changes in bus-travel time as they are to changes in access time.

Effects of Income

Increases in household income have the predicted effect on the log-odds of driving. High-income households favor the drive mode relative to low-income households, when one automobile is available per worker. As regards higher-order effects, when income is interacted with the level-of-service variables, the major distinction that emerges is between the bottom- and top-income categories: The middle-income category usually proves not to add significantly to goodness of fit.

Automobile-Passenger Mode and IIA Assumption

The theory of the conditional logit model as well as the model employed here assumes that the introduction of a third mode (for example, the shared-ride option for a commuter) will not affect the parameters of the model. If the attributes of such a mode are designated $X_3$, then according to theory the substitution of $X_3$ for $X_2$ in Equations 1 and 1a should leave the $\beta$-or $w$-terms unaffected.

The best way to test for the validity of the IIA assumption here would be to perform a test analogous to a Chow test. This would involve the entire set of 5080 observations, collapsed across all alternatives. A dummy variable would be introduced (call it variable 6) that would take a value of zero if the drive-bus option is described by a particular observation and one if the drive-passenger option is relevant. At issue in the test would be whether this new variable has any two-way interaction with variable 1 and one of the remaining variables. If such interactions emerge as significant then we would reject the hypothesis of IIA, that is, that the $w$ coefficients are the same for the two pairs of alternatives.

An alternative to such a test, which is vastly cruder, involves simply comparing the parameters of the drive or bus alternative with those of the drive or shared-ride alternative. We performed such a test. Because the $G$ statistic for the latter data set was excessively high when the model of Table 2 was used, we employed for both data sets the least complicated higher-order model (a) that passed the standard significance test and (b) of which the model of Table 2 is a proper subset. This was the model: 2345 1235 1245 1345. The probability statistic exceeded 0.4 for each of the two data sets to which this model was fitted.

Of concern was the issue of how the difference between the coefficients for the two data sets compared with the sum of the standard deviations. As the ratio of the coefficient differences to the sum of the standard deviations exceeded 2.0 for only 9 percent of the 148 parameters that were estimated, one might be tempted, by using such a test, to reject the hypothesis that the coefficients for the two alternatives are different—that IIA is not a valid assumption.

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Small-Area Trip-Distribution Model

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A model for predicting trip tables for small areas based on the access and land development travel function is described along with the results of an initial test of the model. The model provides trip tables required for regional analyses without the need for windowing into a regional data set. The model requires minimum-path friction skim trees and trip-end data for the small area as input. Trip-end data can be derived from ground

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counts on links that enter the small area or from the results of an assignment of a regional trip table. Test results from a small area in Hudson County, New Jersey, suggest the validity of the model. The need for further refinements to the model is discussed in the paper.

In recent years, emphasis has been placed on short-range, small-scale, non-capital-intensive solutions to transportation problems and needs. This emphasis has made development of new tools for quick, inexpensive analysis of subregional alternative actions necessary. Regional models are not always cost effective nor sufficiently detailed for analyzing small-scale or subregional alternatives. Some examples of new tools are windowing and network-aggregation programs and small-area and microassignment programs (1-3).

One need for many subregional or small-area analyses is a trip table. This paper describes a model for developing a small-area trip table. The model provides a powerful tool for the analysis of subregional plans since it does not require data from outside the area of interest.

The small-area trip-distribution model was developed to provide an initial starting point for a process to estimate a trip table based on observed link volumes. The model can be used independently.

PURPOSE OF THE MODEL

The small-area trip-distribution model (SMALD) is based on the access and land development (ALD) travel function (4). It produces a small-area trip table based on link volumes at boundary points of entry and exit (boundary load nodes), productions and attractions at points internal to the small area (internal load nodes), and minimum-path friction skim trees within the area of interest.

A number of definitions are necessary for a clear understanding of the model:

1. Small area—an area where a major portion of the trips have one or both trip ends outside of the area under consideration;
2. Boundary load node—a point where a link crosses the cordon line that defines the area (also referred to as a point of entry or exit);
3. Internal load node—a point internal to the small area where trips originate or terminate; represents an analysis zone;
4. Skim tree—a matrix that gives the generalized cost (friction) of the shortest paths between all load nodes (customarily, friction is a linear combination of travel time and travel cost); and
5. Domain—the part of the region served by a point on the network (usually a load node).

