairs that had more than 60 trips by both modes in the expanded matrix (3) of trips by both modes. The main reason for adopting this procedure is that the modal split for origin-destination pairs with smaller demand would have far more variability due to the relatively small number of trips in the sample.

The functional form that we chose for the calibration is that of the logistic function. Although this form achieved recent fame in its use as a disaggregate probabilistic-choice function, we use it with aggregate data due to its ease of manipulation and its property of predicting choice values with a smooth ogive-type curve. The form that we used is

$$p_{au} = 1/1 + \exp(k_0 + \sum_i k_i x_i)$$
 (16)

where

p<sub>su</sub> = the proportion of trips that occur by
 automobile,
 k<sub>0</sub> = a constant, and

$$k_i$$
,  $i = 1, ..., m$  = the coefficients associated with the  $x_i$ ,  $i = 1, ..., n$  explanatory variables.

A simple algebraic manipulation results in the form  $\ell_{n} (1 - p_{nu}/p_{nu}) = k_0 + \sum_i k_i x_i$ , which is used for calibrating  $k_0$ ,  $k_i$ ,  $i = 1, \ldots, n$  by simple linear regression. This method of estimation is often referred to as Berkson-Theil estimation to acknowledge their early work (8,9) in aggregate logistic-function calibration.

Another functional form that we tried is the so-called "dogit" proposed recently by Gaudry  $(\underline{10})$ , which adds to the logit form modal constant  $\theta_{au}$ ,  $\theta_{tr}$  as follows:

$$p_{au} = (1/1 + \theta_{au} + \theta_{tr})[1/1 + \exp(k_0 + \sum_i k_i x_i)] + \theta_{au}$$
 (17)

However, in all of the trials that we performed, the best values for  $\theta_{au}$ ,  $\theta_{tr}$  were always zero; that is, the logistic function was satisfactory and neither of the two modes considered had a fixed proportion  $(\theta_{au} \text{ or } \theta_{tr})$  of the modal split as an advantage.

The actual calibration test spanned a period of eight months, during which several hundred regressions were run by also using transformations of the explanatory variables. The best modal-split model for all the considered origin-destination pairs is given in Table 3.

We were not entirely satisfied with this model because the best fit obtained with a transformation of variables was not much better, as can be seen in Table 4.

We then subdivided the origins into subgroups by using a criterion related to the error introduced by the modal-split function. We reasoned that errors on individual origin-destination pairs were unavoidable; however, the model should not distort the origin-destination matrix.

That is, there should not be too much bias introduced on demand totals by origins and destinations. Thus, we subdivided the origins into subgroups according to the error introduced by the model on origin totals; that is, origins that had negative deviations were grouped together and origins that had positive errors and origins that had acceptable error formed a second and third subgrouping. Finally, we obtained four modal-split models as shown in Table 5.

# BASE-YEAR CALIBRATION—BIMODAL MODEL

The execution of a bimodal assignment in EMME requires the simultaneous use of the vehicle assignment, the transit assignment, and the modal-split function. Each is calibrated independently and then used jointly in the computations. Since each introduces a certain error by its calibration, there will be some differences between the observed values and the output of the bimodal computations for the base year. Fortunately, these differences are not large and are well within the variances that are acceptable in calibration of transportation models.

The staff of the city of Winnipeg asked that we apply the modal-split function to all of the origin-destination

Table 3. Model 1 parameter values.

Variable	Parameter Value	95 Percent Confidence Interval			
Constant (k <sub>0</sub> )	2.563	1,758	to	3,369	
Transit impedance	-0.0122	-0,220	to	-0.002 42	
Automobile time	0.0220	0.001 92	to	0.0422	
Proportion men	-3.279	-4.117	to	-2.441	
Parking cost	0.0745	0.0532	to	0.0957	
Automobile availability	-1.904	-2.726	to	-1.082	
R <sup>2</sup>	0.60				
R	0.77				

Table 4. Model 2 parameter values.

Variable	Parameter Value	95 Percent Confidence Interval				
Constant (ko)	16.566	10.000	to -23.131			
(Transit impedance)2	-0.000 758	-0.000 127	to	-0.000	242	
(Automobile time)2	0.000 242	-0.000 180	to	0.000	665	
(Proportion men)2	-2,256 654	-3.478	to	-2.234		
Parking cost	0.363 40	0.276	to	0.444		
in (income)	-1.752	-2.446	to	-1.058		
R <sup>a</sup>	0.64					
R	0.80					

pairs, even though we had calibrated the model by using only origin-destination pairs that had more than 60 trips in the expanded total trip matrix. This became necessary because only about 28 percent of the total trips were represented by that sample. Thus we applied, to all origins that were not represented in the calibration data, the initial modal-split model (model 1). For all other origins, we applied the corresponding modal-split models to all of the relevant destinations. The results were surprisingly good. Only 307 trips (or 0.5 percent of the total number of trips) are the difference between the observed total number of trips by automobile and the differences that result on trip ends (that is, origin and destination totals) are mostly of the order of up to 8 percent. The predicted origin-destination matrix is plotted versus the observed origin-destination matrix in Fig-

We judged these demand differences acceptable in view of the general consideration that the true demand varies daily and differences of the order of 10 percent between various days of the week are accepted to be commonplace. Further, these differences were not sufficiently high to materially change the orders of magnitude of the link flows on the important arteries.

The computation times on the CDC-Cyber-176 of the University of Montreal for the base year bimodal run are as follows:

Computation Step	Time (s)		
Generate transit network	3.66		
Calculate bus frequency	1.23		
Calculate transit impedance	1110.55		
Initialize road traffic demand	3.44		
Perform road traffic assignment	2509.44		
Modify transit link times	67.61		
Calculate fixed transit demand	1.77		
Calculate demand function (transit)	0.00		
Perform transit assignment	423.91		
Modify transit capacity	0.00		

The costs are given below.

Function	Cost (\$)
Central processor	189.20
Input-output	22.80
Fast memory	560.40
Total	772.40

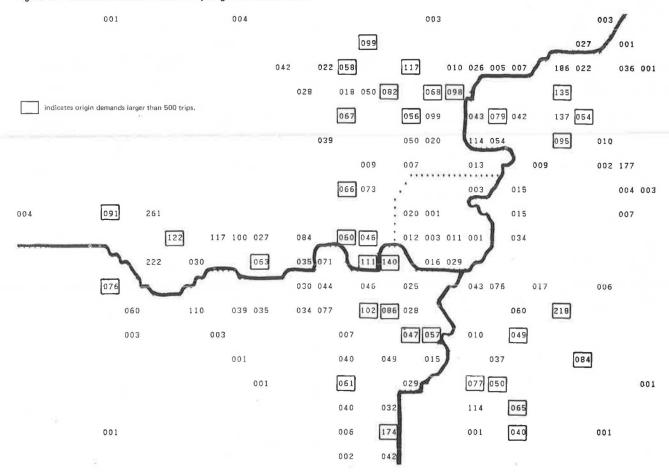
## CONCLUSION

There are several ways in which EMME may be used to simulate the impact of contemplated improvement scenarios. One may use the single-mode assignment mod-

Table 5. Model 3 parameter values.

Variable	Model 3a		Model 3b			Model 3c		Model 3d		
	Value	95 Percent Confi- dence Interval	Value	95 Percent dence Inter		nfi-	Value	95 Percent Confidence Interval	Value	95 Percent Confidence Interval
Constant Transit	2.352	1.004 to 5.708	1.516	0.297	to	2.735	3.071	-0.014 0 to 6.156	2.935	1.860 to 4.010
impedance Automobile	-0.0133	-0.0532 to 0.0265	-0.0101	-0.022 1	to	0.001 78	-0.0315	-0.051 3 to -0.0118	-0.0139	-0.0408 to -0.131
time Proportion	0.0334	-0.0503 to 0.117	0.0253	0.000 186	to	0.050 4	0.0733	0.034 3 to 0.112	0.0323	-0.0302 to 0.0949
men	-2.799	-5.646 to 0.0491	-3.719	-4.827	to	-2.611	-3.101	-5.043 to -1.159	-2.499	-4,020 to -0.978
Parking cost Automobile	0.0959	0.0225 to 0.169	0.0968	0.069 4	to	0.124	0.471	0.004 71 to 0.0895	0.0332	-0.0124 to 0.078
availability R <sup>2</sup> R	-3.023 0.72 0.85	-6.359 to 0.312	-1.232 0.67 0.82	-2,394	to	-0.069 8	-1.502 0.78 0.88	-4.686 to 1.682	-1.844 0.64 0.80	-3.139 to -0.548
Number of origin- destination pairs	26		135				36		45	

Figure 4. Predicted automobile demand by origin for bimodal run.



ules and thus simulate the impact of the scenario without changing the modal shares of the demand. This may be appropriate for some situations where only marginal improvements are made and the only interest is to anticipate the changes in route choice that result due to the modifications. However, most current transportation planning methods have this capability. The other way to use EMME is to simulate the impact of each scenario with a full bimodal run, which would predict the anticipated changes in modal share of demand as well. This capability is so far unique to EMME.