It may be possible to use a standard trip-distribution model to produce a small-area trip table, but the process is conceptually inferior to SMALD. Regional trip-distribution models work on the premise that a major portion of the trips have both trip ends within the area of interest. However, by definition, this assumption is violated in small areas. In addition, standard trip distribution does not consider a source of valuable information on the characteristics of trips that enter at boundary load nodes; that is, the type of service provided by the link as characterized by its functional class at the boundary load node. A trip that enters the small area on an expressway will have different characteristics from a trip that enters the area on an arterial or local street or a trip that is internally generated. This information is essential to SMALD. SMALD explicitly considers that the small area is surrounded by more region that attracts trips (see Figure 1).

It is possible to extract a small-area trip table from a regional trip table by using the Urban Transportation Planning System (UTPS) program NAG (5). NAG performs an all-or-nothing assignment of a regional trip table on the regional network, traces the trips, and records them as they pass through the area of interest. If the regional trip table is unavailable or unreliable, NAG cannot be used for deriving a small-area trip table.

Thus, SMALD has been developed to fill a void. It finds reasonable trip tables for small areas based on data from only the small area. However, it accounts for and uses the fact that the area is surrounded by more region that attracts trips.

SMALD THEORY

SMALD is based on the ALD travel function and a gravity-type distribution process:

\[ V_{ij} = F_i R_j / \beta \]

where

\[ V_{ij} \] = the interchange between point i and point j,
\[ F_i \] = the productions at point i,
\[ R_j \] = the attractions at point j,
\[ \beta \] = the decay function, and
\[ \sum_{i} F_i \] = the sum of \( F_i \) and \( R_j \).

Decay functions are based on the single-mode ALD travel function:

\[ F_i = K_2 (2 \sqrt{x_i}) / A_i \]

where

\[ A \] = a system constant,
\[ t \] = any measure of separation (e.g., distance, time, or friction), and
\[ K_2 \] = the modified Bessel function of the second type and second order for the argument (2/\( x_i \)).

In SMALD the travel function is different for each type of interchange based on the domains at different types of facilities. The various travel functions are derived by integrating the basic travel function (Equation 2) over the respective domains of the facilities at the points of entry and exit. The following assumptions are made about the domains of points of entry and exit on different facility types:

1. For expressway boundary load nodes, domains expand two-dimensionally into the region external to the area of interest;
2. For arterial boundary load nodes, domains expand in only one dimension into the region external to the area of interest; and
3. For local street boundary load nodes, domains are bounded and small enough to be treated as ordinary point zones (internal load nodes).

Figure 2 shows domains for the different types of load nodes. It is also assumed that the region external to the area of interest is uniform in its accessibility and trip-end density.

The form of the effective travel function is given in the table below. Note that \( K_1 \) (where \( 1 = 0, 1, \) or 2) is the modified Bessel function of the second type, \( i \)th
Derivation of the Decay Functions

In the derivation that follows, the region is assumed to be uniform—that is, it has about the same trip-end density and accessibility everywhere. Domains shown in Figure 2 assume that local streets always compete with arterial roads, arterial roads always compete with other arterial roads (but not with expressways), and expressways compete only with an arterial infrastructure of some sort. Domains have not been, and cannot be, drawn carefully. Their main and only generally stable feature is their dimensionality (i.e., point-like, linear, or geometric).

The following notations are used in the derivation that follows:

- $Q$ = a quantity proportional to trip interchange volume (i.e., the trip decay function);
- $P, p$ = productions, production density;
- $R, r$ = attractions, attraction density;
- $F = \text{the ALD travel function, } K_a \frac{(2 \sqrt{A} t)}{At};$
- $dS$ = an element of area (see Figure 2);
- $C_a$ = a factor related to arterial domain width and travel friction such that $C_a dt = dS$;
- $\bar{P}$ = a quantity similar to, but not necessarily identical with, entry volume at a point of entry ($P = C_p \frac{p}{A}$ for arterial roads and $P = C_p \frac{p}{A^2}$ for expressways);
- $\bar{R}$ = a quantity similar to, but not necessarily identical with, exit volume at a point of exit ($R = C_r \frac{r}{A}$ for arterial roads and $R = C_r \frac{r}{A^2}$ for expressways);
- $t = \text{travel friction internal to small area } \{\text{with a subscript it signifies friction on external segments } (t_i \text{ for external before entrance to area, } t_e \text{ for external after exit from area})\};$ and
- $A = \text{sensitivity to friction.}$

For internal-internal trips, no extended domains are involved and the function follows the base ALD travel function (see UTPS manual 5 for extended derivation):

$$Q = PFR$$

$$= PRK_a \left(2 \sqrt{A} t \right) / At$$

(3)

For internal-arterial trips, the interchange between an internal zone and an element of area within an arterial domain is measured by

$$dQ = PF(rdS)$$

(4)

so that the interchange to the entire domain is

$$Q = Pr \int_{-\infty}^{\infty} FdS$$

$$= PC_a \int_{-\infty}^{\infty} (t + t_i) dt_i$$

$$= PC_a t K_a (2 \sqrt{A} t) / A \sqrt{A} t$$

$$= \bar{P} \bar{R} K_a (2 \sqrt{A} t) / \sqrt{A} t$$

(5)

For internal-expressway trips, the element of area is proportional to $t_i dt_i$, rather than just $dt_i$, so that
For arterial-internal trips, production density and the elemental size of the production domain measure the interchange:

\[ \frac{dQ}{dt} = pdSFR \]
so that

\[ Q = R_C p \int_0^t F(t + t_i) dt_i \]

\[ = R_C p K_1 (2\sqrt{At}) \]

\[ = P R K_1 (2\sqrt{At}) \]

For arterial-internal trips, production density and the elemental size of the production domain measure the interchange:

\[ \frac{dQ}{dt} = pdSFR \]
so that

\[ Q = R_C p \int_0^t F(t + t_i) dt_i \]

\[ = R_C p K_1 (2\sqrt{At}) \]

\[ = P R K_1 (2\sqrt{At}) \]

For arterial-arterial trips, both production and attraction domains measure the interchange:

\[ \frac{dQ}{dt} = (pdS)F(r dS) \]
so that

\[ Q = C_p R K_0 (2VAf) \]

\[ = P R K_0 (2VAf) \]

For arterial-expressway trips, the attraction domain element of area is, again, proportional to \( t_i dt_i \), so that

\[ Q = C_p R \int_0^t t_i dt_i \int_0^t F(t + t_i + t_j) dt_j \]

\[ = P R K_0 (2\sqrt{At}) \]

For expressway-expressway trips, both the production and attraction domains are proportional to \( t_i dt_i \), so that

\[ Q = C_p R \int_0^t F(t + t_i + t_j) dt_j \]

\[ = P R K_0 (2\sqrt{At}) \]

For expressway-arterial trips

\[ Q = C_p R \int_0^t F(t + t_i + t_j) dt_j \]

\[ = P R K_0 (2\sqrt{At}) \]

Note, again, that local street domains are assumed to be bounded and small enough to be treated as ordinary point zones (internals); wherever the word internal appears, local street boundary load node can be substituted without much damage to the mathematics.

Limitations to the Theory

Perhaps the most severe simplification in the functions listed in the preceding table is that nothing is said about competition among domains, which occurs, for example, when

1. An expressway or arterial emerges from the internal area at a different angle than it enters, so that its entry and exit domains overlap;
2. Two parallel expressways through the area or in its neighborhood cut the domain of each;
3. More than two expressways are involved, so that at least one of the domains tends to grow only linearly with distance from its boundary load node rather than geometrically;
4. An expressway domain pinches off an arterial domain only a short distance from the boundary load node; or
5. Any peculiarity of network geometry or performance causes a facility's domain to have an anomalous shape.