The main conclusion that we draw from this project is that the use of sophisticated models, such as EMME, is feasible and the simulation of scenarios results in refined and fully detailed evaluations, which would not be possible otherwise. The main obstacles are the quality of the available data and the calibration of the demand model. Fortunately, we had access to very good data and we succeeded to calibrate a satisfactory modal-split model.

The costs of building up the necessary data base and calibrating the model are relatively high; however, the use of the model is not expensive. The figure of \$800 for each bimodal simulation is reasonable, when one considers that the analyst's time to set up a scenario and analyze the EMME output is one to two days.

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# Confidence Intervals for Choice Probabilities of the Multinomial Logit Model

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This paper describes three methods for developing confidence intervals for the choice probabilities in multinomial logit models. The confidence intervals reflect the effects of sampling errors in the parameters of the models. The first method is based on the asymptotic sampling distribution of the choice probabilities and leads to a joint confidence region for these probabilities. This confidence region is not rectangular and is useful mainly for testing hypotheses about the values of the choice probabilities. The second method is based on an asymptotic linear approximation of the relation between errors in models' parameters and errors in choice probabilities. The method yields confidence intervals for individual choice probabilities as well as rectangular joint confidence regions for all of the choice probabilities. However, the linear approximation on which the method is based can yield erroneous results, thus limiting the applicability of the method. A procedure for setting an upper bound on the error caused by the linear approximation is described. The third method is based on nonlinear programming. This method also leads to rectangular joint confidence regions for the choice probabilities. The nonlinear programming method is exact and, therefore, more generally applicable than the linear approximation method. However, when the linear approximation is accurate, it tends to produce narrower confidence intervals than does the nonlinear programming method, except in cases where the number of alternatives in the choice set is either two or very large. Several numerical examples are given in which the nonlinear programming method is illustrated and compared with the linear approximation method.

The multinomial logit formulation of urban travel-demand models has a variety of theoretical and computational advantages over other demand-model formulations and is receiving widespread use both for research purposes and as a practical demand-forecasting tool (1-3). However, travel-demand forecasts derived from logit models, like forecasts derived from other types of econometric models, are subject to errors that arise from several sources, including sampling errors in the estimated values of parameters of the models, errors in the values of explanatory variables, and errors in the functional specifications of the models. Knowledge of the magnitudes of forecasting errors can be important in practice, particularly if either the errors themselves or the costs of making erroneous decisions are large. This paper deals with the problem of estimating the magnitudes of forecasting errors that result from sampling errors in the estimated values of the parameters of logit models. Specifically, the paper describes techniques for developing confidence intervals for choice probabilities and functions of choice probabilities (e.g., aggregate market shares, changes in choice probabilities caused by changes in independent variables) derived from

logit models, conditional on correct functional specification of the models and use of correct values of the explanatory variables.

A model's forecasting error can be characterized in a variety of ways, including average forecasting error and root-mean-square forecasting error, in addition to confidence intervals for the forecast. Among the various error characterizations, only the confidence interval provides a range in which the true value of the forecast quantity is likely to lie. Methods for developing confidence intervals for the forecasts of linear econometric models are well known (4). However, these methods are not applicable to logit models, which are nonlinear in parameters. Koppelman (5,6) has analyzed the forecasting errors of logit models and has described the ways in which various sources of error contribute to total error in forecasts in choice probabilities. Koppelman's error measures do not include confidence intervals for the choice probabilities although, as will be shown later in this paper, one of his error measures can be used to derive approximate confidence intervals.

Three methods for estimating confidence intervals for the choice probabilities of logit models are described in this paper. All of the methods lead to asymptotic confidence intervals in that they are based on the large-sample properties of the estimated parameters of the models. The first method is based on the exact asymptotic sampling distribution of the choice probabilities and leads to a joint confidence region for these probabilities. This region is useful mainly for testing hypotheses about the values of the choice probabilities. The region is not rectangular and, therefore, is difficult to use in practical forecasting. Moreover, the methods used to derive the confidence region cannot be readily extended to functions of the choice probabilities.

The second method is based on an asymptotic linear approximation of the relation between sampling errors in models' parameters and sampling errors in choice probabilities. The linear approximation method yields confidence intervals for individual choice probabilities as well as rectangular joint confidence regions for all of the choice probabilities. The method can easily be extended to functions of the choice probabilities. However, the linear approximation on which the method is based can yield erroneous results, thus limiting the method's applicability. A procedure for placing an upper bound