Problems typified by examples 1 and 2 above can be dealt with in an approximate manner by factoring the attractiveness between two boundary load nodes based on the type and severity of the anomaly. If there is an anomaly, the attractiveness between two load nodes can be expected to decrease from what would be expected in a perfect world. Attractiveness factors can be applied in a logically consistent manner. In effect, they modify the size of the domains of the facilities in question.

Problems typified by examples 3 and 4 above may be dealt with on an individual basis. In small areas, links that cut the boundary must be identified by their actual function at that point. If there are two expressways in an area, one may easily serve as an arterial for most trips around that area even though in the region it serves as an expressway. SMALD will perform satisfactorily without taking into account the actual traffic-carrying function of links at boundary load nodes, but results will be improved if this information is known.

The model's performance deteriorates as the size of the area of interest decreases, since the problem becomes increasingly dominated by the specific structure of the network external to the area. This is not considered explicitly in the model.

The present formulation of the model is unimodal (automobile trips only) and does not consider explicitly competition with walking trips. As a result, the model's performance is questionable when zones represented by internal load nodes are extremely small. In practice, this should limit zone sizes to a minimum of about 0.65 km² (0.25 mile²).
Table 1. Equilibrium assignment summary results.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Observed</th>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
<th>Run 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume (vehicle-h)</td>
<td>15,350.61</td>
<td>20,866.05</td>
<td>18,959.47</td>
<td>17,659.33</td>
<td>17,325.30</td>
</tr>
<tr>
<td>of travel</td>
<td>23,086.67</td>
<td>23,086.67</td>
<td>23,086.67</td>
<td>23,086.67</td>
<td>23,086.67</td>
</tr>
<tr>
<td>Net error (h)</td>
<td>5,515.39</td>
<td>3,608.07</td>
<td>3,608.07</td>
<td>3,608.07</td>
<td>3,608.07</td>
</tr>
<tr>
<td>Percentage error</td>
<td>35.9</td>
<td>23.5</td>
<td>15.0</td>
<td>12.9</td>
<td>12.9</td>
</tr>
<tr>
<td>Absolute error (h)</td>
<td>6,323.76</td>
<td>4,711.11</td>
<td>3,661.09</td>
<td>3,409.27</td>
<td>3,409.27</td>
</tr>
<tr>
<td>Percentage error</td>
<td>41.5</td>
<td>30.7</td>
<td>23.3</td>
<td>22.2</td>
<td>22.2</td>
</tr>
<tr>
<td>Sensitivity to friction*</td>
<td>0.003</td>
<td>0.007</td>
<td>0.011</td>
<td>0.007</td>
<td>0.007</td>
</tr>
<tr>
<td>Average trip length (min)</td>
<td>2.05</td>
<td>2.73</td>
<td>2.49</td>
<td>2.32</td>
<td>2.28</td>
</tr>
</tbody>
</table>

*All nonexpressway boundary load nodes considered as local streets.

TEST APPLICATION OF THE MODEL

Data for a 15.5-km² (6-mile²) test area in Hudson County, New Jersey (see Figure 4), were extracted from a regional data set that describes the New York metropolitan area. The area was rather unique since it is parallel to the Hudson River and includes the approaches to both the Lincoln and Holland Tunnels. The data include 24 internal load nodes and 34 boundary load nodes connected by 369 unidirectional links. Two-way ground counts were available for all actual network links in the small area, including all boundary crossings. Production and attractions for internal load nodes were obtained by using a program similar to the UTPS program NAG. Specifically, the regional trip table was assigned by an all-or-nothing process, and trips were traced and recorded as they crossed the area's boundary. It was discovered, however, that the trip table obtained by this process was subject to the pathological quirks of all-or-nothing assignment and, as a result, was not very good.

Since no trip table was available as a standard for comparison, abstract criteria were used to test the quality of the model and calibration. The main criteria used were average trip length in the small area and total absolute link-volume error (observed versus assigned volumes) after five iterations of an equilibrium assignment of the trip table. The average trip length was determined to be 2.05 min from the total vehicle hours of travel on the links (15,350.61 vehicle-h) and the total trips (448,864 trips) in the small area. Note that the trip length was based on friction (a linear combination of time and distance expressed in minutes).

Results

Four runs of SMALD were made. All runs used the same friction skim tree. In the first three runs, the value of the system constant (parameter A in Equation 2) was varied. System constants were chosen to be equal to, two times, and three times the A value used in the trip-distribution model (ALDGRAV) for the region. In the fourth run, the functional class of all arterial boundary load nodes was assumed to be local to more accurately describe their operation due to the unique location of the data set. The A value for the fourth run was twice the regional A value.

Table 1 summarizes the results of the equilibrium assignment of the trip tables from the four tests. Net error is the sum of differences between observed and assigned volumes on all links in the network. Absolute error is the sum of absolute differences in observed and assigned volumes on all links in the network.

Based on the summaries shown in Table 1, the model tends to overpredict average trip lengths even at very high A values (theoretically, the sensitivity to friction used in SMALD should be the same as that used in the regional distribution model).

The size of absolute volume errors is encouraging. Although the errors seem high, they are about one-half of the errors that resulted when the trip table extracted from the regional trip table was assigned. The absolute volume error from the extracted trip table was 94.9 percent of the observed volume.

Although they are not shown here, the resulting trip tables appear intuitively reasonable. Specific interchanges are reasonable in their relative magnitudes. The number of right-angle and U-turn movements through the area appears to be reasonable—about 3 percent of the boundary-to-boundary trips are U-turn movements, and about 30 percent are right-angle movements.

CONCLUSIONS

The results of SMALD are encouraging. SMALD is capable of building a reasonable trip table for a small area, based on data from only that area. There are several areas for possible enhancements to the model, the most promising being methods for specifying routes through the small area. Additional improvements to and testing of the model are currently in progress. The model, in its present form, is applicable in planning studies.

ACKNOWLEDGMENT

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Disaggregate Travel Models: How Strong Are the Foundations?

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This paper presents a review and analysis of disaggregate travel-demand modeling founded on an examination of the published literature. This analysis is directed to the conceptual foundations of the modeling process, which appear to be somewhat obscurely covered by the literature. The analysis is at two levels: (a) a review of where the modeling structure fits into the overall travel process and (b) an analysis of the foundations of the specific models and how they relate to the target processes.

The literature provides much confusion in definitions and terminology at the conceptual level. Many of the foundations of the modeling approach are subjectively derived without testing of the underlying assumptions. Some of the most important concepts are left to the references, which also remain obscure.

A clear response to these questions, including clarification of the concepts, would broaden the understanding and acceptance of DDMs. The conceptual base of the current econometric thrust is so narrow, however, as to preclude the confirmation of the strong empirical results claimed. The usefulness of the methodology is thus restricted to fairly limited applications.

BACKGROUND

DDMs are widely reported in the literature. A series of conference proceedings provide the most exhaustive reviews (1-3). Specific modeling developments are provided by Ben Akiva (4), Charles River Associates (5), Domencich and McFadden (6), and Manski (7).

DDMs were originally developed to gain greater insight into travel behavior, particularly at the individual level. This fundamental understanding was found lacking in the aggregate forecasting models generally used in the Urban Transportation Planning (UTP) process. Critiques of traditional aggregate models are abundant (1), pp. 13-19.

The initial modeling work was of an empirical nature, developing logit models of mode choice. Later theoretical work of Charles River Associates (5), McFadden (8), and Domencich and McFadden (6) provided a behavioral interpretation and foundation for the preceding empirical work. Despite this formulation, however, the nature of the DDM methodology remains overwhelmingly empirical. Conceptual difficulties and behavioral inconsistencies have arisen from time to time, and the underlying theory has often been adjusted in an ad hoc manner to account for discrepancies (7).

Empirical and technical work has dominated; less attention has been given to theoretical understanding, and, unfortunately, this has led to what seems to be lack of concern for the modeling foundations.

DDMs have been looked on as accurate, inexpensive replacements for traditional forecasting models; they are capable of dealing with policy questions that the earlier methodology could not handle. Yet, with the few exceptions, DDMs have not become a standard tool for analysis in practical settings. This is despite their virtues over the UTP models (4, 6).

From time to time questions of a conceptual and theoretical nature have been raised about DDMs. These